estat aroots — Check the stability condition of ARIMA estimates

Description

`estat aroots` checks the eigenvalue stability condition after estimating the parameters of an ARIMA model using `arima`. A graph of the eigenvalues of the companion matrices for the AR and MA polynomials is also produced.

`estat aroots` is available only after `arima`; see [TS] **arima**.

Quick start

Verify that all eigenvalues of the autoregressive polynomial lie inside the unit circle after `arima`

```
estat aroots
```

Same as above, but suppress the graph

```
estat aroots, nograph
```

Label each plotted eigenvalue with its distance from the unit circle

```
estat aroots, dlabel
```

Menu for estat

Statistics > Postestimation

Syntax

estat aroots [, *options*]

options	Description
nograph	suppress graph of eigenvalues for the companion matrices
dlabel	label eigenvalues with the distance from the unit circle
modlabel	label eigenvalues with the modulus
Grid	
nogrid	suppress polar grid circles
pgrid([...])	specify radii and appearance of polar grid circles; see Options for details
Plot	
marker_options	change look of markers (color, size, etc.)
Reference unit circle	
rlopts(*cline_options*)	affect rendition of reference unit circle
Y axis, X axis, Titles, Legend, Overall	
twoway_options	any options other than by() documented in [G-3] ***twoway_options***

collect is allowed; see **[U] 11.1.10 Prefix commands**.

Options

nograph specifies that no graph of the eigenvalues of the companion matrices be drawn.

dlabel labels each eigenvalue with its distance from the unit circle. dlabel cannot be specified with modlabel.

modlabel labels the eigenvalues with their moduli. modlabel cannot be specified with dlabel.

Grid

nogrid suppresses the polar grid circles.

pgrid([*numlist*] [, *line_options*]) determines the radii and appearance of the polar grid circles. By default, the graph includes nine polar grid circles with radii 0.1, 0.2, ..., 0.9 that have the grid line style. The *numlist* specifies the radii for the polar grid circles. The *line_options* determine the appearance of the polar grid circles; see [G-3] ***line_options***. Because the pgrid() option can be repeated, circles with different radii can have distinct appearances.

Plot

marker_options specify the look of markers. This look includes the marker symbol, the marker size, and its color and outline; see [G-3] ***marker_options***.

Reference unit circle

rlopts(*cline_options*) affect the rendition of the reference unit circle; see [G-3] ***cline_options***.

Y axis, X axis, Titles, Legend, Overall

twoway_options are any of the options documented in [G-3] ***twoway_options***, except `by()`. These include options for titling the graph (see [G-3] ***title_options***) and for saving the graph to disk (see [G-3] ***saving_option***).

Remarks and examples

Inference after `arima` requires that the variable y_t be covariance stationary. The variable y_t is covariance stationary if its first two moments exist and are time invariant. More explicitly, y_t is covariance stationary if

1. $E(y_t)$ is finite and not a function of t;
2. Var(y_t) is finite and independent of t; and
3. Cov(y_t, y_s) is a finite function of $|t - s|$ but not of t or s alone.

The stationarity of an ARMA process depends on the autoregressive (AR) parameters. If the inverse roots of the AR polynomial all lie inside the unit circle, the process is stationary, invertible, and has an infinite-order moving-average (MA) representation. Hamilton (1994, chap. 1) shows that if the modulus of each eigenvalue of the matrix $\mathbf{F}(\boldsymbol{\rho})$ is strictly less than 1, the estimated ARMA is stationary; see *Methods and formulas* for the definition of the matrix $\mathbf{F}(\boldsymbol{\rho})$.

The MA part of an ARMA process can be rewritten as an infinite-order AR process provided that the MA process is invertible. Hamilton (1994, chap. 1) shows that if the modulus of each eigenvalue of the matrix $\mathbf{F}(\boldsymbol{\theta})$ is strictly less than 1, the estimated ARMA is invertible; see *Methods and formulas* for the definition of the matrix $\mathbf{F}(\boldsymbol{\theta})$.

▷ Example 1

In this example, we check the stability condition of the SARIMA model that we fit in example 3 of [TS] **arima**. We begin by reestimating the parameters of the model.

```
. use https://www.stata-press.com/data/r19/air2
(TIMESLAB: Airline passengers)
. generate lnair = ln(air)
```

```
. arima lnair, arima(0,1,1) sarima(0,1,1,12) noconstant
(setting optimization to BHHH)
Iteration 0:  Log likelihood =  223.8437
Iteration 1:  Log likelihood = 239.80405
Iteration 2:  Log likelihood = 244.10265
Iteration 3:  Log likelihood = 244.65895
Iteration 4:  Log likelihood = 244.68945
(switching optimization to BFGS)
Iteration 5:  Log likelihood = 244.69431
Iteration 6:  Log likelihood = 244.69647
Iteration 7:  Log likelihood = 244.69651
Iteration 8:  Log likelihood = 244.69651
ARIMA regression
Sample: 14 thru 144                             Number of obs     =        131
                                                Wald chi2(2)      =      84.53
Log likelihood = 244.6965                       Prob > chi2       =     0.0000
```

DS12.lnair	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
ARMA						
ma						
L1.	-.4018324	.0730307	-5.50	0.000	-.5449698	-.2586949
ARMA12						
ma						
L1.	-.5569342	.0963129	-5.78	0.000	-.745704	-.3681644
/sigma	.0367167	.0020132	18.24	0.000	.0327708	.0406625

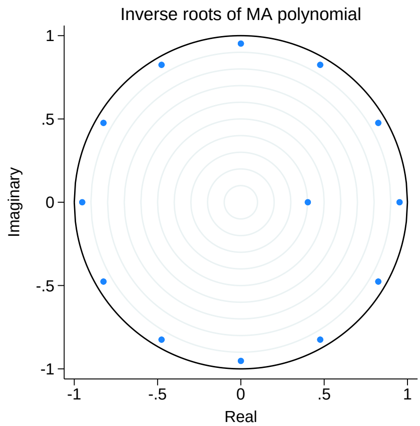
```
Note: The test of the variance against zero is one sided, and the two-sided
      confidence interval is truncated at zero.
```

We can now use `estat aroots` to check the stability condition of the MA part of the model.

```
. estat aroots
   Eigenvalue stability condition
```

Eigenvalue	Modulus
.824798 + .4761974*i*	.952395
.824798 - .4761974*i*	.952395
.9523947	.952395
-.824798 + .4761974*i*	.952395
-.824798 - .4761974*i*	.952395
-.4761974 + .824798*i*	.952395
-.4761974 - .824798*i*	.952395
2.776e-16 + .9523947*i*	.952395
2.776e-16 - .9523947*i*	.952395
.4761974 + .824798*i*	.952395
.4761974 - .824798*i*	.952395
-.9523947	.952395
.4018324	.401832

```
   All the eigenvalues lie inside the unit circle.
   MA parameters satisfy invertibility condition.
```



Because the modulus of each eigenvalue is strictly less than 1, the MA process is invertible and can be represented as an infinite-order AR process.

The graph produced by estat aroots displays the eigenvalues with the real components on the x axis and the imaginary components on the y axis. The graph indicates visually that these eigenvalues are just inside the unit circle.

◁

Stored results

estat aroots stores the following in r():

Matrices

r(Re_ar)	real part of the eigenvalues of $F(\rho)$
r(Im_ar)	imaginary part of the eigenvalues of $F(\rho)$
r(Modulus_ar)	modulus of the eigenvalues of $F(\rho)$
r(ar)	$F(\rho)$, the AR companion matrix
r(Re_ma)	real part of the eigenvalues of $F(\theta)$
r(Im_ma)	imaginary part of the eigenvalues of $F(\theta)$
r(Modulus_ma)	modulus of the eigenvalues of $F(\theta)$
r(ma)	$F(\theta)$, the MA companion matrix

Methods and formulas

Recall the general form of the ARMA model,

$$\boldsymbol{\rho}(L^p)(y_t - \mathbf{x}_t\boldsymbol{\beta}) = \boldsymbol{\theta}(L^q)\epsilon_t$$

where

$$\boldsymbol{\rho}(L^p) = 1 - \rho_1 L - \rho_2 L^2 - \cdots - \rho_p L^p$$
$$\boldsymbol{\theta}(L^q) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$$

and $L^j y_t = y_{t-j}$.

estat aroots forms the companion matrix

$$\mathbf{F}(\boldsymbol{\gamma}) = \begin{pmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_{r-1} & \gamma_r \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{pmatrix}$$

where $\boldsymbol{\gamma} = \boldsymbol{\rho}$ and $r = p$ for the AR part of ARMA, and $\boldsymbol{\gamma} = -\boldsymbol{\theta}$ and $r = q$ for the MA part of ARMA. aroots obtains the eigenvalues of $\mathbf{F}$ by using matrix eigenvalues. The modulus of the complex eigenvalue $r + c\mathrm{i}$ is $\sqrt{r^2 + c^2}$. As shown by Hamilton (1994, chap. 1), a process is stable and invertible if the modulus of each eigenvalue of $\mathbf{F}$ is strictly less than 1.

Reference

Hamilton, J. D. 1994. *Time Series Analysis*. Princeton, NJ: Princeton University Press. https://doi.org/10.2307/j.ctv14jx6sm.

Also see

[TS] **arima** — ARIMA, ARMAX, and other dynamic regression models