var svar — Structural vector autoregressive models

Description

svar fits a vector autoregressive model subject to short- or long-run constraints you place on the resulting impulse–response functions (IRFs). Economic theory typically motivates the constraints, allowing a causal interpretation of the IRFs to be made. See [TS] var intro for a list of commands that are used in conjunction with svar.

Quick start

Structural VAR for \( y_1, y_2, \) and \( y_3 \) using \texttt{tsset} data with short-run constraints on impulse responses given by predefined matrices \( A \) and \( B \)
\[
\texttt{svar y1 y2 y3, aeq(A) beq(B)}
\]
Structural VAR for \( y_1, y_2, \) and \( y_3 \) with long-run constraint on impulse responses given by the predefined matrix \( C \)
\[
\texttt{svar y1 y2 y3, lreq(C)}
\]
Add exogenous variables \( x_1 \) and \( x_2 \)
\[
\texttt{svar y1 y2 y3, lreq(C) exog(x1 x2)}
\]
Same as above, but include third and fourth lags of the dependent variables instead of first and second
\[
\texttt{svar y1 y2 y3, lreq(C) exog(x1 x2) lags(3 4)}
\]

Menu

Statistics > Multivariate time series > Structural vector autoregression (SVAR)

Syntax

Short-run constraints
\[
\texttt{svar depvarlist [if] [in], \{acns(matrix_{acns}) aeq(matrix_{aeq}) \}
\]
\[
\texttt{\{aconstrains(constraints_a) aeq(matrix_{aeq}) \}}
\]
\[
\texttt{[\ short\_run\_options]}\]

Long-run constraints
\[
\texttt{svar depvarlist [if] [in], \{lrcns(matrix_{lrcns}) \}}
\]
\[
\texttt{\lreq(matrix_{lreq}) \}} [long\_run\_options] \]

1
**short-run_options**

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>noconstant</code></td>
<td>suppress constant term</td>
</tr>
<tr>
<td><code>aconstraints(constraints_a)</code></td>
<td>apply previously defined <code>constraints_a</code> to A</td>
</tr>
<tr>
<td><code>aeq(matrix_aeq)</code></td>
<td>define and apply to A equality constraint matrix <code>matrix_aeq</code></td>
</tr>
<tr>
<td><code>acns(matrix_acns)</code></td>
<td>define and apply to A cross-parameter constraint matrix <code>matrix_acns</code></td>
</tr>
<tr>
<td><code>bconstraints(constraints_b)</code></td>
<td>apply previously defined <code>constraints_b</code> to B</td>
</tr>
<tr>
<td><code>beq(matrix_beq)</code></td>
<td>define and apply to B equality constraint matrix <code>matrix_beq</code></td>
</tr>
<tr>
<td><code>bcns(matrix_bcns)</code></td>
<td>define and apply to B cross-parameter constraint matrix <code>matrix_bcns</code></td>
</tr>
<tr>
<td><code>lags(numlist)</code></td>
<td>use lags <code>numlist</code> in the underlying VAR</td>
</tr>
</tbody>
</table>

**Model 2**

| exog(varlist_exog) | use exogenous variables `varlist` |
| varconstraints(constraints_v) | apply `constraints_v` to underlying VAR |
| noiselog | suppress SURE iteration log |
| `isiterate(#)` | set maximum number of iterations for SURE; default is `isiterate(1600)` |
| `istolerance(#)` | set convergence tolerance of SURE |
| noisure | use one-step SURE |
| dfk | make small-sample degrees-of-freedom adjustment |
| small | report small-sample t and F statistics |
| noidencheck | do not check for local identification |
| nobigf | do not compute parameter vector for coefficients implicitly set to zero |

**Reporting**

| level(#) | set confidence level; default is `level(95)` |
| full | show constrained parameters in table |
| var | display underlying `var` output |
| lutstats | report Lütkepohl lag-order selection statistics |
| noconsreport | do not display constraints |
| `display_options` | control columns and column formats |

**Maximization**

| maximize_options | control the maximization process; seldom used |
| coeflegend | display legend instead of statistics |

* `aconstraints(constraints_a), aeq(matrix_aeq), acns(matrix_acns), bconstraints(constraints_b), beq(matrix_beq), bcns(matrix_bcns)`: at least one of these options must be specified.

dcoeflegend does not appear in the dialog box.
**long-run_options**

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>noconstant</td>
<td>suppress constant term</td>
</tr>
<tr>
<td><em>lrconstraints(constraints&lt;sub&gt;lr&lt;/sub&gt;)</em></td>
<td>apply previously defined constraints&lt;sub&gt;lr&lt;/sub&gt; to C</td>
</tr>
<tr>
<td><em>lreq(matrix&lt;sub&gt;lreq&lt;/sub&gt;)</em></td>
<td>define and apply to C equality constraint matrix matrix&lt;sub&gt;lreq&lt;/sub&gt;</td>
</tr>
<tr>
<td><em>lrcns(matrix&lt;sub&gt;lrcns&lt;/sub&gt;)</em></td>
<td>define and apply to C cross-parameter constraint matrix matrix&lt;sub&gt;lrcns&lt;/sub&gt;</td>
</tr>
<tr>
<td>lags(numlist)</td>
<td>use lags numlist in the underlying VAR</td>
</tr>
</tbody>
</table>

**Model 2**

| exog(varlist<sub>exog</sub>) | use exogenous variables varlist |
| varconstraints(constraints<sub>v</sub>) | apply constraints<sub>v</sub> to underlying VAR |
| noislog | suppress SURE iteration log |
| isiterate(#) | set maximum number of iterations for SURE; default is isiterate(1600) |
| istolerance(#) | set convergence tolerance of SURE |
| noisure | use one-step SURE |
| dfk | make small-sample degrees-of-freedom adjustment |
| small | report small-sample t and F statistics |
| noidencheck | do not check for local identification |
| nobigf | do not compute parameter vector for coefficients implicitly set to zero |

**Reporting**

| level(#) | set confidence level; default is level(95) |
| full | show constrained parameters in table |
| var | display underlying var output |
| lutstats | report Lütkepohl lag-order selection statistics |
| nocnsreport | do not display constraints |
| display_options | control columns and column formats |

**Maximization**

| maximize_options | control the maximization process; seldom used |
| coeflegend | display legend instead of statistics |

* lrconstraints(constraints<sub>lr</sub>), lreq(matrix<sub>lreq</sub>), lrcns(matrix<sub>lrcns</sub>): at least one of these options must be specified. coeflegend does not appear in the dialog box.

You must *tsset* your data before using *svar*; see [TS] tsset.

depvarlist and varlist<sub>exog</sub> may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, collect, fp, rolling, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
Options

noconstant; see [R] Estimation options.

aconstraints(constraints_a), aeq(matrix_aeq), acns(matrix_acns)
bconstraints(constraints_b), beq(matrix_beq), bcns(matrix(bcns))

These options specify the short-run constraints in an SVAR. To specify a short-run SVAR model, you must specify at least one of these options. The first list of options specifies constraints on the parameters of the $A$ matrix; the second list specifies constraints on the parameters of the $B$ matrix (see Short-run SVAR models). If at least one option is selected from the first list and none are selected from the second list, svar sets $B$ to the identity matrix. Similarly, if at least one option is selected from the second list and none are selected from the first list, svar sets $A$ to the identity matrix.

None of these options may be specified with any of the options that define long-run constraints.

aconstraints(constraints_a) specifies a numlist of previously defined Stata constraints to be applied to $A$ during estimation.

aeq(matrix_aeq) specifies a matrix that defines a set of equality constraints. This matrix must be square with dimension equal to the number of equations in the underlying VAR. The elements of this matrix must be missing or real numbers. A missing value in the $(i, j)$ element of this matrix specifies that the $(i, j)$ element of $A$ is a free parameter. A real number in the $(i, j)$ element of this matrix constrains the $(i, j)$ element of $A$ to this real number. For example, 

$$A = \begin{bmatrix} 1 & 0 \\ 1.5 & \end{bmatrix}$$


acns(matrix_acns) specifies a matrix that defines a set of exclusion or cross-parameter equality constraints on $A$. This matrix must be square with dimension equal to the number of equations in the underlying VAR. Each element of this matrix must be missing, 0, or a positive integer. A missing value in the $(i, j)$ element of this matrix specifies that no constraint be placed on this element of $A$. A zero in the $(i, j)$ element of this matrix constrains the $(i, j)$ element of $A$ to be zero. Any strictly positive integers must be in two or more elements of this matrix. A strictly positive integer in the $(i, j)$ element of this matrix constrains the $(i, j)$ element of $A$ to be equal to all the other elements of $A$ that correspond to elements in this matrix that contain the same integer. For example, consider the matrix 

$$A = \begin{bmatrix} \cdot & 1 \\ 1 & \cdot \end{bmatrix}$$


bconstraints(constraints_b) specifies a numlist of previously defined Stata constraints to be applied to $B$ during estimation.

beq(matrix_beq) specifies a matrix that defines a set of equality constraints. This matrix must be square with dimension equal to the number of equations in the underlying VAR. The elements of this matrix must be either missing or real numbers. The syntax of implied constraints is analogous to the one described in aeq(), except that it applies to $B$ rather than to $A$. 

**bcns**(matrix$bcns$) specifies a matrix that defines a set of exclusion or cross-parameter equality constraints on $B$. This matrix must be square with dimension equal to the number of equations in the underlying VAR. Each element of this matrix must be *missing*, 0, or a positive integer. The format of the implied constraints is the same as the one described in the acns() option above.

**lrconstraints**(constraints$_lr$), **lreq**(matrix$lreq$), **lrcns**(matrix$lrcns$)

These options specify the long-run constraints in an SVAR. To specify a long-run SVAR model, you must specify at least one of these options. The list of options specifies constraints on the parameters of the long-run $C$ matrix (see Long-run SVAR models for the definition of $C$). None of these options may be specified with any of the options that define short-run constraints.

**lrconstraints**(constraints$_lr$) specifies a numlist of previously defined Stata constraints to be applied to $C$ during estimation.

**lreq**(matrix$lreq$) specifies a matrix that defines a set of equality constraints on the elements of $C$. This matrix must be square with dimension equal to the number of equations in the underlying VAR. The elements of this matrix must be either *missing* or real numbers. The syntax of implied constraints is analogous to the one described in option aeq(), except that it applies to $C$.

**lrcns**(matrix$lrcns$) specifies a matrix that defines a set of exclusion or cross-parameter equality constraints on $C$. This matrix must be square with dimension equal to the number of equations in the underlying VAR. Each element of this matrix must be *missing*, 0, or a positive integer. The syntax of the implied constraints is the same as the one described for the acns() option above.

**lags**(numlist) specifies the lags to be included in the underlying VAR model. The default is lags(1 2). This option takes a numlist and not simply an integer for the maximum lag. For instance, lags(2) would include only the second lag in the model, whereas lags(1/2) would include both the first and second lags in the model. See [U] 11.1.8 numlist and [U] 11.4.4 Time-series varlists for further discussion of numlists and lags.

**varconstraints**(constraints$_v$) specifies a list of constraints to be applied to the coefficients in the underlying VAR. Because svar estimates multiple equations, the constraints must specify the equation name for all but the first equation.

noislog prevents svar from displaying the iteration log from the iterated seemingly unrelated regression algorithm. When the varconstraints() option is not specified, the VAR coefficients are estimated via OLS, a noniterative procedure. As a result, noislog may be specified only with varconstraints(). Similarly, noislog may not be combined with noisure.

isiterate(#) sets the maximum number of iterations for the iterated seemingly unrelated regression algorithm. The default limit is 1,600. When the varconstraints() option is not specified, the VAR coefficients are estimated via OLS, a noniterative procedure. As a result, isiterate() may be specified only with varconstraints(). Similarly, isiterate() may not be combined with noisure.

istolerance(#) specifies the convergence tolerance of the iterated seemingly unrelated regression algorithm. The default tolerance is 1e-6. When the varconstraints() option is not specified, the VAR coefficients are estimated via OLS, a noniterative procedure. As a result, istolerance() may be specified only with varconstraints(). Similarly, istolerance() may not be combined with noisure.
noisure specifies that the VAR coefficients be estimated via one-step seemingly unrelated regression when \texttt{varconstraints()} is specified. By default, \texttt{svar} estimates the coefficients in the VAR via iterated seemingly unrelated regression when \texttt{varconstraints()} is specified. When the \texttt{varconstraints()} option is not specified, the VAR coefficient estimates are obtained via OLS, a noniterative procedure. As a result, \texttt{noisure} may be specified only with \texttt{varconstraints()}.

dfk specifies that a small-sample degrees-of-freedom adjustment be used when estimating $\Sigma$, the covariance matrix of the VAR disturbances. Specifically, $1/(T - \bar{m})$ is used instead of the large-sample divisor $1/T$, where $\bar{m}$ is the average number of parameters in the functional form for $y_t$ over the $K$ equations.

small causes \texttt{svar} to calculate and report small-sample $t$ and $F$ statistics instead of the large-sample normal and $\chi^2$ statistics.

noidencheck requests that the Amisano and Giannini (1997) check for local identification not be performed. This check is local to the starting values used. Because of this dependence on the starting values, you may wish to suppress this check by specifying the \texttt{noidencheck} option. However, be careful in specifying this option. Models that are not structurally identified can still converge, thereby producing meaningless results that only appear to have meaning.

nobigf requests that \texttt{svar} not compute the estimated parameter vector that incorporates coefficients that have been implicitly constrained to be zero, such as when some lags have been omitted from a model. \texttt{e(bf)} is used for computing asymptotic standard errors in the postestimation commands \texttt{irf create} and \texttt{fcast compute}. Therefore, specifying \texttt{nobigf} implies that the asymptotic standard errors will not be available from \texttt{irf create} and \texttt{fcast compute}. See \textit{Fitting models with some lags excluded} in [TS] \texttt{var}.

\begin{itemize}
\item [\texttt{level(#)}; see [R] Estimation options.] 
full shows constrained parameters in table.
\item \texttt{var} specifies that the output from \texttt{var} also be displayed. By default, the underlying VAR is fit quietly.
\item \texttt{lutstats} specifies that the Lütkepohl versions of the lag-order selection statistics be computed. See \textit{Methods and formulas} in [TS] \texttt{varsoc} for a discussion of these statistics.
\item \texttt{nocnsreport}; see [R] Estimation options.
\item \texttt{display_options}: \texttt{noci, nopvalues, cformat(\%fmt), pformat(\%fmt), and sformat(\%fmt)}; see [R] Estimation options.
\end{itemize}

\begin{itemize}
\item [\texttt{maximize_options}: difficult, technique(algorithm\_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init\_specs); see [R] Maximize.] These options are seldom used.
\end{itemize}

The following option is available with \texttt{svar} but is not shown in the dialog box:
\texttt{coeflegend}; see [R] Estimation options.
Remarks and examples

Remarks are presented under the following headings:

Introduction
Short-run SVAR models
Long-run SVAR models

Introduction

This entry assumes that you have already read [TS] var intro and [TS] var; if not, please do. Here we illustrate how to fit SVARs in Stata subject to short-run and long-run restrictions. For more detailed information on SVARs, see Amisano and Giannini (1997) and Hamilton (1994). For good introductions to VARs, see Lütkepohl (2005), Hamilton (1994), Stock and Watson (2001), and Becketti (2020).

Short-run SVAR models

A short-run SVAR model without exogenous variables can be written as

\[ A(I_K - A_1 L - A_2 L^2 - \cdots - A_p L^p) y_t = A \epsilon_t = B e_t \]

where \( L \) is the lag operator, \( A, B, \) and \( A_1, \ldots, A_p \) are \( K \times K \) matrices of parameters, \( \epsilon_t \) is a \( K \times 1 \) vector of innovations with \( \epsilon_t \sim N(0, \Sigma) \) and \( E[\epsilon_t \epsilon_s'] = 0_K \) for all \( s \neq t \), and \( e_t \) is a \( K \times 1 \) vector of orthogonalized disturbances; that is, \( e_t \sim N(0, I_K) \) and \( E[e_t e_s'] = 0_K \) for all \( s \neq t \). These transformations of the innovations allow us to analyze the dynamics of the system in terms of a change to an element of \( e_t \). In a short-run SVAR model, we obtain identification by placing restrictions on \( A \) and \( B \), which are assumed to be nonsingular.

Example 1: Short-run just-identified SVAR model

Following Sims (1980), the Cholesky decomposition is one method of identifying the impulse–response functions in a VAR; thus, this method corresponds to an SVAR. There are several sets of constraints on \( A \) and \( B \) that are easily manipulated back to the Cholesky decomposition, and the following example illustrates this point.

One way to impose the Cholesky restrictions is to assume an SVAR model of the form

\[ \tilde{A}(I_K - A_1 - A_2 L^2 - \cdots - A_p L^p) y_t = \tilde{B} e_t \]

where \( \tilde{A} \) is a lower triangular matrix with ones on the diagonal and \( \tilde{B} \) is a diagonal matrix. Because the \( P \) matrix for this model is \( P_{sr} = \tilde{A}^{-1} \tilde{B} \), its estimate, \( \hat{P}_{sr} \), obtained by plugging in estimates of \( \tilde{A} \) and \( \tilde{B} \), should equal the Cholesky decomposition of \( \hat{\Sigma} \).

To illustrate, we use the German macroeconomic data discussed in Lütkepohl (2005) and used in [TS] var. In this example, \( y_t = (d\ln \text{inv}, d\ln \text{inc}, d\ln \text{consump}) \), where \( d\ln \text{inv} \) is the first difference of the log of investment, \( d\ln \text{inc} \) is the first difference of the log of income, and \( d\ln \text{consump} \) is the first difference of the log of consumption. Because the first difference of the natural log of a variable can be treated as an approximation of the percentage change in that variable, we will refer to these variables as percentage changes in \( \text{inv}, \text{inc}, \) and \( \text{consump} \), respectively.

We will impose the Cholesky restrictions on this system by applying equality constraints with the constraint matrices

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
.1 & 1 & 0 \\
.1 & .1 & 1
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
. & 0 & 0 \\
0 & . & 0 \\
0 & 0 & .
\end{bmatrix}
\]
With these structural restrictions, we assume that the percentage change in \( \text{inv} \) is not contemporaneously affected by the percentage changes in either \( \text{inc} \) or \( \text{consump} \). We also assume that the percentage change of \( \text{inc} \) is affected by contemporaneous changes in \( \text{inv} \) but not \( \text{consump} \). Finally, we assume that percentage changes in \( \text{consump} \) are affected by contemporaneous changes in both \( \text{inv} \) and \( \text{inc} \).

The following commands fit an SVAR model with these constraints.

```
. use https://www.stata-press.com/data/r18/lutkepohl2
(Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)
. matrix A = (1,0,0\,1,0,\,1)
. matrix B = (.\,0,0,0\,0,\,0)
. svar dln_inv dln_inc dln_consump if qtr<=tq(1978q4), aeq(A) beq(B)
```

Estimating short-run parameters
(output omitted)

Structural vector autoregression

```plaintext
(1) \[ A \]_{1_1} = 1
(2) \[ A \]_{1_2} = 0
(3) \[ A \]_{1_3} = 0
(4) \[ A \]_{2_2} = 1
(5) \[ A \]_{2_3} = 0
(6) \[ A \]_{3_3} = 1
(7) \[ B \]_{1_2} = 0
(8) \[ B \]_{1_3} = 0
(9) \[ B \]_{2_1} = 0
(10) \[ B \]_{2_3} = 0
(11) \[ B \]_{3_1} = 0
(12) \[ B \]_{3_2} = 0
```

Sample: 1960q4 thru 1978q4  Number of obs = 73
Exactly identified model  Log likelihood = 606.307

|   | Coefficient | Std. err. | z    | P>|z| | [95% conf. interval] |
|---|-------------|-----------|------|-----|---------------------|
|/A|             |           |      |     |                     |
|1_1| 1 (constrained) |
|2_1| -.0336288 | .0294605  | -1.14 | 0.254 | -.0913702 | .0241126 |
|3_1| -.0435846 | .0194408  | -2.24 | 0.025 | -.0816879 | -.0054812 |
|1_2| 0 (constrained) |
|2_2| 1 (constrained) |
|3_2| -.424774 | .0765548  | -5.55 | 0.000 | -.5748187 | -.2747293 |
|1_3| 0 (constrained) |
|2_3| 0 (constrained) |
|3_3| 1 (constrained) |

/B
```
|   | Coefficient | Std. err. | z    | P>|z| | [95% conf. interval] |
|---|-------------|-----------|------|-----|---------------------|
|1_1| .0438796  | .0036315  | 12.08 | 0.000 | .036762 | .0509972 |
|2_1| 0 (constrained) |
|3_1| 0 (constrained) |
|1_2| 0 (constrained) |
|2_2| .0110449 | .0009141  | 12.08 | 0.000 | .0092534 | .0128365 |
|3_2| 0 (constrained) |
|1_3| 0 (constrained) |
|2_3| 0 (constrained) |
|3_3| .0072243 | .0005979  | 12.08 | 0.000 | .0060525 | .0083962 |
```

The SVAR output has four parts: an iteration log, a display of the constraints imposed, a header with sample and SVAR log-likelihood information, and a table displaying the estimates of the parameters from the \( A \) and \( B \) matrices. From the output above, we can see that the equality constraint matrices
supplied to \texttt{svar} imposed the intended constraints and that the SVAR header informs us that the model we fit is just identified. The estimates of $/A:2\_1$, $/A:3\_1$, and $/A:3\_2$ are all negative. Because the off-diagonal elements of the $A$ matrix contain the negative of the actual contemporaneous effects, the estimated effects are positive, as expected.

The estimates $\hat{A}$ and $\hat{B}$ are stored in $e(A)$ and $e(B)$, respectively, allowing us to compute the estimated Cholesky decomposition.

\begin{verbatim}
. matrix Aest = e(A)
. matrix Best = e(B)
. matrix chol_est = inv(Aest)*Best
. matrix list chol_est
chol_est[3,3]    dln_inv   dln_inc   dln_consump
   dln_inv   .04387957   0   0
   dln_inc   .00147562  .01104494   0
   dln_consump   .00253928  .0046916  .00722432
\end{verbatim}

\texttt{svar} stores the estimated $\Sigma$ from the underlying \texttt{var} in $e(Sigma)$. The output below illustrates the computation of the Cholesky decomposition of $e(Sigma)$. It is the same as the output computed from the SVAR estimates.

\begin{verbatim}
. matrix sig_var = e(Sigma)
. matrix chol_var = cholesky(sig_var)
. matrix list chol_var
chol_var[3,3]    dln_inv   dln_inc   dln_consump
   dln_inv   .04387957   0   0
   dln_inc   .00147562  .01104494   0
   dln_consump   .00253928  .0046916  .00722432
\end{verbatim}

We might now wonder why we bother obtaining parameter estimates via nonlinear estimation if we can obtain them simply by a transform of the estimates produced by \texttt{var}. When the model is just identified, as in the previous example, the SVAR parameter estimates can be computed via a transform of the VAR estimates. However, when the model is overidentified, such is not the case.

\section*{Example 2: Short-run overidentified SVAR model}

The Cholesky decomposition example above fit a just-identified model. This example considers an overidentified model. In example 1, the $/A:2\_1$ parameter was not significant, which is consistent with a theory in which changes in our measure of investment affect only changes in income with a lag. We can impose the restriction that $/A:2\_1$ is zero and then test this overidentifying restriction. Our $A$ and $B$ matrices are now

\[
A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ . & . & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} . & 0 & 0 \\ 0 & . & 0 \\ 0 & 0 & . \end{bmatrix}
\]

The output below contains the commands and results we obtained by fitting this model on the Lütkepohl data.

\begin{verbatim}
. matrix B = (.0,.0\0,.0\0,.0\0,.0)
. matrix A = (1,0\0,1,0\0,.\0,.1)
\end{verbatim}
. svar dln_inv dln_inc dln_consump if qtr<=tq(1978q4), aeq(A) beq(B)

Estimating short-run parameters

(output omitted)

Structural vector autoregression

\begin{align*}
(1) & \beta_{11} = 1 \\
(2) & \beta_{12} = 0 \\
(3) & \beta_{13} = 0 \\
(4) & \beta_{21} = 0 \\
(5) & \beta_{22} = 1 \\
(6) & \beta_{23} = 0 \\
(7) & \beta_{33} = 1 \\
(8) & \beta_{31} = 0 \\
(9) & \beta_{32} = 0 \\
(10) & \beta_{31} = 0 \\
(11) & \beta_{32} = 0 \\
(12) & \beta_{33} = 0 \\
\end{align*}

Sample: 1960q4 thru 1978q4
Number of obs = 73
Overidentified model
Log likelihood = 605.6613

| Coefficient | Std. err. | z | P>|z| | [95% conf. interval] |
|-------------|-----------|---|-------|-------------------|
| /A 1_1 | 1 (constrained) |
| 2_1 | 0 (constrained) |
| 3_1 | -.0435911 .0192696 -2.26 0.024 -.0813589 -.0058233 |
| 1_2 | 0 (constrained) |
| 2_2 | 1 (constrained) |
| 3_2 | -.4247741 .0758806 -5.60 0.000 -.5734973 -.2760508 |
| 1_3 | 0 (constrained) |
| 2_3 | 0 (constrained) |
| 3_3 | 1 (constrained) |
| /B 1_1 | .0438796 .0036315 12.08 0.000 .036762 .0509972 |
| 2_1 | 0 (constrained) |
| 3_1 | 0 (constrained) |
| 1_2 | 0 (constrained) |
| 2_2 | .0111431 .0009222 12.08 0.000 .0093356 .0129506 |
| 3_2 | 0 (constrained) |
| 1_3 | 0 (constrained) |
| 2_3 | 0 (constrained) |
| 3_3 | .0072243 .0005979 12.08 0.000 .0060525 .0083962 |

LR test of identifying restrictions: chi2(1) = 1.292 Prob > chi2 = 0.256

The footer in this example reports a test of the overidentifying restriction. The null hypothesis of this test is that any overidentifying restrictions are valid. In the case at hand, we cannot reject this null hypothesis at any of the conventional levels.

Example 3: Short-run SVAR model with constraints

\texttt{svar} also allows us to place constraints on the parameters of the underlying \texttt{VAR}. We begin by looking at the underlying \texttt{VAR} for the SVARs that we have used in the previous examples.
. var dln_inv dln_inc dln_consump if qtr<=tq(1978q4)

Vector autoregression

Sample: 1960q4 thru 1978q4
Number of obs = 73

Log likelihood = 606.307
AIC = -16.03581
FPE = 2.18e-11
HQIC = -15.77323
SBIC = -15.37691

Det(Sigma_ml) = 1.23e-11

<table>
<thead>
<tr>
<th>Equation</th>
<th>Parms</th>
<th>RMSE</th>
<th>R-sq</th>
<th>chi2</th>
<th>P&gt;chi2</th>
</tr>
</thead>
<tbody>
<tr>
<td>dln_inv</td>
<td>7</td>
<td>.046148</td>
<td>0.1286</td>
<td>10.76961</td>
<td>0.0958</td>
</tr>
<tr>
<td>dln_inc</td>
<td>7</td>
<td>.011719</td>
<td>0.1142</td>
<td>9.410683</td>
<td>0.1518</td>
</tr>
<tr>
<td>dln_consump</td>
<td>7</td>
<td>.009445</td>
<td>0.2513</td>
<td>24.50031</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

| Coefficient | Std. err. | z     | P>|z| | [95% conf. interval] |
|-------------|-----------|-------|-------|----------------------|
| dln_inv     |           |       |       |                      |
| L1.         | -.3196318 | .1192898 | -2.68 | 0.007 | -0.5534355 | -0.0858282 |
| L2.         | -.1605508 | .118767 | -1.35 | 0.176 | -.39333 | .0722283 |
| dln_inc     |           |       |       |                      |
| L1.         | .1459851  | .5188451 | 0.28  | 0.778 | -.8709326 | 1.162903 |
| L2.         | .1146009  | .508295 | 0.23  | 0.822 | -.881639 | 1.110841 |
| dln_consump |           |       |       |                      |
| L1.         | .9612288  | .6316557 | 1.52  | 0.128 | -.2767936 | 2.199251 |
| L2.         | .9344001  | .6324034 | 1.48  | 0.140 | -.3050877 | 2.173888 |
| _cons       | -.0167221 | .0163796 | -1.02 | 0.307 | -.0488257 | .0153814 |
| dln_inc     |           |       |       |                      |
| L1.         | .0439309  | .0302933 | 1.45  | 0.147 | -.0154427 | .1033046 |
| L2.         | .0500302  | .0301605 | 1.66  | 0.097 | -.0090833 | .1091437 |
| dln_inc     |           |       |       |                      |
| L1.         | -.1527311 | .131759 | -1.16 | 0.246 | -.4109741 | .1055118 |
| L2.         | .0191634  | .1290799 | 0.15  | 0.882 | -.2338285 | .2721552 |
| dln_consump |           |       |       |                      |
| L1.         | .2884992  | .1604069 | 1.80  | 0.072 | -.258926 | .6028909 |
| L2.         | -.0102    | .1605968 | -0.06 | 0.949 | -.3249639 | .3045639 |
| _cons       | .0157672  | .0041596 | 3.79  | 0.000 | .0076146 | .0239198 |
| dln_consump |           |       |       |                      |
| L1.         | -.002423  | .0244142 | -0.10 | 0.921 | -.050274 | .045428 |
| L2.         | .0338806  | .0243072 | 1.39  | 0.163 | -.0137607 | .0815219 |
| dln_inc     |           |       |       |                      |
| L1.         | .2248134  | .1061884 | 2.12  | 0.034 | .0166879 | .4329389 |
| L2.         | .3549135  | .1040292 | 3.41  | 0.001 | .1510199 | .558807 |
| dln_consump |           |       |       |                      |
| L1.         | -.2639695 | .1292766 | -2.04 | 0.041 | -.517347 | -.010592 |
| L2.         | -.0222264 | .1294296 | -0.17 | 0.864 | -.2759039 | .231451 |
| _cons       | .0129258  | .0033523 | 3.86  | 0.000 | .0063554 | .0194962 |
The equation-level model tests reported in the header indicate that we cannot reject the null hypotheses that all the coefficients in the first equation are zero, nor can we reject the null that all the coefficients in the second equation are zero at the 5% significance level. We use a combination of theory and the p-values from the output above to place some exclusion restrictions on the underlying VAR(2). Specifically, in the equation for the percentage change of inv, we constrain the coefficients on L2.dln_inv, L.dln_inc, L2.dln_inc, and L2.dln_consump to be zero. In the equation for dln_inc, we constrain the coefficients on L2.dln_inv, L2.dln_inc, and L2.dln_consump to be zero. Finally, in the equation for dln_consump, we constrain L.dln_inv and L2.dln_consump to be zero. We then refit the SVAR from the previous example.

```plaintext
.svar dln_inv dln_inc dln_consump if qtr<=tq(1978q4), aeq(A) beq(B) > varconst(1/9) noislog
Estimating short-run parameters
(output omitted)
```

Structural vector autoregression

(1) \[ A_{11} = 1 \]
(2) \[ A_{12} = 0 \]
(3) \[ A_{13} = 0 \]
(4) \[ A_{21} = 0 \]
(5) \[ A_{22} = 1 \]
(6) \[ A_{23} = 0 \]
(7) \[ A_{33} = 1 \]
(8) \[ B_{12} = 0 \]
(9) \[ B_{13} = 0 \]
(10) \[ B_{21} = 0 \]
(11) \[ B_{23} = 0 \]
(12) \[ B_{31} = 0 \]
(13) \[ B_{32} = 0 \]
### Structural vector autoregressive models

Sample: 1960q4 thru 1978q4  
Number of obs = 73  
Log likelihood = 601.8591

| Coefficient | Std. err. | z     | P>|z| | [95% conf. interval] |
|-------------|-----------|-------|------|----------------------|

**A**  
1 \(1\)  
2 \(1\)  
3 \(1\) \(-.0418708\) \(.0187579\) \(-2.23\) \(0.026\) \(-.0786356\) \(-.0051061\)  
1 \(2\)  
2 \(2\)  
3 \(2\) \(-.4255808\) \(.0745298\) \(-5.71\) \(0.000\) \(-.5716565\) \(-.2795051\)  
1 \(3\)  
2 \(3\)  
3 \(3\)  

**B**  
1 \(1\)  
2 \(1\)  
3 \(1\)  
1 \(2\)  
2 \(2\) \(.0113723\) \(.0009412\) \(12.08\) \(0.000\) \(.0095276\) \(.013217\)  
3 \(2\)  
1 \(3\)  
2 \(3\)  
3 \(3\) \(.0072417\) \(.0005993\) \(12.08\) \(0.000\) \(.006067\) \(.0084164\)

LR test of identifying restrictions: chi2(1) = .8448  
Prob > chi2 = 0.358

If we displayed the underlying VAR(2) results by using the var option, we would see that most of the unconstrained coefficients are now significant at the 10% level and that none of the equation-level model statistics fail to reject the null hypothesis at the 10% level. The svar output reveals that the \(p\)-value of the overidentification test rose and that the coefficient on /A:3_1 is still insignificant at the 1% level but not at the 5% level.

Before moving on to models with long-run constraints, consider these limitations. We cannot place constraints on the elements of \(A\) in terms of the elements of \(B\), or vice versa. This limitation is imposed by the form of the check for identification derived by Amisano and Giannini (1997). As noted in Methods and formulas, this test requires separate constraint matrices for the parameters in \(A\) and \(B\). Another limitation is that we cannot mix short-run and long-run constraints.

### Long-run SVAR models

As discussed in [TS] var intro, a long-run SVAR has the form

\[
y_t = C e_t
\]

In long-run models, the constraints are placed on the elements of \(C\), and the free parameters are estimated. These constraints are often exclusion restrictions. For instance, constraining \(C[1,2]\) to be zero can be interpreted as setting the long-run response of variable 1 to the structural shocks driving variable 2 to be zero.

Similar to the short-run model, the \(P_{lr}\) matrix such that \(P_{lr}P_{lr}' = \Sigma\) identifies the structural impulse–response functions. \(P_{lr} = C\) is identified by the restrictions placed on the parameters in \(C\). There are \(K^2\) parameters in \(C\), and the order condition for identification requires that there be
at least $K^2 - K(K + 1)/2$ restrictions placed on those parameters. As in the short-run model, this order condition is necessary but not sufficient, so the Amisano and Giannini (1997) check for local identification is performed by default.

### Example 4: Long-run SVAR model

Suppose that we have a theory in which unexpected changes to the money supply have no long-run effects on changes in output and, similarly, that unexpected changes in output have no long-run effects on changes in the money supply. The $C$ matrix implied by this theory is

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
. use https://www.stata-press.com/data/r18/m1gdp
. matrix lr = (.,0\0,.)
. svar d.nl_m1 d.nl_gdp, lreq(lr)
```

```
 Estimating long-run parameters
(output omitted)
```

Structural vector autoregression

1. $[C]_{1,2} = 0$
2. $[C]_{2,1} = 0$

Sample: 1959q4 thru 2002q2
Number of obs = 171
Overidentified model Log likelihood = 1151.614

| Coefficient Std. err.  z  P>|z|  [95% conf. interval] |
|------------------|-----------------|------|--------|----------------|
| $/C$             |                 |      |        |                |
| 1_1              | .0301007        | .0016277 | 18.49 | 0.000 | .0269106 .0332909 |
| 2_1              | 0 (constrained) |      |        |        |                |
| 1_2              | 0 (constrained) |      |        |        |                |
| 2_2              | .0129691        | .0007013 | 18.49 | 0.000 | .0115946 .0143436 |

```
LR test of identifying restrictions: chi2(1) = .1368  Prob > chi2 = 0.712
```

We have assumed that the underlying VAR has 2 lags; four of the five selection-order criteria computed by `varsoc` (see `[TS] varsoc`) recommended this choice. The test of the overidentifying restrictions provides no indication that it is not valid.
Stored results

`svar` stores the following in `e()`:

Scalars

- `e(N)` number of observations
- `e(N_cns)` number of constraints
- `e(k_eq)` number of equations in `e(b)`
- `e(k_dv)` number of dependent variables
- `e(ll)` log likelihood from `svar`
- `e(N_gaps_var)` number of gaps in the sample
- `e(k_var)` number of coefficients in VAR
- `e(k_eq_var)` number of equations in underlying VAR
- `e(k_dv_var)` number of dependent variables in underlying VAR
- `e(df_eq_var)` average number of parameters in an equation
- `e(df_r_var)` if `small`, residual degrees of freedom
- `e(obs_#_var)` number of observations on equation `#`
- `e(k_#_var)` number of coefficients in equation `#`
- `e(df_eq_#_var)` number of equations in underlying VAR
- `e(df_dv_#_var)` number of dependent variables in underlying VAR
- `e(ll_#_var)` log likelihood for equation `#`
- `e(chi2_#_var)` $\chi^2$ statistic for equation `#`
- `e(r2_#_var)` $R^2$ for equation `#`
- `e(chi2_oid)` overidentification test
- `e(oid_df)` number of overidentifying restrictions
- `e(rank)` rank of `e(V)`
- `e(rc_ml)` return code from `ml`
- `e(fpe)` final prediction error
- `e(mlag) ml` highest lag in VAR
- `e(tmin)` first time period in the sample
- `e(tmax)` maximum time
- `e(eqnames_var)` names of equations
- `e(lutstats_var)` `lutstats`, if specified
- `e(constraints_var)` `constraints_var`, if there are constraints on VAR
- `e(small)` small, if specified
- `e(properties)` `b V` program used to implement `predict`

Macros

- `e(cmd)` `svar`
- `e(cmdline)` command as typed
- `e(lrmodel)` long-run model, if specified
- `e(lags_var)` lags in model
- `e(depvar_var)` names of dependent variables
- `e(endog_var)` names of endogenous variables
- `e(exog_var)` names of exogenous variables, if specified
- `e(nocons) nocons`, if `noconstant` specified
- `e(cns_lr)` long-run constraints
- `e(cns_a)` cross-parameter equality constraints on A
- `e(cns_b)` cross-parameter equality constraints on B
- `e(dfk_var)` alternate divisor (dfk), if specified
- `e(eqnames_var)` names of equations
- `e(lutstats_var)` `lutstats`, if specified
- `e(constraints_var)` `constraints_var`, if there are constraints on VAR
- `e(small)` `small`, if specified
- `e(tsfmt)` format of `timevar`
- `e(timevar)` name of `timevar`
- `e(title)` title in estimation output
- `e(properties)` `b V` program used to implement `predict`
Matrices
- \( e(b) \) \text{ coefficient vector}
- \( e(Cns) \) \text{ constraints matrix}
- \( e(Sigma) \) \( \Sigma \) \text{ matrix of estimators}
- \( e(V) \) \text{ variance–covariance matrix of the estimators}
- \( e(b\_var) \) \text{ coefficient vector of underlying VAR model}
- \( e(V\_var) \) \text{ VCE of underlying VAR model}
- \( e(bf\_var) \) \text{ full coefficient vector with zeros in dropped lags}
- \( e(G\_var) \) \text{ Gamma matrix stored by \texttt{var}; see \textit{Methods and formulas} in [TS] \texttt{var}}
- \( e(aeq) \) \( \texttt{aeq(matrix)}, \text{if specified} \)
- \( e(acns) \) \( \texttt{acns(matrix)}, \text{if specified} \)
- \( e(beq) \) \( \texttt{beq(matrix)}, \text{if specified} \)
- \( e(bcns) \) \( \texttt{bcns(matrix)}, \text{if specified} \)
- \( e(lreq) \) \( \texttt{lreq(matrix)}, \text{if specified} \)
- \( e(lrcns) \) \( \texttt{lrcns(matrix)}, \text{if specified} \)
- \( e(Cns\_var) \) \text{ constraint matrix from \texttt{var}, if \texttt{varconstraints()} is specified}
- \( e(A) \) \text{ estimated A matrix, if a short-run model}
- \( e(B) \) \text{ estimated B matrix}
- \( e(C) \) \text{ estimated C matrix, if a long-run model}
- \( e(A1) \) \text{ estimated \( \tilde{A} \) matrix, if a long-run model}

Functions
- \( e(sample) \) \text{ marks estimation sample}

In addition to the above, the following is stored in \( r() \):

Matrices
- \( r(table) \) \text{ matrix containing the coefficients with their standard errors, test statistics, \( p \)-values, and confidence intervals}

Note that results stored in \( r() \) are updated when the command is replayed and will be replaced when any \texttt{r-class} command is run after the estimation command.

**Methods and formulas**

The log-likelihood function for models with short-run constraints is

\[
L(A, B) = -\frac{NK}{2} \ln(2\pi) + \frac{N}{2} \ln(|W|^2) - \frac{N}{2} \text{tr}(W'W\hat{\Sigma})
\]

where \( W = B^{-1}A \).

When there are long-run constraints, because \( C = \tilde{A}^{-1}B \) and \( A = I_K \), \( W = B^{-1} = C^{-1}\tilde{A}^{-1} = (\tilde{A}C)^{-1} \). Substituting the last term for \( W \) in the short-run log likelihood produces the long-run log likelihood

\[
L(C) = -\frac{NK}{2} \ln(2\pi) + \frac{N}{2} \ln(|\tilde{W}|^2) - \frac{N}{2} \text{tr}(\tilde{W}'\tilde{W}\hat{\Sigma})
\]

where \( \tilde{W} = (\tilde{A}C)^{-1} \).

For both the short-run and the long-run models, the maximization is performed by the scoring method. See Harvey (1990) for a discussion of this method.
Based on results from Amisano and Giannini (1997), the score vector for the short-run model is
\[
\frac{\partial L(A, B)}{\partial \text{vec}(A), \text{vec}(B)} = N \left[ \{ \text{vec}(W^{-1}) \}' - \{ \text{vec}(W) \}' (\hat{\Sigma} \otimes I_K) \right] \times \\
\left[ (I_K \otimes B^{-1}), -(A'B^{-1} \otimes B^{-1}) \right]
\]
and the expected information matrix is
\[
I[\text{vec}(A), \text{vec}(B)] = N \left[ \frac{(W^{-1} \otimes B^{-1})}{(I_K \otimes B^{-1})} \right] (I_{K^2} + \oplus) \left[ (W^{-1} \otimes B^{-1}), -(I_K \otimes B^{-1}) \right]
\]
where \( \oplus \) is the commutation matrix defined in Magnus and Neudecker (2019, 54–55).

Using results from Amisano and Giannini (1997), we can derive the score vector and the expected information matrix for the case with long-run restrictions. The score vector is
\[
\frac{\partial L(C)}{\partial \text{vec}(C)} = N \left[ \{ \text{vec}(W'^{-1}) \}' - \{ \text{vec}(W) \}' (\hat{\Sigma} \otimes I_K) \right] \left[ -(A'^{-1}C'^{-1} \otimes C^{-1}) \right]
\]
and the expected information matrix is
\[
I[\text{vec}(C)] = N(I_K \otimes C'^{-1})(I_{K^2} + \oplus)(I_K \otimes C'^{-1})
\]

### Checking for identification

This section describes the methods used to check for identification of models with short-run or long-run constraints. Both methods depend on the starting values. By default, \texttt{svar} uses starting values constructed by taking a vector of appropriate dimension and applying the constraints. If there are \( m \) parameters in the model, the \( j \)th element of the \( 1 \times m \) vector is \( 1 + m / 100 \). \texttt{svar} also allows the user to provide starting values.

For the short-run case, the model is identified if the matrix
\[
V_{sr}^* = \begin{bmatrix}
N_K & N_K \\
N_K & N_K \\
R_a(W' \otimes B) & 0_{K^2} \\
0_{K^2} & R_a(I_K \otimes B)
\end{bmatrix}
\]
has full column rank of \( 2K^2 \), where \( N_K = (1/2)(I_{K^2} + \oplus) \), \( R_a \) is the constraint matrix for the parameters in \( A \) (that is, \( R_a \text{vec}(A) = r_a \)), and \( R_b \) is the constraint matrix for the parameters in \( B \) (that is, \( R_b \text{vec}(B) = r_b \)).

For the long-run case, based on results from the \( C \) model in Amisano and Giannini (1997), the model is identified if the matrix
\[
V_{lr}^* = \begin{bmatrix}
(I \otimes C'^{-1})(2N_K)(I \otimes C^{-1}) \\
R_c
\end{bmatrix}
\]
has full column rank of \( K^2 \), where \( R_c \) is the constraint matrix for the parameters in \( C \); that is, \( R_c \text{vec}(C) = r_c \).
The test of the overidentifying restrictions is computed as

\[ LR = 2(LL_{\text{var}} - LL_{\text{svar}}) \]

where \( LR \) is the value of the test statistic against the null hypothesis that the overidentifying restrictions are valid, \( LL_{\text{var}} \) is the log likelihood from the underlying \( \text{VAR}(p) \) model, and \( LL_{\text{svar}} \) is the log likelihood from the SVAR model. The test statistic is asymptotically distributed as \( \chi^2(q) \), where \( q \) is the number of overidentifying restrictions. Amisano and Giannini (1997, 38–39) emphasize that, because this test of the validity of the overidentifying restrictions is an omnibus test, it can be interpreted as a test of the null hypothesis that all the restrictions are valid.

Because constraints might not be independent either by construction or because of the data, the number of restrictions is not necessarily equal to the number of constraints. The rank of \( e(V) \) gives the number of parameters that were independently estimated after applying the constraints. The maximum number of parameters that can be estimated in an identified short-run or long-run SVAR is \( K(K + 1)/2 \). This implies that the number of overidentifying restrictions, \( q \), is equal to \( K(K + 1)/2 \) minus the rank of \( e(V) \).

The number of overidentifying restrictions is also linked to the order condition for each model. In a short-run SVAR model, there are \( 2K^2 \) parameters. Because no more than \( K(K + 1)/2 \) parameters may be estimated, the order condition for a short-run SVAR model is that at least \( 2K^2 - K(K + 1)/2 \) restrictions be placed on the model. Similarly, there are \( K^2 \) parameters in long-run SVAR model. Because no more than \( K(K + 1)/2 \) parameters may be estimated, the order condition for a long-run SVAR model is that at least \( K^2 - K(K + 1)/2 \) restrictions be placed on the model.

Acknowledgment

We thank Gianni Amisano of the Board of Governors of the Federal Reserve System for his helpful comments.

References


Also see

[TS] **var svar postestimation** — Postestimation tools for svar

[TS] **tsset** — Declare data to be time-series data

[TS] **var** — Vector autoregressive models

[TS] **var intro** — Introduction to vector autoregressive models

[TS] **varbasic** — Fit a simple VAR and graph IRFs or FEVDs

[TS] **vec** — Vector error-correction models

[U] **20 Estimation and postestimation commands**

*Stata Dynamic Stochastic General Equilibrium Models Reference Manual*