

## mgarch — Multivariate GARCH models

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## Description

`mgarch` estimates the parameters of multivariate generalized autoregressive conditional-heteroskedasticity (MGARCH) models. MGARCH models allow both the conditional mean and the conditional covariance to be dynamic.

The general MGARCH model is so flexible that not all the parameters can be estimated. For this reason, there are many MGARCH models that parameterize the problem more parsimoniously.

`mgarch` implements four commonly used parameterizations: the diagonal vech model, the constant conditional correlation model, the dynamic conditional correlation model, and the time-varying conditional correlation model.

## Syntax

```
mgarch model eq [eq ... eq] [if] [in] [, ...]
```

Family	<i>model</i>
Vech	
diagonal vech	<code>dvech</code>
Conditional correlation	
constant conditional correlation	<code>ccc</code>
dynamic conditional correlation	<code>dcc</code>
varying conditional correlation	<code>vcc</code>

## Remarks and examples

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## An introduction to MGARCH models

Multivariate GARCH models allow the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure and allow the conditional mean to follow a vector-autoregressive (VAR) structure.

The general MGARCH model is too flexible for most problems. There are many restricted MGARCH models in the literature because there is no parameterization that always provides an optimal trade-off between flexibility and parsimony.

`mgarch` implements four commonly used parameterizations: the diagonal vech (DVECH) model, the constant conditional correlation (CCC) model, the dynamic conditional correlation (DCC) model, and the time-varying conditional correlation (VCC) model.

Bollerslev, Engle, and Wooldridge (1988); Bollerslev, Engle, and Nelson (1994); Bauwens, Laurent, and Rombouts (2006); Silvennoinen and Teräsvirta (2009); and Engle (2009) provide general introductions to MGARCH models. We provide a quick introduction organized around the models implemented in `mgarch`.

We give a formal definition of the general MGARCH model to establish notation that facilitates comparisons of the models. The general MGARCH model is given by

$$\begin{aligned} \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t &= \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t \end{aligned}$$

where

$\mathbf{y}_t$  is an  $m \times 1$  vector of dependent variables;

$\mathbf{C}$  is an  $m \times k$  matrix of parameters;

$\mathbf{x}_t$  is a  $k \times 1$  vector of independent variables, which may contain lags of  $\mathbf{y}_t$ ;

$\mathbf{H}_t^{1/2}$  is the Cholesky factor of the time-varying conditional covariance matrix  $\mathbf{H}_t$ ; and

$\boldsymbol{\nu}_t$  is an  $m \times 1$  vector of zero-mean, unit-variance, and independent and identically distributed innovations.

In the general MGARCH model,  $\mathbf{H}_t$  is a matrix generalization of univariate GARCH models. For example, in a general MGARCH model with one autoregressive conditional heteroskedastic (ARCH) term and one GARCH term,

$$\text{vech}(\mathbf{H}_t) = \mathbf{s} + \mathbf{A}\text{vech}(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}) + \mathbf{B}\text{vech}(\mathbf{H}_{t-1}) \quad (1)$$

where the `vech()` function stacks the unique elements that lie on or below the main diagonal in a symmetric matrix into a vector,  $\mathbf{s}$  is a vector of parameters, and  $\mathbf{A}$  and  $\mathbf{B}$  are conformable matrices of parameters. Because this model uses the `vech()` function to extract and model the unique elements of  $\mathbf{H}_t$ , it is also known as the VEC model.

Because it is a conditional covariance matrix,  $\mathbf{H}_t$  must be positive definite. Equation (1) can be used to show that the parameters in  $\mathbf{s}$ ,  $\mathbf{A}$ , and  $\mathbf{B}$  are not uniquely identified and that further restrictions must be placed on  $\mathbf{s}$ ,  $\mathbf{A}$ , and  $\mathbf{B}$  to ensure that  $\mathbf{H}_t$  is positive definite for all  $t$ .

The various MGARCH models proposed in the literature differ in how they trade off flexibility and parsimony in their specifications for  $\mathbf{H}_t$ . Increased flexibility allows a model to capture more complex  $\mathbf{H}_t$  processes. Increased parsimony makes parameter estimation feasible for more datasets. An important measure of the flexibility–parsimony trade-off is how fast the number of model parameters increases with the number of time series  $m$ , because many applied models use multiple time series.

## Diagonal vech MGARCH models

Bollerslev, Engle, and Wooldridge (1988) derived the diagonal vech (DVECH) model by restricting  $\mathbf{A}$  and  $\mathbf{B}$  to be diagonal. Although the DVECH model is much more parsimonious than the general model, it can only handle a few series because the number of parameters grows quadratically with the number of series. For example, there are  $3m(m+1)/2$  parameters in a DVECH(1,1) model for  $\mathbf{H}_t$ .

Despite the large number of parameters, the diagonal structure implies that each conditional variance and each conditional covariance depends on its own past but not on the past of the other conditional variances and covariances. Formally, in the DVECH(1,1) model each element of  $\mathbf{H}_t$  is modeled by

$$h_{ij,t} = s_{ij} + a_{ij}\epsilon_{i,(t-1)}\epsilon_{j,(t-1)} + b_{ij}h_{ij,(t-1)}$$

Parameter estimation can be difficult because it requires that  $\mathbf{H}_t$  be positive definite for each  $t$ . The requirement that  $\mathbf{H}_t$  be positive definite for each  $t$  imposes complicated restrictions on the off-diagonal elements.

See [TS] [mgarch dvech](#) for more details about this model.

## Conditional correlation MGARCH models

Conditional correlation (CC) models use nonlinear combinations of univariate GARCH models to represent the conditional covariances. In each of the conditional correlation models, the conditional covariance matrix is positive definite by construction and has a simple structure, which facilitates parameter estimation. CC models have a slower parameter growth rate than DVECH models as the number of time series increases.

In CC models,  $\mathbf{H}_t$  is decomposed into a matrix of conditional correlations  $\mathbf{R}_t$  and a diagonal matrix of conditional variances  $\mathbf{D}_t$ :

$$\mathbf{H}_t = \mathbf{D}_t^{1/2}\mathbf{R}_t\mathbf{D}_t^{1/2} \quad (2)$$

where each conditional variance follows a univariate GARCH process and the parameterizations of  $\mathbf{R}_t$  vary across models.

Equation (2) implies that

$$h_{ij,t} = \rho_{ij,t}\sigma_{i,t}\sigma_{j,t} \quad (3)$$

where  $\sigma_{i,t}^2$  is modeled by a univariate GARCH process. Equation (3) highlights that CC models use nonlinear combinations of univariate GARCH models to represent the conditional covariances and that the parameters in the model for  $\rho_{ij,t}$  describe the extent to which the errors from equations  $i$  and  $j$  move together.

Comparing (1) and (2) shows that the number of parameters increases more slowly with the number of time series in a CC model than in a DVECH model.

The three CC models implemented in `mgarch` differ in how they parameterize  $\mathbf{R}_t$ .

### Constant conditional correlation MGARCH model

Bollerslev (1990) proposed a CC MGARCH model in which the correlation matrix is time invariant. It is for this reason that the model is known as a constant conditional correlation (CCC) MGARCH model. Restricting  $\mathbf{R}_t$  to a constant matrix reduces the number of parameters and simplifies the estimation but may be too strict in many empirical applications.

See [TS] `mgarch ccc` for more details about this model.

### Dynamic conditional correlation MGARCH model

Engle (2002) introduced a dynamic conditional correlation (DCC) MGARCH model in which the conditional quasicorrelations  $\mathbf{R}_t$  follow a GARCH(1,1)-like process. (As described by Engle [2009] and Aielli [2009], the parameters in  $\mathbf{R}_t$  are not standardized to be correlations and are thus known as quasicorrelations.) To preserve parsimony, all the conditional quasicorrelations are restricted to follow the same dynamics. The DCC model is significantly more flexible than the CCC model without introducing an unestimable number of parameters for a reasonable number of series.

See [TS] `mgarch dcc` for more details about this model.

### Varying conditional correlation MGARCH model

Tse and Tsui (2002) derived the varying conditional correlation (VCC) MGARCH model in which the conditional correlations at each period are a weighted sum of a time-invariant component, a measure of recent correlations among the residuals, and last period's conditional correlations. For parsimony, all the conditional correlations are restricted to follow the same dynamics.

See [TS] `mgarch vcc` for more details about this model.

### Error distributions and quasimaximum likelihood

By default, `mgarch dvech`, `mgarch ccc`, `mgarch dcc`, and `mgarch vcc` estimate the parameters of MGARCH models by maximum likelihood (ML), assuming that the errors come from a multivariate normal distribution. Both the ML estimator and the quasi-maximum likelihood (QML) estimator, which drops the normality assumption, are assumed to be consistent and normally distributed in large samples; see Jeantheau (1998), Berkes and Horváth (2003), Comte and Lieberman (2003), Ling and McAleer (2003), and Fiorentini and Sentana (2007). Specify `vce(robust)` to estimate the parameters by QML. The QML parameter estimates are the same as the ML estimates, but the VCEs are different.

Based on low-level assumptions, Jeantheau (1998), Comte and Lieberman (2003), and Ling and McAleer (2003) prove that some of the ML and QML estimators implemented in `mgarch` are consistent and asymptotically normal. Based on higher-level assumptions, Fiorentini and Sentana (2007) prove that all the ML and QML estimators implemented in `mgarch` are consistent and asymptotically normal. The low-level assumption proofs specify the technical restrictions on the data-generating processes more precisely than the high-level proofs, but they do not cover as many models or cases as the high-level proofs.

It is generally accepted that there could be more low-level theoretical work done to substantiate the claims that the ML and QML estimators are consistent and asymptotically normally distributed. These widely applied estimators have been subjected to many Monte Carlo studies that show that the large-sample theory performs well in finite samples.

The `distribution(t)` option causes the `mgarch` commands to estimate the parameters of the corresponding model by ML assuming that the errors come from a multivariate Student  $t$  distribution.

The choice between the multivariate normal and the multivariate  $t$  distributions is one between robustness and efficiency. If the disturbances come from a multivariate Student  $t$ , then the ML estimates based on the multivariate Student  $t$  assumption will be consistent and efficient, while the QML estimates based on the multivariate normal assumption will be consistent but not efficient. In contrast, if the disturbances come from a well-behaved distribution that is neither multivariate Student  $t$  nor multivariate normal, then the ML estimates based on the multivariate Student  $t$  assumption will not be consistent, while the QML estimates based on the multivariate normal assumption will be consistent but not efficient.

Fiorentini and Sentana (2007) compare the ML and QML estimators implemented in `mgarch` and provide many useful technical results pertaining to the estimators.

## Treatment of missing data

`mgarch` allows for gaps due to missing data. The unconditional expectations are substituted for the dynamic components that cannot be computed because of gaps. This method of handling gaps can only handle the case in which  $g/T$  goes to zero as  $T$  goes to infinity, where  $g$  is the number of observations lost to gaps in the data and  $T$  is the number of nonmissing observations.

## References

- Aielli, G. P. 2009. Dynamic Conditional Correlations: On Properties and Estimation. Working paper, Dipartimento di Statistica, University of Florence, Florence, Italy. <http://dx.doi.org/10.2139/ssrn.1507743>.
- Baum, C. F., and S. Hurn. 2021. *Environmental Econometrics Using Stata*. College Station, TX: Stata Press.
- Bauwens, L., S. Laurent, and J. V. K. Rombouts. 2006. Multivariate GARCH models: A survey. *Journal of Applied Econometrics* 21: 79–109. <https://doi.org/10.1002/jae.842>.
- Berkes, I., and L. Horváth. 2003. The rate of consistency of the quasi-maximum likelihood estimator. *Statistics and Probability Letters* 61: 133–143. [https://doi.org/10.1016/S0167-7152\(02\)00342-5](https://doi.org/10.1016/S0167-7152(02)00342-5).
- Boffelli, S., and G. Urga. 2016. *Financial Econometrics Using Stata*. College Station, TX: Stata Press.
- Bollerslev, T. 1990. Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics* 72: 498–505. <https://doi.org/10.2307/2109358>.
- Bollerslev, T., R. F. Engle, and D. B. Nelson. 1994. ARCH models. In Vol. 4 of *Handbook of Econometrics*, ed. R. F. Engle and D. L. McFadden. Amsterdam: Elsevier. [https://doi.org/10.1016/S1573-4412\(05\)80018-2](https://doi.org/10.1016/S1573-4412(05)80018-2).
- Bollerslev, T., R. F. Engle, and J. M. Wooldridge. 1988. A capital asset pricing model with time-varying covariances. *Journal of Political Economy* 96: 116–131. <https://doi.org/10.1086/261527>.
- Comte, F., and O. Lieberman. 2003. Asymptotic theory for multivariate GARCH processes. *Journal of Multivariate Analysis* 84: 61–84. [https://doi.org/10.1016/s0047-259x\(02\)00009-x](https://doi.org/10.1016/s0047-259x(02)00009-x).
- Engle, R. F. 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics* 20: 339–350. <https://doi.org/10.1198/073500102288618487>.
- . 2009. *Anticipating Correlations: A New Paradigm for Risk Management*. Princeton, NJ: Princeton University Press.

- Fiorentini, G., and E. Sentana. 2007. On the efficiency and consistency of likelihood estimation in multivariate conditionally heteroskedastic dynamic regression models. Working paper 0713, CEMFI, Madrid, Spain. <ftp://ftp.cemfi.es/wp/07/0713.pdf>.
- Jeantheau, T. 1998. Strong consistency of estimators for multivariate ARCH models. *Economic Theory* 14: 70–86. <https://doi.org/10.1017/S0266466698141038>.
- Ling, S., and M. McAleer. 2003. Asymptotic theory for a vector ARM–GARCH model. *Economic Theory* 19: 280–310. <https://doi.org/10.1017/S0266466603192092>.
- Silvennoinen, A., and T. Teräsvirta. 2009. Multivariate GARCH models. In *Handbook of Financial Time Series*, ed. T. G. Andersen, R. A. Davis, J.-P. Kreiß, and T. Mikosch, 201–229. New York: Springer. [https://doi.org/10.1007/978-3-540-71297-8\\_9](https://doi.org/10.1007/978-3-540-71297-8_9).
- Tse, Y. K., and A. K. C. Tsui. 2002. A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business and Economic Statistics* 20: 351–362. <https://doi.org/10.1198/073500102288618496>.

### Also see

- [TS] **arch** — Autoregressive conditional heteroskedasticity (ARCH) family of estimators
- [TS] **var** — Vector autoregressive models
- [U] **20 Estimation and postestimation commands**