**Description**

`prais` uses the generalized least-squares method to estimate the parameters in a linear regression model in which the errors are serially correlated. Specifically, the errors are assumed to follow a first-order autoregressive process.

**Quick start**

Prais–Winsten regression of `y` on `x` estimating the autocorrelation parameter by a single-lag OLS regression of residuals using `tsset` data

```
prais y x
```

As above, but estimate the autocorrelation parameter using a single-lead OLS of residuals

```
prais y x, rhotype(freg)
```

As above, but estimate the autocorrelation parameter using autocorrelation of residuals

```
prais y x, rhotype(tscorr)
```

Cochrane–Orcutt regression of `y` on `x` with first-order serial correlation

```
prais y x, corc
```

**Menu**

Statistics > Time series > Prais-Winsten regression
Syntax

```
prais depvar [ indepvars ] [ if ] [ in ] [ , options ]
```

<table>
<thead>
<tr>
<th>options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>rho</code></td>
<td>Model</td>
</tr>
<tr>
<td><code>rho(type(regress))</code></td>
<td>base $\rho$ on single-lag OLS of residuals; the default</td>
</tr>
<tr>
<td><code>rho(type(freg))</code></td>
<td>base $\rho$ on single-lead OLS of residuals</td>
</tr>
<tr>
<td><code>rho(type(tscorr))</code></td>
<td>base $\rho$ on autocorrelation of residuals</td>
</tr>
<tr>
<td><code>rho(type(dw))</code></td>
<td>base $\rho$ on autocorrelation based on Durbin–Watson</td>
</tr>
<tr>
<td><code>rho(type(theil))</code></td>
<td>base $\rho$ on adjusted autocorrelation</td>
</tr>
<tr>
<td><code>rho(type(nagar))</code></td>
<td>base $\rho$ on adjusted Durbin–Watson</td>
</tr>
<tr>
<td><code>corc</code></td>
<td>use Cochrane–Orcutt transformation</td>
</tr>
<tr>
<td><code>sse</code></td>
<td>search for $\rho$ that minimizes SSE</td>
</tr>
<tr>
<td><code>twostep</code></td>
<td>stop after the first iteration</td>
</tr>
<tr>
<td><code>noconstant</code></td>
<td>suppress constant term</td>
</tr>
<tr>
<td><code>hascons</code></td>
<td>has user-defined constant</td>
</tr>
<tr>
<td><code>save</code></td>
<td>conserve memory during estimation</td>
</tr>
<tr>
<td><code>vce(vcetype)</code></td>
<td><code>vcetype</code> may be <code>ols</code>, <code>robust</code>, <code>cluster clustvar</code>, <code>hc2</code>, or <code>hc3</code></td>
</tr>
<tr>
<td><code>level(#)</code></td>
<td>set confidence level; default is <code>level(95)</code></td>
</tr>
<tr>
<td><code>nodw</code></td>
<td>do not report the Durbin–Watson statistic</td>
</tr>
<tr>
<td><code>display_options</code></td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td><code>optimize_options</code></td>
<td>control the optimization process; seldom used</td>
</tr>
<tr>
<td><code>coeflegend</code></td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>

You must `tsset` your data before using `prais`; see [TS] `tsset`.

`indepvars` may contain factor variables; see [U] 11.4.3 Factor variables.

`depvar` and `indepvars` may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, `fp`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
Options

\texttt{rhomethod} selects a specific computation for the autocorrelation parameter \( \rho \), where \texttt{rhomethod} can be

\begin{align*}
\texttt{regress} & : \quad \rho_{\text{reg}} = \beta \text{ from the residual regression } \epsilon_t = \beta \epsilon_{t-1} \\
\texttt{freg} & : \quad \rho_{\text{freg}} = \beta \text{ from the residual regression } \epsilon_t = \beta \epsilon_{t+1} \\
\texttt{tscorr} & : \quad \rho_{\text{tscorr}} = \epsilon' \epsilon_{t-1} / \epsilon' \epsilon, \text{ where } \epsilon \text{ is the vector of residuals} \\
\texttt{dw} & : \quad \rho_{\text{dw}} = 1 - \text{dw}/2, \text{ where dw is the Durbin–Watson } d \text{ statistic} \\
\texttt{theil} & : \quad \rho_{\text{theil}} = \rho_{\text{tscorr}} (N - k)/N \\
\texttt{nagar} & : \quad \rho_{\text{nagar}} = (\rho_{\text{dw}} * N^2 + k^2) / (N^2 - k^2)
\end{align*}

The \texttt{prais} estimator can use any consistent estimate of \( \rho \) to transform the equation, and each of these estimates meets that requirement. The default is \texttt{regress}, which produces the minimum sum-of-squares solution (\texttt{sse}) option for the Cochrane–Orcutt transformation—none of these computations will produce the minimum sum-of-squares solution for the full Prais–Winsten transformation. See Judge et al. (1985) for a discussion of each estimate of \( \rho \).

corc specifies that the Cochrane–Orcutt transformation be used to estimate the equation. With this option, the Prais–Winsten transformation of the first observation is not performed, and the first observation is dropped when estimating the transformed equation; see \textit{Methods and formulas} below.

\texttt{sse} specifies that a search be performed for the value of \( \rho \) that minimizes the sum-of-squared errors of the transformed equation (Cochrane–Orcutt or Prais–Winsten transformation). The search method is a combination of quadratic and modified bisection searches using golden sections.

twostep specifies that \texttt{prais} stop on the first iteration after the equation is transformed by \( \rho \)—the two-step efficient estimator. Although iterating these estimators to convergence is customary, they are efficient at each step.

\texttt{noconstant}; see [R] \textit{Estimation options}.

\texttt{hascons} indicates that a user-defined constant, or a set of variables that in linear combination forms a constant, has been included in the regression. For some computational concerns, see the discussion in [R] \texttt{regress}.

\texttt{savespace} specifies that \texttt{prais} attempt to save as much space as possible by retaining only those variables required for estimation. The original data are restored after estimation. This option is rarely used and should be used only if there is insufficient space to fit a model without the option.

\texttt{vce(vcetype)} specifies the estimator for the variance–covariance matrix of the estimator; see [R] \textit{vce_option}.

\texttt{vce(ols)}, the default, uses the standard variance estimator for ordinary least-squares regression.
\texttt{vce(robust)} specifies to use the Huber/White/sandwich estimator.
\texttt{vce(cluster clustvar)} specifies to use the intragroup correlation estimator.
\texttt{vce(hc2)} and \texttt{vce(hc3)} specify an alternative bias correction for the \texttt{vce(robust)} variance calculation; for more information, see [R] \texttt{regress}. You may specify only one of \texttt{vce(hc2)}, \texttt{vce(hc3)}, or \texttt{vce(robust)}. 

All estimates from \texttt{prais} are conditional on the estimated value of $\rho$. Robust variance estimates here are robust only to heteroskedasticity and are not generally robust to misspecification of the functional form or omitted variables. The estimation of the functional form is intertwined with the estimation of $\rho$, and all estimates are conditional on $\rho$. Thus estimates cannot be robust to misspecification of functional form. For these reasons, it is probably best to interpret \texttt{vce(robust)} in the spirit of White’s (1980) original paper on estimation of heteroskedastic-consistent covariance matrices.

\texttt{Reporting} \texttt{level(#)}; see \texttt{[R] Estimation options}. \texttt{nodw} suppresses reporting of the Durbin–Watson statistic. \texttt{display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch}; see \texttt{[R] Estimation options}.

\texttt{Optimization} \texttt{optimize_options: iterate(#), [no]log, tolerance(#)}. \texttt{iterate()} specifies the maximum number of iterations. \texttt{log/nolog} specifies whether to show the iteration log (see \texttt{set iterlog} in \texttt{[R] set iter}). \texttt{tolerance()} specifies the tolerance for the coefficient vector; \texttt{tolerance(1e-6)} is the default. These options are seldom used.

The following option is available with \texttt{prais} but is not shown in the dialog box: \texttt{coflegend}; see \texttt{[R] Estimation options}.

\textbf{Remarks and examples} \hfill \texttt{stata.com}

\texttt{prais} fits a linear regression of \texttt{depvar on indepvars} that is corrected for first-order serially correlated residuals by using the Prais–Winsten (1954) transformed regression estimator, the Cochrane–Orcutt (1949) transformed regression estimator, or a version of the search method suggested by Hildreth and Lu (1960). Davidson and MacKinnon (1993) provide theoretical details on the three methods (see pages 333–335 for the latter two and pages 343–351 for Prais–Winsten). See Becketti (2020) for more examples showing how to use \texttt{prais}.

The most common autocorrelated error process is the first-order autoregressive process. Under this assumption, the linear regression model can be written as

$$y_t = x_t \beta + u_t$$

where the errors satisfy

$$u_t = \rho u_{t-1} + e_t$$

and the $e_t$ are independent and identically distributed as $N(0, \sigma^2)$. The covariance matrix $\Psi$ of the error term $u$ can then be written as

$$\Psi = \frac{1}{1 - \rho^2} \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{T-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{bmatrix}$$
The Prais–Winsten estimator is a generalized least-squares (GLS) estimator. The Prais–Winsten method (as described in Judge et al. 1985) is derived from the AR(1) model for the error term described above. Whereas the Cochrane–Orcutt method uses a lag definition and loses the first observation in the iterative method, the Prais–Winsten method preserves that first observation. In small samples, this can be a significant advantage.

Technical note

To fit a model with autocorrelated errors, you must specify your data as time series and have (or create) a variable denoting the time at which an observation was collected. The data for the regression should be equally spaced in time.

Example 1

Say that we wish to fit a time-series model of `usr` on `idle` but are concerned that the residuals may be serially correlated. We will declare the variable `t` to represent time by typing

```
    . use https://www.stata-press.com/data/r16/idle
    . tsset t
        time variable:  t, 1 to 30
        delta:  1 unit
```

We can obtain Cochrane–Orcutt estimates by specifying the `corc` option:

```
    . prais usr idle, corc
    Iteration 0:  rho = 0.0000
    Iteration 1:  rho = 0.3518
      (output omitted)
    Iteration 13: rho = 0.5708

    Cochrane-Orcutt AR(1) regression -- iterated estimates
    Number of obs = 29
    F(1, 27) = 6.49
    Prob > F = 0.0168
    R-squared = 0.1938
    Adj R-squared = 0.1640
    Root MSE = 2.4862

    usr Coef. Std. Err. t P>|t| [95% Conf. Interval]
    idle  -.1254511  .0492356 -2.55 0.017 -.2264742 -.024428
    _cons  14.54641  4.272299 3.40 0.002 5.78038 23.31245

    rho  .5707918

    Durbin-Watson statistic (original) 1.295766
    Durbin-Watson statistic (transformed) 1.466222
```

The fitted model is

\[ \text{usr}_t = -0.1255 \text{idle}_t + 14.55 + u_t \quad \text{and} \quad u_t = 0.5708 u_{t-1} + e_t \]
We can also fit the model with the Prais–Winsten method,

```stata
.prais usr idle
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.3518
(output omitted)
Iteration 14:  rho = 0.5535
Prais-Winsten AR(1) regression -- iterated estimates

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F(1, 28) = 7.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>43.0076941</td>
<td>1</td>
<td>43.0076941</td>
<td>Prob &gt; F = 0.0125</td>
</tr>
<tr>
<td>Residual</td>
<td>169.165739</td>
<td>28</td>
<td>6.04163545</td>
<td>R-squared = 0.2027</td>
</tr>
<tr>
<td></td>
<td>Adj R-squared = 0.1742</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>212.173433</td>
<td>29</td>
<td>7.31632528</td>
<td>Root MSE = 2.458</td>
</tr>
</tbody>
</table>

| usr     | Coef.    | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|--------|----------|-----------|-------|-----|---------------------|
| idle   | -.1356522 | .0472195  | -2.87 | 0.008 | -.2323769 -.1389275 |
| _cons  | 15.20415   | 4.160391  | 3.65  | 0.001 | 6.681978 23.72633  |

| rho     | .5535476 |

Durbin-Watson statistic (original) 1.295766
Durbin-Watson statistic (transformed) 1.476004

where the Prais–Winsten fitted model is

\[
\text{usr}_t = -0.1357 \text{idle}_t + 15.20 + u_t \quad \text{and} \quad u_t = 0.5535 u_{t-1} + e_t
\]

As the results indicate, for these data there is little difference between the Cochrane–Orcutt and Prais–Winsten estimators, whereas the OLS estimate of the slope parameter is substantially different.

> Example 2

We have data on quarterly sales, in millions of dollars, for 5 years, and we would like to use this information to model sales for company X. First, we fit a linear model by OLS and obtain the Durbin–Watson statistic by using `estat dwatson`; see [R] `regress postestimation time series`.

```stata
.use https://www.stata-press.com/data/r16/qsales
.regress csales isales

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F(1, 18) = 14888.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>110.256901</td>
<td>1</td>
<td>110.256901</td>
<td>Prob &gt; F = 0.000</td>
</tr>
<tr>
<td>Residual</td>
<td>.133302302</td>
<td>18</td>
<td>.007405683</td>
<td>R-squared = 0.9988</td>
</tr>
<tr>
<td></td>
<td>Adj R-squared = 0.9987</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>110.390204</td>
<td>19</td>
<td>5.81001072</td>
<td>Root MSE = .08606</td>
</tr>
</tbody>
</table>

| csales  | Coef.    | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|--------|----------|-----------|-------|-----|---------------------|
| isales | .1762828 | .0014447  | 122.02 | 0.000 | .1732475 .1793181  |
| _cons  | -1.454753 | .2141461 | -6.79  | 0.000 | -1.904657 -1.004849 |

<table>
<thead>
<tr>
<th></th>
<th>.estat dwatson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durbin-Watson d-statistic( 2, 20) = .7347276</td>
<td></td>
</tr>
</tbody>
</table>
```
Because the Durbin–Watson statistic is far from 2 (the expected value under the null hypothesis of no serial correlation) and well below the 5% lower limit of 1.2, we conclude that the disturbances are serially correlated. (Upper and lower bounds for the $d$ statistic can be found in most econometrics texts; for example, Harvey [1990]. The bounds have been derived for only a limited combination of regressors and observations.) To reinforce this conclusion, we use two other tests to test for serial correlation in the error distribution.

```
. estat bgodfrey, lags(1)
Breusch-Godfrey LM test for autocorrelation

lags(p) | chi2  | df  | Prob > chi2
-------|------|-----|-------------
   1   | 7.998| 1   | 0.0047      

H0: no serial correlation
```

```
. estat durbinalt
Durbin's alternative test for autocorrelation

lags(p) | chi2  | df  | Prob > chi2
-------|------|-----|-------------
   1   | 11.329| 1   | 0.0008      

H0: no serial correlation
```

`estat bgodfrey` reports the Breusch–Godfrey Lagrange multiplier test statistic, and `estat durbinalt` reports the Durbin’s alternative test statistic. Both tests give a small $p$-value and thus reject the null hypothesis of no serial correlation. These two tests are asymptotically equivalent when testing for AR(1) process. See [R] `regress postestimation time series` if you are not familiar with these two tests.

We correct for autocorrelation with the `ssesearch` option of `prais` to search for the value of $\rho$ that minimizes the sum-of-squared residuals of the Cochrane–Orcutt transformed equation. Normally, the default Prais–Winsten transformations is used with such a small dataset, but the less-efficient Cochrane–Orcutt transformation allows us to demonstrate an aspect of the estimator’s convergence.

```
. prais csales isales, corc ssesearch
Iteration 1: rho = 0.8944 , criterion = -.07298558
Iteration 2: rho = 0.8944 , criterion = -.07298558
(output omitted)
Iteration 15: rho = 0.9588 , criterion = -.07167037

Cochrane-Orcutt AR(1) regression -- SSE search estimates

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>2.33199178</td>
<td>1</td>
<td>2.33199178</td>
<td>F(1, 17) = 553.14</td>
</tr>
<tr>
<td>Residual</td>
<td>.071670369</td>
<td>17</td>
<td>.004215904</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>2.40366215</td>
<td>18</td>
<td>.133536786</td>
<td>R-squared = 0.9702</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Model</td>
<td>2.33199178</td>
<td>1</td>
<td>2.33199178</td>
<td>F(1, 17) = 553.14</td>
</tr>
<tr>
<td>Residual</td>
<td>.071670369</td>
<td>17</td>
<td>.004215904</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>2.40366215</td>
<td>18</td>
<td>.133536786</td>
<td>R-squared = 0.9702</td>
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<td>Model</td>
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<td>2.33199178</td>
<td>F(1, 17) = 553.14</td>
</tr>
<tr>
<td>Residual</td>
<td>.071670369</td>
<td>17</td>
<td>.004215904</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>2.40366215</td>
<td>18</td>
<td>.133536786</td>
<td>R-squared = 0.9702</td>
</tr>
</tbody>
</table>

| csales     | Coef. | Std. Err. | t    | P>|t| | [95% Conf. Interval] |
|------------|-------|-----------|------|------|---------------------|
| isales     | .1605233 | .0068253 | 23.52 | 0.000 | .1461233 .1749234 |
| _cons      | 1.738946 | 1.432674 | 1.21 | 0.241 | -1.283732 .4761624 |
| rho        | .9588209 |           |      |      |                     |
```

Durbin-Watson statistic (original) 0.734728
Durbin-Watson statistic (transformed) 1.724419
We noted in *Options* that, with the default computation of $\rho$, the Cochrane–Orcutt method produces an estimate of $\rho$ that minimizes the sum-of-squared residuals—the same criterion as the `ssesearch` option. Given that the two methods produce the same results, why would the search method ever be preferred? It turns out that the back-and-forth iterations used by Cochrane–Orcutt may have difficulty converging if the value of $\rho$ is large. Using the same data, the Cochrane–Orcutt iterative procedure requires more than 350 iterations to converge, and a higher tolerance must be specified to prevent premature convergence:

```
. prais csales isales, corc tol(1e-9) iterate(500)
```

```
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.6312
Iteration 2:  rho = 0.6866
(output omitted)
Iteration 377: rho = 0.9588
Iteration 378: rho = 0.9588
Iteration 379: rho = 0.9588
```

Cochrane-Orcutt AR(1) regression -- iterated estimates

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>2.33199171</td>
<td>1</td>
<td>2.33199171</td>
<td>F(1, 17) = 553.14</td>
</tr>
<tr>
<td>Residual</td>
<td>.071670369</td>
<td>17</td>
<td>.004215904</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>2.40366208</td>
<td>18</td>
<td>.133536782</td>
<td>R-squared = 0.9702</td>
</tr>
</tbody>
</table>

| Source | Coef. Std. Err. t P>|t|  [95% Conf. Interval] |
|--------|-----------------|-------|---------|-------------------|
| csales | .1605233 .0068253 23.52 0.000 .1461233 .1749234 |
| isales | 1.738946 1.432674 1.21 0.241 -1.283732 4.761625 |
| _cons | .9588209 |

Durbin-Watson statistic (original) 0.734728
Durbin-Watson statistic (transformed) 1.724419

Once convergence is achieved, the two methods produce identical results.
Stored results

`prais` stores the following in `e()`:

Scalars

- `e(N)`: number of observations
- `e(N_gaps)`: number of gaps
- `e(mss)`: model sum of squares
- `e(df_m)`: model degrees of freedom
- `e(rss)`: residual sum of squares
- `e(df_r)`: residual degrees of freedom
- `e(r2)`: $R^2$
- `e(r2_adj)`: adjusted $R^2$
- `e(F)`: $F$ statistic
- `e(rmse)`: root mean squared error
- `e(ll)`: log likelihood
- `e(N_clust)`: number of clusters
- `e(rho)`: autocorrelation parameter $\rho$
- `e(dw)`: Durbin–Watson $d$ statistic for transformed regression
- `e(dw_0)`: Durbin–Watson $d$ statistic of untransformed regression
- `e(rank)`: rank of $e(V)$
- `e(tol)`: target tolerance
- `e(max_ic)`: maximum number of iterations
- `e(ic)`: number of iterations

Macros

- `e(cmd)`: `prais`
- `e(cmdline)`: command as typed
- `e(depvar)`: name of dependent variable
- `e(title)`: title in estimation output
- `e(clustvar)`: name of cluster variable
- `e(cons)`: `noconstant` or not reported
- `e(method)`: `twostep`, `iterated`, or `SSE search`
- `e(tranmeth)`: `corc` or `prais`
- `e(rhotype)`: method specified in `rhotype()` option
- `e(vce)`: `vcetype` specified in `vce()`
- `e(vcetype)`: title used to label Std. Err.
- `e(properties)`: `b V` or `b V(modelbased)`
- `e(predict)`: program used to implement `predict`
- `e(marginsok)`: predictions allowed by `margins`
- `e(asbalanced)`: factor variables `fvset` as `asbalanced`
- `e(asobserved)`: factor variables `fvset` as `asobserved`

Matrices

- `e(b)`: coefficient vector
- `e(V)`: variance–covariance matrix of the estimators
- `e(V_modelbased)`: model-based variance

Functions

- `e(sample)`: estimation sample

Methods and formulas

Consider the command `prais y x z`. The 0th iteration is obtained by estimating $a$, $b$, and $c$ from the standard linear regression:

$$y_t = ax_t + bz_t + c + u_t$$

An estimate of the correlation in the residuals is then obtained. By default, `prais` uses the auxiliary regression:

$$u_t = \rho u_{t-1} + e_t$$

This can be changed to any computation noted in the `rhotype()` option.
Next we apply a Cochrane–Orcutt transformation (1) for observations \( t = 2, \ldots, n \)
\[
y_t - \rho y_{t-1} = a(x_t - \rho x_{t-1}) + b(z_t - \rho z_{t-1}) + c(1 - \rho) + v_t
\] (1)

and the transformation (1′) for \( t = 1 \)
\[
\sqrt{1 - \rho^2} y_1 = a(\sqrt{1 - \rho^2} x_1) + b(\sqrt{1 - \rho^2} z_1) + c\sqrt{1 - \rho^2} + \sqrt{1 - \rho^2} v_1
\] (1′)

Thus the differences between the Cochrane–Orcutt and the Prais–Winsten methods are that the latter uses (1′) in addition to (1), whereas the former uses only (1), necessarily decreasing the sample size by one.

Equations (1) and (1′) are used to transform the data and obtain new estimates of \( a, b, \) and \( c \).

When the `twostep` option is specified, the estimation process stops at this point and reports these estimates. Under the default behavior of iterating to convergence, this process is repeated until the change in the estimate of \( \rho \) is within a specified tolerance.

The new estimates are used to produce fitted values
\[
\hat{y}_t = \hat{a}x_t + \hat{b}z_t + \hat{c}
\]
and then \( \rho \) is reestimated using, by default, the regression defined by
\[
y_t - \hat{y}_t = \rho(y_{t-1} - \hat{y}_{t-1}) + u_t
\] (2)

We then reestimate (1) by using the new estimate of \( \rho \) and continue to iterate between (1) and (2) until the estimate of \( \rho \) converges.

Convergence is declared after `iterate()` iterations or when the absolute difference in the estimated correlation between two iterations is less than `tol()`; see [R] `Maximize`. Sargan (1964) has shown that this process will always converge.

Under the `sesearch` option, a combined quadratic and bisection search using golden sections searches for the value of \( \rho \) that minimizes the sum-of-squared residuals from the transformed equation. The transformation may be either the Cochrane–Orcutt (1 only) or the Prais–Winsten (1 and 1′).

All reported statistics are based on the \( \rho \)-transformed variables, and \( \rho \) is assumed to be estimated without error. See Judge et al. (1985) for details.

The Durbin–Watson \( d \) statistic reported by `prais` and `estat dwatson` is
\[
d = \frac{\sum_{j=1}^{n-1} (u_{j+1} - u_j)^2}{\sum_{j=1}^{n} u_j^2}
\]
where \( u_j \) represents the residual of the \( j \)th observation.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] `_robust`, particularly Introduction and Methods and formulas.
All estimates from `prais` are conditional on the estimated value of $\rho$. Robust variance estimates here are robust only to heteroskedasticity and are not generally robust to misspecification of the functional form or omitted variables. The estimation of the functional form is intertwined with the estimation of $\rho$, and all estimates are conditional on $\rho$. Thus estimates cannot be robust to misspecification of functional form. For these reasons, it is probably best to interpret `vce(robust)` in the spirit of White’s original paper on estimation of heteroskedastic-consistent covariance matrices.

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Sigbert Jon Prais (1928–2014) was born in Frankfurt and moved to Britain in 1934 as a refugee. After earning degrees at the universities of Birmingham and Cambridge and serving in various posts in research and industry, he settled at the National Institute of Economic and Social Research. Prais’s interests extended widely across economics, including studies of the influence of education on economic progress.

Christopher Blake Winsten (1923–2005) was born in Welwyn Garden City, England; the son of the writer Stephen Winsten and the painter and sculptress Clare Blake. He was educated at the University of Cambridge and worked with the Cowles Commission at the University of Chicago and at the universities of Oxford, London (Imperial College) and Essex, making many contributions to economics and statistics, including the Prais–Winsten transformation and joint authorship of a celebrated monograph on transportation economics.

Donald Cochrane (1917–1983) was an Australian economist and econometrician. He was born in Melbourne and earned degrees at Melbourne and Cambridge. After wartime service in the Royal Australian Air Force, he held chairs at Melbourne and Monash, being active also in work for various international organizations and national committees.

Guy Henderson Orcutt (1917–2006) was born in Michigan and earned degrees in physics and economics at the University of Michigan. He worked at Harvard, the University of Wisconsin, and Yale. He contributed to econometrics and economics in several fields, most distinctively in developing microanalytical models of economic behavior.

References


Also see

[TS] prais postestimation — Postestimation tools for prais
[TS] arima — ARIMA, ARMAX, and other dynamic regression models
[TS] mswitch — Markov-switching regression models
[TS] tsset — Declare data to be time-series data
[RS] regress — Linear regression
[RS] regress postestimation time series — Postestimation tools for regress with time series
[U] 20 Estimation and postestimation commands