prais — Prais–Winsten and Cochrane–Orcutt regression

Description

prais uses the generalized least-squares method to estimate the parameters in a linear regression model in which the errors are serially correlated. Specifically, the errors are assumed to follow a first-order autoregressive process.

Quick start

Prais–Winsten regression of y on x estimating the autocorrelation parameter by a single-lag OLS regression of residuals using tsset data

```
prais y x
```

Same as above, but estimate the autocorrelation parameter using a single-lead OLS of residuals

```
prais y x, rhotype(freg)
```

Same as above, but estimate the autocorrelation parameter using autocorrelation of residuals

```
prais y x, rhotype(tscorr)
```

Cochrane–Orcutt regression of y on x with first-order serial correlation

```
prais y x, corc
```

Menu

Statistics > Time series > Prais–Winsten regression

Syntax

prais *depvar* [*indepvars*] [*if*] [*in*] [, *options*]

options	Description
Model	
rhotype(regress)	base ρ on single-lag OLS of residuals; the default
rhotype(freg)	base ρ on single-lead OLS of residuals
rhotype(tscorr)	base ρ on autocorrelation of residuals
rhotype(dw)	base ρ on autocorrelation based on Durbin–Watson
rhotype(theil)	base ρ on adjusted autocorrelation
rhotype(nagar)	base ρ on adjusted Durbin–Watson
corc	use Cochrane–Orcutt transformation
ssesearch	search for ρ that minimizes SSE
twostep	stop after the first iteration
noconstant	suppress constant term
hascons	has user-defined constant
savespace	conserve memory during estimation
SE/Robust	
vce(*vcetype*)	*vcetype* may be ols, robust, cluster *clustvar*, hc2, or hc3
Reporting	
level(#)	set confidence level; default is level(95)
nodw	do not report the Durbin–Watson statistic
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
optimize_options	control the optimization process; seldom used
coeflegend	display legend instead of statistics

You must tsset your data before using prais; see [TS] **tsset**.

indepvars may contain factor variables; see **[U] 11.4.3 Factor variables**.

depvar and *indepvars* may contain time-series operators; see **[U] 11.4.4 Time-series varlists**.

by, collect, fp, rolling, and statsby are allowed; see **[U] 11.1.10 Prefix commands**.

coeflegend does not appear in the dialog box.

See **[U] 20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

`rhotype(`*rhomethod*`)` selects a specific computation for the autocorrelation parameter ρ, where *rhomethod* can be

`regress`	$\rho_{\text{reg}} = \beta$ from the residual regression $\epsilon_t = \beta\epsilon_{t-1}$
`freg`	$\rho_{\text{freg}} = \beta$ from the residual regression $\epsilon_t = \beta\epsilon_{t+1}$
`tscorr`	$\rho_{\text{tscorr}} = \boldsymbol{\epsilon}'\boldsymbol{\epsilon}_{t-1}/\boldsymbol{\epsilon}'\boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon}$ is the vector of residuals
`dw`	$\rho_{\text{dw}} = 1 - \text{dw}/2$, where dw is the Durbin–Watson d statistic
`theil`	$\rho_{\text{theil}} = \rho_{\text{tscorr}}(N-k)/N$
`nagar`	$\rho_{\text{nagar}} = (\rho_{\text{dw}} * N^2 + k^2)/(N^2 - k^2)$

The `prais` estimator can use any consistent estimate of ρ to transform the equation, and each of these estimates meets that requirement. The default is `regress`, which produces the minimum sum-of-squares solution (`ssesearch` option) for the Cochrane–Orcutt transformation—none of these computations will produce the minimum sum-of-squares solution for the full Prais–Winsten transformation. See Judge et al. (1985) for a discussion of each estimate of ρ.

`corc` specifies that the Cochrane–Orcutt transformation be used to estimate the equation. With this option, the Prais–Winsten transformation of the first observation is not performed, and the first observation is dropped when estimating the transformed equation; see *Methods and formulas* below.

`ssesearch` specifies that a search be performed for the value of ρ that minimizes the sum-of-squared errors of the transformed equation (Cochrane–Orcutt or Prais–Winsten transformation). The search method is a combination of quadratic and modified bisection searches using golden sections.

`twostep` specifies that `prais` stop on the first iteration after the equation is transformed by ρ—the two-step efficient estimator. Although iterating these estimators to convergence is customary, they are efficient at each step.

`noconstant`; see [R] **Estimation options**.

`hascons` indicates that a user-defined constant, or a set of variables that in linear combination forms a constant, has been included in the regression. For some computational concerns, see the discussion in [R] **regress**.

`savespace` specifies that `prais` attempt to save as much space as possible by retaining only those variables required for estimation. The original data are restored after estimation. This option is rarely used and should be used only if there is insufficient space to fit a model without the option.

SE/Robust

`vce(`*vcetype*`)` specifies the estimator for the variance–covariance matrix of the estimator; see [R] ***vce_option***.

`vce(ols)`, the default, uses the standard variance estimator for ordinary least-squares regression.

`vce(robust)` specifies to use the Huber/White/sandwich estimator.

`vce(cluster` *clustvar*`)` specifies to use the intragroup correlation estimator.

`vce(hc2)` and `vce(hc3)` specify an alternative bias correction for the `vce(robust)` variance calculation; for more information, see [R] **regress**. You may specify only one of `vce(hc2)`, `vce(hc3)`, or `vce(robust)`.

All estimates from prais are conditional on the estimated value of ρ. Robust variance estimates here are robust only to heteroskedasticity and are not generally robust to misspecification of the functional form or omitted variables. The estimation of the functional form is intertwined with the estimation of ρ, and all estimates are conditional on ρ. Thus estimates cannot be robust to misspecification of functional form. For these reasons, it is probably best to interpret vce(robust) in the spirit of White's (1980) original paper on estimation of heteroskedastic-consistent covariance matrices.

Reporting

level(#); see [R] **Estimation options**.

nodw suppresses reporting of the Durbin–Watson statistic.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(*style*), cformat(%*fmt*), pformat(%*fmt*), sformat(%*fmt*), and nolstretch; see [R] **Estimation options**.

Optimization

optimize_options: iterate(#), [no]log, tolerance(#). iterate() specifies the maximum number of iterations. log/nolog specifies whether to show the iteration log (see set iterlog in [R] ***set iter***). tolerance() specifies the tolerance for the coefficient vector; tolerance(1e-6) is the default. These options are seldom used.

The following option is available with prais but is not shown in the dialog box:

coeflegend; see [R] **Estimation options**.

Remarks and examples

prais fits a linear regression of *depvar* on *indepvars* that is corrected for first-order serially correlated residuals by using the Prais–Winsten (1954) transformed regression estimator, the Cochrane–Orcutt (1949) transformed regression estimator, or a version of the search method suggested by Hildreth and Lu (1960). Davidson and MacKinnon (1993) provide theoretical details on the three methods (see pages 333–335 for the latter two and pages 343–351 for Prais–Winsten). See Becketti (2020) for more examples showing how to use prais.

The most common autocorrelated error process is the first-order autoregressive process. Under this assumption, the linear regression model can be written as

$$y_t = \mathbf{x}_t\boldsymbol{\beta} + u_t$$

where the errors satisfy

$$u_t = \rho\, u_{t-1} + e_t$$

and the e_t are independent and identically distributed as $N(0, \sigma^2)$. The covariance matrix $\boldsymbol{\Psi}$ of the error term u can then be written as

$$\boldsymbol{\Psi} = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{T-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{bmatrix}$$

The Prais–Winsten estimator is a generalized least-squares (GLS) estimator. The Prais–Winsten method (as described in Judge et al. 1985) is derived from the AR(1) model for the error term described above. Whereas the Cochrane–Orcutt method uses a lag definition and loses the first observation in the iterative method, the Prais–Winsten method preserves that first observation. In small samples, this can be a significant advantage.

❑ Technical note

To fit a model with autocorrelated errors, you must specify your data as time series and have (or create) a variable denoting the time at which an observation was collected. The data for the regression should be equally spaced in time.

❑

▷ Example 1

Say that we wish to fit a time-series model of `usr` on `idle` but are concerned that the residuals may be serially correlated. We will declare the variable `t` to represent time by typing

```
. use https://www.stata-press.com/data/r19/idle
. tsset t
Time variable: t, 1 to 30
        Delta: 1 unit
```

We can obtain Cochrane–Orcutt estimates by specifying the `corc` option:

```
. prais usr idle, corc
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.3518
  (iteration log omitted)
Iteration 13: rho = 0.5708
Cochrane-Orcutt AR(1) regression with iterated estimates
      Source |       SS           df       MS      Number of obs   =        29
-------------+----------------------------------   F(1, 27)        =      6.49
       Model |  40.1309584         1  40.1309584   Prob > F        =    0.0168
    Residual |  166.898474        27  6.18142498   R-squared       =    0.1938
-------------+----------------------------------   Adj R-squared   =    0.1640
       Total |  207.029433        28  7.39390831   Root MSE        =    2.4862

------------------------------------------------------------------------------
         usr | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        idle |  -.1254511   .0492356    -2.55   0.017    -.2264742    -.024428
       _cons |   14.54641   4.272299     3.40   0.002      5.78038    23.31245
-------------+----------------------------------------------------------------
         rho |   .5707918
------------------------------------------------------------------------------
Durbin-Watson statistic (original)    = 1.295766
Durbin-Watson statistic (transformed) = 1.466222
```

The fitted model is

$$\texttt{usr}_t = -0.1255\,\texttt{idle}_t + 14.55 + u_t \qquad \text{and} \qquad u_t = 0.5708\,u_{t-1} + e_t$$

We can also fit the model with the Prais–Winsten method,

```
. prais usr idle
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.3518
  (iteration log omitted)
Iteration 14: rho = 0.5535
Prais-Winsten AR(1) regression with iterated estimates
      Source |       SS           df       MS      Number of obs   =        30
-------------+----------------------------------   F(1, 28)        =      7.12
       Model |  43.0076941         1  43.0076941   Prob > F        =    0.0125
    Residual |  169.165739        28  6.04163354   R-squared       =    0.2027
-------------+----------------------------------   Adj R-squared   =    0.1742
       Total |  212.173433        29  7.31632528   Root MSE        =     2.458

------------------------------------------------------------------------------
         usr | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        idle |  -.1356522   .0472195    -2.87   0.008    -.2323769   -.0389275
       _cons |   15.20415   4.160391     3.65   0.001     6.681978    23.72633
-------------+----------------------------------------------------------------
         rho |   .5535476
------------------------------------------------------------------------------
Durbin-Watson statistic (original)    = 1.295766
Durbin-Watson statistic (transformed) = 1.476004
```

where the Prais–Winsten fitted model is

$$\texttt{usr}_t = -.1357\,\texttt{idle}_t + 15.20 + u_t \qquad \text{and} \qquad u_t = .5535\,u_{t-1} + e_t$$

As the results indicate, for these data there is little difference between the Cochrane–Orcutt and Prais–Winsten estimators, whereas the OLS estimate of the slope parameter is substantially different.

◁

▷ Example 2

We have data on quarterly sales, in millions of dollars, for 5 years, and we would like to use this information to model sales for company X. First, we fit a linear model by OLS and obtain the Durbin–Watson statistic by using `estat dwatson`; see [R] **regress postestimation time series**.

```
. use https://www.stata-press.com/data/r19/qsales
. regress csales isales
      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(1, 18)        =  14888.15
       Model |  110.256901         1  110.256901   Prob > F        =    0.0000
    Residual |  .133302302        18  .007405683   R-squared       =    0.9988
-------------+----------------------------------   Adj R-squared   =    0.9987
       Total |  110.390204        19  5.81001072   Root MSE        =    .08606

------------------------------------------------------------------------------
      csales | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      isales |   .1762828   .0014447   122.02   0.000     .1732475    .1793181
       _cons |  -1.454753   .2141461    -6.79   0.000    -1.904657   -1.004849
------------------------------------------------------------------------------
. estat dwatson
Durbin-Watson d-statistic(  2,    20) =  .7347276
```

Because the Durbin–Watson statistic is far from 2 (the expected value under the null hypothesis of no serial correlation) and well below the 5% lower limit of 1.2, we conclude that the disturbances are serially correlated. (Upper and lower bounds for the d statistic can be found in most econometrics texts; for example, Harvey [1990]. The bounds have been derived for only a limited combination of regressors and observations.) To reinforce this conclusion, we use two other tests to test for serial correlation in the error distribution.

```
. estat bgodfrey, lags(1)
Breusch-Godfrey LM test for autocorrelation
```

lags(p)	chi2	df	Prob > chi2
1	7.998	1	0.0047

```
                        H0: no serial correlation
. estat durbinalt
Durbin's alternative test for autocorrelation
```

lags(p)	chi2	df	Prob > chi2
1	11.329	1	0.0008

```
                        H0: no serial correlation
```

`estat bgodfrey` reports the Breusch–Godfrey Lagrange multiplier test statistic, and `estat durbinalt` reports the Durbin's alternative test statistic. Both tests give a small p-value and thus reject the null hypothesis of no serial correlation. These two tests are asymptotically equivalent when testing for AR(1) process. See [R] **regress postestimation time series** if you are not familiar with these two tests.

We correct for autocorrelation with the `ssesearch` option of `prais` to search for the value of ρ that minimizes the sum-of-squared residuals of the Cochrane–Orcutt transformed equation. Normally, the default Prais–Winsten transformations is used with such a small dataset, but the less-efficient Cochrane–Orcutt transformation allows us to demonstrate an aspect of the estimator's convergence.

```
. prais csales isales, corc ssesearch
Iteration 1:  rho =  0.8944, criterion = -.07298558
Iteration 2:  rho =  0.8944, criterion = -.07298558
  (iteration log omitted)
Iteration 15: rho =  0.9588, criterion = -.07167037
Cochrane-Orcutt AR(1) regression with SSE search estimates
```

Source	SS	df	MS
Model	2.33199178	1	2.33199178
Residual	.071670369	17	.004215904
Total	2.40366215	18	.133536786

Number of obs	=	19
F(1, 17)	=	553.14
Prob > F	=	0.0000
R-squared	=	0.9702
Adj R-squared	=	0.9684
Root MSE	=	.06493

csales	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]	
isales	.1605233	.0068253	23.52	0.000	.1461233	.1749234
_cons	1.738946	1.432674	1.21	0.241	-1.283732	4.761624
rho	.9588209					

```
Durbin-Watson statistic (original)    = 0.734728
Durbin-Watson statistic (transformed) = 1.724419
```

We noted in *Options* that, with the default computation of ρ, the Cochrane–Orcutt method produces an estimate of ρ that minimizes the sum-of-squared residuals—the same criterion as the `ssesearch` option. Given that the two methods produce the same results, why would the search method ever be preferred? It turns out that the back-and-forth iterations used by Cochrane–Orcutt may have difficulty converging if the value of ρ is large. Using the same data, the Cochrane–Orcutt iterative procedure requires more than 350 iterations to converge, and a higher tolerance must be specified to prevent premature convergence:

```
. prais csales isales, corc tol(1e-9) iterate(500)
Iteration 0:   rho = 0.0000
Iteration 1:   rho = 0.6312
Iteration 2:   rho = 0.6866
  (iteration log omitted)
Iteration 377: rho = 0.9588
Iteration 378: rho = 0.9588
Iteration 379: rho = 0.9588
Cochrane-Orcutt AR(1) regression with iterated estimates

      Source |       SS           df       MS      Number of obs   =        19
-------------+----------------------------------   F(1, 17)        =    553.14
       Model |  2.33199171         1  2.33199171   Prob > F        =    0.0000
    Residual |  .071670369        17  .004215904   R-squared       =    0.9702
-------------+----------------------------------   Adj R-squared   =    0.9684
       Total |  2.40366208        18  .133536782   Root MSE        =    .06493

------------------------------------------------------------------------------
      csales | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      isales |   .1605233   .0068253    23.52   0.000     .1461233    .1749234
       _cons |   1.738946   1.432674     1.21   0.241    -1.283732    4.761625
-------------+----------------------------------------------------------------
         rho |   .9588209
------------------------------------------------------------------------------
Durbin-Watson statistic (original)    = 0.734728
Durbin-Watson statistic (transformed) = 1.724419
```

Once convergence is achieved, the two methods produce identical results.

◁

Stored results

prais stores the following in e():

Scalars

e(N)	number of observations
e(N_gaps)	number of gaps
e(mss)	model sum of squares
e(df_m)	model degrees of freedom
e(rss)	residual sum of squares
e(df_r)	residual degrees of freedom
e(r2)	R^2
e(r2_a)	adjusted R^2
e(F)	F statistic
e(rmse)	root mean squared error
e(ll)	log likelihood
e(N_clust)	number of clusters
e(rho)	autocorrelation parameter ρ
e(dw)	Durbin–Watson d statistic for transformed regression
e(dw_0)	Durbin–Watson d statistic of untransformed regression
e(rank)	rank of e(V)
e(tol)	target tolerance
e(max_ic)	maximum number of iterations
e(ic)	number of iterations

Macros

e(cmd)	prais
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(title)	title in estimation output
e(clustvar)	name of cluster variable
e(cons)	noconstant or not reported
e(method)	twostep, iterated, or SSE search
e(tranmeth)	corc or prais
e(rhotype)	method specified in rhotype() option
e(vce)	*vcetype* specified in vce()
e(vcetype)	title used to label Std. err.
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices

e(b)	coefficient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions

e(sample)	estimation sample

In addition to the above, the following is stored in r():

Matrices

r(table)	matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

Consider the command '`prais` y x z'. The 0th iteration is obtained by estimating a, b, and c from the standard linear regression:

$$y_t = ax_t + bz_t + c + u_t$$

An estimate of the correlation in the residuals is then obtained. By default, `prais` uses the auxiliary regression:

$$u_t = \rho u_{t-1} + e_t$$

This can be changed to any computation noted in the `rhotype()` option.

Next we apply a Cochrane–Orcutt transformation (1) for observations $t = 2, \ldots, n$

$$y_t - \rho y_{t-1} = a(x_t - \rho x_{t-1}) + b(z_t - \rho z_{t-1}) + c(1-\rho) + v_t \tag{1}$$

and the transformation ($1'$) for $t = 1$

$$\sqrt{1-\rho^2}y_1 = a(\sqrt{1-\rho^2}x_1) + b(\sqrt{1-\rho^2}z_1) + c\sqrt{1-\rho^2} + \sqrt{1-\rho^2}v_1 \tag{$1'$}$$

Thus the differences between the Cochrane–Orcutt and the Prais–Winsten methods are that the latter uses ($1'$) in addition to (1), whereas the former uses only (1), necessarily decreasing the sample size by one.

Equations (1) and ($1'$) are used to transform the data and obtain new estimates of a, b, and c.

When the `twostep` option is specified, the estimation process stops at this point and reports these estimates. Under the default behavior of iterating to convergence, this process is repeated until the change in the estimate of ρ is within a specified tolerance.

The new estimates are used to produce fitted values

$$\widehat{y}_t = \widehat{a}x_t + \widehat{b}z_t + \widehat{c}$$

and then ρ is reestimated using, by default, the regression defined by

$$y_t - \widehat{y}_t = \rho(y_{t-1} - \widehat{y}_{t-1}) + u_t \tag{2}$$

We then reestimate (1) by using the new estimate of ρ and continue to iterate between (1) and (2) until the estimate of ρ converges.

Convergence is declared after `iterate()` iterations or when the absolute difference in the estimated correlation between two iterations is less than `tol()`; see [R] **Maximize**. Sargan (1964) has shown that this process will always converge.

Under the `ssesearch` option, a combined quadratic and bisection search using golden sections searches for the value of ρ that minimizes the sum-of-squared residuals from the transformed equation. The transformation may be either the Cochrane–Orcutt (1 only) or the Prais–Winsten (1 and $1'$).

All reported statistics are based on the ρ-transformed variables, and ρ is assumed to be estimated without error. See Judge et al. (1985) for details.

The Durbin–Watson d statistic reported by `prais` and `estat dwatson` is

$$d = \frac{\sum_{j=1}^{n-1}(u_{j+1} - u_j)^2}{\sum_{j=1}^{n} u_j^2}$$

where u_j represents the residual of the jth observation.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster` *clustvar*`)`, respectively. See [P] **_robust**, particularly *Introduction* and *Methods and formulas*.

All estimates from `prais` are conditional on the estimated value of ρ. Robust variance estimates here are robust only to heteroskedasticity and are not generally robust to misspecification of the functional form or omitted variables. The estimation of the functional form is intertwined with the estimation of ρ, and all estimates are conditional on ρ. Thus estimates cannot be robust to misspecification of functional form. For these reasons, it is probably best to interpret `vce(robust)` in the spirit of White's original paper on estimation of heteroskedastic-consistent covariance matrices.

Acknowledgment

We thank Richard Dickens of the Department of Economics, University of Sussex, UK, for testing and assistance with an early version of this command.

Sigbert Jon Prais (1928–2014) was born in Frankfurt and moved to Britain in 1934 as a refugee. After earning degrees at the universities of Birmingham and Cambridge and serving in various posts in research and industry, he settled at the National Institute of Economic and Social Research. Prais's interests extended widely across economics, including studies of the influence of education on economic progress.

Christopher Blake Winsten (1923–2005) was born in Welwyn Garden City, England; the son of the writer Stephen Winsten and the painter and sculptress Clare Blake. He was educated at the University of Cambridge and worked with the Cowles Commission at the University of Chicago and at the universities of Oxford, London (Imperial College) and Essex, making many contributions to economics and statistics, including the Prais–Winsten transformation and joint authorship of a celebrated monograph on transportation economics.

Donald Cochrane (1917–1983) was an Australian economist and econometrician. He was born in Melbourne and earned degrees at Melbourne and Cambridge. After wartime service in the Royal Australian Air Force, he held chairs at Melbourne and Monash, being active also in work for various international organizations and national committees.

Guy Henderson Orcutt (1917–2006) was born in Michigan and earned degrees in physics and economics at the University of Michigan. He worked at Harvard, the University of Wisconsin, and Yale. He contributed to econometrics and economics in several fields, most distinctively in developing microanalytical models of economic behavior.

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Also see

[TS] **prais postestimation** — Postestimation tools for prais

[TS] **arima** — ARIMA, ARMAX, and other dynamic regression models

[TS] **mswitch** — Markov-switching regression models

[TS] **tsset** — Declare data to be time-series data

[R] **regress** — Linear regression

[R] **regress postestimation time series** — Postestimation tools for regress with time series

[U] 20 Estimation and postestimation commands

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