## dfgls — DF-GLS unit-root test

Description Options Acknowledgments Quick start Remarks and examples References Menu Stored results Also see Syntax Methods and formulas

## Description

dfgls performs a modified Dickey–Fuller t test for a unit root in which the series has been transformed by a generalized least-squares regression.

# **Quick start**

Modified Dickey-Fuller unit-root test for y1 using GLS-transformed series using tsset data dfgls y1

Same as above, for series y2 that has no linear time trend

dfgls y2, notrend

Same as above, but with at most 2 lags dfgls y2, notrend maxlag(2)

## Menu

Statistics > Time series > Tests > DF-GLS test for a unit root

# Syntax

dfgls <i>varnar</i>	ne [if] [in] [, options]
options	Description
Vain	
<pre>maxlag(#)</pre>	use # as the highest lag order for Dickey–Fuller GLS regressions
notrend	series is stationary around a mean instead of around a linear time trend
ers	present interpolated critical values from Elliott, Rothenberg, and Stock (1996)

You must tsset your data before using dfgls; see [TS] tsset. *varname* may contain time-series operators; see [U] 11.4.4 Time-series varlists. collect is allowed; see [U] 11.1.10 Prefix commands.

# Options

Main

- maxlag(#) sets the value of k, the highest lag order for the first-differenced, detrended variable in the Dickey-Fuller regression. By default, dfgls sets k according to the method proposed by Schwert (1989); that is, dfgls sets  $k_{\text{max}} = \text{floor}[12\{(T+1)/100\}^{0.25}]$ .
- notrend specifies that the alternative hypothesis be that the series is stationary around a mean instead of around a linear time trend. By default, a trend is included.
- ers specifies that dfgls should present interpolated critical values from tables presented by Elliott, Rothenberg, and Stock (1996), which they obtained from simulations. See *Critical values* under *Methods and formulas* for details.

## **Remarks and examples**

dfgls tests for a unit root in a time series. It performs the modified Dickey–Fuller t test (known as the DF-GLS test) proposed by Elliott, Rothenberg, and Stock (1996). Essentially, the test is an augmented Dickey–Fuller test, similar to the test performed by Stata's dfuller command, except that the time series is transformed via a generalized least squares (GLS) regression before performing the test. Elliott, Rothenberg, and Stock and later studies have shown that this test has significantly greater power than the previous versions of the augmented Dickey–Fuller test.

dfgls performs the DF-GLS test for the series of models that include 1 to k lags of the first-differenced, detrended variable, where k can be set by the user or by the method described in Schwert (1989). Stock and Watson (2019, 622–624) provide an excellent discussion of the approach.

As discussed in [TS] **dfuller**, the augmented Dickey–Fuller test involves fitting a regression of the form

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta t + \zeta_1 \Delta y_{t-1} + \zeta_2 \Delta y_{t-2} + \dots + \zeta_k \Delta y_{t-k} + \epsilon_t$$

and then testing the null hypothesis  $H_0$ :  $\beta = 0$ . The DF-GLS test is performed analogously but on GLSdetrended data. The null hypothesis of the test is that  $y_t$  is a random walk, possibly with drift. There are two possible alternative hypotheses:  $y_t$  is stationary about a linear time trend or  $y_t$  is stationary with a possibly nonzero mean but with no linear time trend. The default is to use the former. To specify the latter alternative, use the notrend option.

#### Example 1

Here we use the German macroeconomic dataset and test whether the natural log of investment exhibits a unit root. We use the default options with dfgls.

-	s://www.stata-pres SA West German ma _inv		-		Table E.1)
	DF-GLS test for unit root Variable: ln_inv			obs = 80	
Lag select	Lag selection: Schwert criterion		Maximum 1	lag = 11	
		Cr	itical value		
[lags]	DF-GLS tau	1%	5%	10%	
11	-2.925	-3.610	-2.763	-2.489	
10	-2.671	-3.610	-2.798	-2.523	
9	-2.766	-3.610	-2.832	-2.555	
8	-3.259	-3.610	-2.865	-2.587	
7	-3.536	-3.610	-2.898	-2.617	
6	-3.115	-3.610	-2.929	-2.646	
5	-3.054	-3.610	-2.958	-2.674	
4	-3.016	-3.610	-2.986	-2.699	
3	-2.071	-3.610	-3.012	-2.723	
2	-1.675	-3.610	-3.035	-2.744	
1	-1.752	-3.610	-3.055	-2.762	
Opt lag (N	g-Perron seq t) =	7 with BMSE	= 0388771		
	-6.169137 at lag				
	-6.136692 at lag				

The null hypothesis of a unit root is not rejected for lags 1-3, it is rejected at the 10% level for lags 9-10, and it is rejected at the 5% level for lags 4-8 and 11. For comparison, we also test for a unit root in log of investment by using dfuller with two different lag specifications. We need to use the trend option with dfuller because it is not included by default.

	ln_inv, lag(4) t Dickey-Fuller te Ln_inv		Number of	obs = 87 lags = 4
HO: Random	walk with or wi	thout drift		
	Test statistic	1%	Dickey-Fuller critical value 5%	10%
Z(t)	-3.133	-4.069	-3.463	-3.158
. dfuller ]	approximate p-va Ln_inv, lag(7) t Dickey-Fuller te	rend		
Variable: 1	ln_inv		Number of Number of	
HO: Random	walk with or wi	thout drift		
	Test statistic	1%	Dickey-Fuller critical value 5%	10%
Z(t)	-3.994	-4.075	-3.466	-3.160

MacKinnon approximate p-value for Z(t) = 0.0090.

The critical values and the test statistic produced by dfuller with 4 lags do not support rejecting the null hypothesis, although the MacKinnon approximate *p*-value is less than 0.1. With 7 lags, the critical values and the test statistic reject the null hypothesis at the 5% level, and the MacKinnon approximate *p*-value is less than 0.01.

That the dfuller results are not as strong as those produced by dfgls is not surprising because the DF-GLS test with a trend has been shown to be more powerful than the standard augmented Dickey–Fuller test.

4

# **Stored results**

If maxlag(0) is specified, dfgls stores the following in r():

Scalars

r(N)	number of observations
r(rmse0)	RMSE
r(dft0)	DF-GLS statistic
r(cv_1)	1% critical value
r(cv_5)	5% critical value
r(cv_10)	10% critical value

Otherwise, dfgls stores the following in r():

Scalars

highest lag order $k$
number of observations
lag chosen by SIC
lag chosen by modified AIC
lag chosen by sequential- $t$
lag chosen by SIC

lag chosen by modified AIC
lag chosen by sequential- $t$
minimum SIC
minimum MAIC
RMSE for sequential-t method
RMSE for SIC method
RMSE for MAIC method
<i>k</i> , MAIC, SIC, RMSE, and DF-GLS statistics <i>k</i> , DF-GLS statistics, and critical values

### Methods and formulas

dfgls tests for a unit root. There are two possible alternative hypotheses:  $y_t$  is stationary around a linear trend or  $y_t$  is stationary with no linear time trend. Under the first alternative hypothesis, the DF-GLS test is performed by first estimating the intercept and trend via GLS. The GLS estimation is performed by generating the new variables,  $\tilde{y}_t, x_t$ , and  $z_t$ , where

$$\begin{split} \tilde{y}_1 &= y_1 \\ \tilde{y}_t &= y_t - \alpha^* y_{t-1} \\ x_1 &= 1 \\ x_t &= 1 - \alpha^* \\ z_1 &= 1 \\ z_t &= t - \alpha^* (t-1) \end{split} \quad t = 2, \dots, T \end{split}$$

and  $\alpha^* = 1 - (13.5/T)$ . An OLS regression is then estimated for the equation

$$\tilde{y}_t = \delta_0 x_t + \delta_1 z_t + \epsilon_t$$

The OLS estimators  $\hat{\delta}_0$  and  $\hat{\delta}_1$  are then used to remove the trend from  $y_t$ ; that is, we generate

$$y^* = y_t - (\hat{\delta}_0 + \hat{\delta}_1 t)$$

Finally, we perform an augmented Dickey-Fuller test on the transformed variable by fitting the OLS regression

$$\Delta y_t^* = \alpha + \beta y_{t-1}^* + \sum_{j=1}^k \zeta_j \Delta y_{t-j}^* + \epsilon_t \tag{1}$$

and then test the null hypothesis  $H_0$ :  $\beta = 0$  by using tabulated critical values.

To perform the DF-GLS test under the second alternative hypothesis, we proceed as before but define  $\alpha^* = 1 - (7/T)$ , eliminate z from the GLS regression, compute  $y^* = y_t - \delta_0$ , fit the augmented Dickey–Fuller regression by using the newly transformed variable, and perform a test of the null hypothesis that  $\beta = 0$  by using the tabulated critical values.

dfgls reports the DF-GLS statistic and its critical values obtained from the regression in (1) for  $k \in \{1, 2, \ldots, k_{\max}\}$ . By default, dfgls sets  $k_{\max} = \text{floor}[12\{(T+1)/100\}^{0.25}]$  as proposed by Schwert (1989), although you can override this choice with another value. The sample size available with  $k_{\max}$  lags is used in all the regressions. Because there are  $k_{\max}$  lags of the first-differenced series,  $k_{\max} + 1$  observations are lost, leaving  $T - k_{\max}$  observations. dfgls requires that the sample of T + 1 observations on  $y_t = (y_0, y_1, \ldots, y_T)$  have no gaps.

dfgls reports the results of three different methods for choosing which value of k to use. These are method 1, the Ng–Perron sequential t; method 2, the minimum Schwarz information criterion (SIC); and method 3, the Ng–Perron modified Akaike information criterion (MAIC). Although the SIC has a long history in time-series modeling, the Ng–Perron sequential t was developed by Ng and Perron (1995), and the MAIC was developed by Ng and Perron (2000).

The SIC can be calculated using either the log likelihood or the sum-of-squared errors from a regression; dfgls uses the latter definition. Specifically, for each k

$$\text{SIC} = \ln(\widehat{\text{rmse}}^2) + (k+1)\frac{\ln(T-k_{\max})}{(T-k_{\max})}$$

where

$$\widehat{\text{rmse}} = \frac{1}{(T-k_{\max})} \sum_{t=k_{\max}+1}^{T} \hat{e}_t^2$$

dfgls reports the value of the smallest SIC and the k that produced it.

Ng and Perron (1995) derived a sequential-t algorithm for choosing k:

- i. Set n = 0 and run the regression in method 2 with all  $k_{\max} n$  lags. If the coefficient on  $\beta_{k_{\max}}$  is significantly different from zero at level  $\alpha$ , choose k to  $k_{\max}$ . Otherwise, continue to ii.
- ii. If  $n < k_{max}$ , set n = n + 1 and continue to iii. Otherwise, set k = 0 and stop.
- iii. Run the regression in method 2 with  $k_{\max} n$  lags. If the coefficient on  $\beta_{k_{\max}-n}$  is significantly different from zero at level  $\alpha$ , choose k to  $k_{\max} n$ . Otherwise, return to ii.

Per Ng and Perron (1995), dfgls uses  $\alpha = 10\%$ . dfgls reports the k selected by this sequential-t algorithm and the regression.

Method 3 is based on choosing k to minimize the MAIC. The MAIC is calculated as

$$\mathrm{MAIC}(k) = \ln(\widehat{\mathrm{rmse}}^2) + \frac{2\{\tau(k) + k\}}{T - k_{\mathrm{max}}}$$

where

$$\tau(k) = \frac{1}{\widehat{\mathrm{rmse}}^2} \widehat{\beta}_0^2 \sum_{t=k \max+1}^T \widetilde{y}_t^2$$

and  $\tilde{y}$  was defined previously.

#### Critical values

By default, dfgls uses the 5% and 10% critical values computed from the response surface analysis of Cheung and Lai (1995). Because Cheung and Lai (1995) did not present results for the 1% case, the 1% critical values are always interpolated from the critical values presented by ERS.

ERS presented critical values, obtained from simulations, for the DF-GLS test with a linear trend and showed that the critical values for the mean-only DF-GLS test were the same as those for the ADF test. If dfgls is run with the ers option, dfgls will present interpolated critical values from these tables. The method of interpolation is standard. For the trend case, below 50 observations and above 200 there is no interpolation; the values for 50 and  $\infty$  are reported from the tables. For a value N that lies between two values in the table, say,  $N_1$  and  $N_2$ , with corresponding critical values CV<sub>1</sub> and CV<sub>2</sub>, the critical value

$$\mathbf{C}\mathbf{V}=\mathbf{C}\mathbf{V}_1+\frac{N-N_1}{N_1}(\mathbf{C}\mathbf{V}_2-\mathbf{C}\mathbf{V}_1)$$

is presented. The same method is used for the mean-only case, except that interpolation is possible for values between 50 and 500.

### Acknowledgments

We thank Christopher F. Baum of the Department of Economics at Boston College and author of the Stata Press books An Introduction to Modern Econometrics Using Stata and An Introduction to Stata Programming and coauthor of the Stata Press book Environmental Econometrics Using Stata and the late Richard Sperling for a previous version of dfgls.

### References

Baum, C. F., and S. Hurn. 2021. Environmental Econometrics Using Stata. College Station, TX: Stata Press.

- Cheung, Y.-W., and K. S. Lai. 1995. Lag order and critical values of a modified Dickey–Fuller test. Oxford Bulletin of Economics and Statistics 57: 411–419. https://doi.org/10.1111/j.1468-0084.1995.mp57003008.x.
- del Barrio Castro, T., A. Bodnar, and A. Sansó. 2016. The lag-length selection and detrending methods for HEGY seasonal unit-root tests using Stata. *Stata Journal* 16: 740–760.
- Dickey, D. A., and W. A. Fuller. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal* of the American Statistical Association 74: 427–431. https://doi.org/10.2307/2286348.
- Elliott, G. R., T. J. Rothenberg, and J. H. Stock. 1996. Efficient tests for an autoregressive unit root. *Econometrica* 64: 813–836. https://doi.org/10.2307/2171846.
- Kagalwala, A. 2022. kpsstest: A command that implements the Kwiatkowski, Phillips, Schmidt, and Shin test with sample-specific critical values and reports *p*-values. *Stata Journal* 22: 269–292.
- Ng, S., and P. Perron. 1995. Unit root tests in ARMA models with data-dependent methods for the selection of the truncation lag. Journal of the American Statistical Association 90: 268–281. https://doi.org/10.2307/2291151.
  - 2000. Lag length selection and the construction of unit root tests with good size and power. Econometrica 69: 1519–1554. https://doi.org/10.1111/1468-0262.00256.
- Otero, J., and C. F. Baum. 2017. Response surface models for the Elliott, Rothenberg, and Stock unit-root test. Stata Journal 17: 985–1002.
  - ——. 2018. Unit-root tests based on forward and reverse Dickey–Fuller regressions. Stata Journal 18: 22–28.
- Otero, J., and J. Smith. 2017. Response surface models for OLS and GLS detrending-based unit-root tests in nonlinear ESTAR models. *Stata Journal* 17: 704–722.
- Rajbhandari, A. 2016. Unit-root tests in Stata. The Stata Blog: Not Elsewhere Classified. https://blog.stata.com/2016/06/ 21/unit-root-tests-in-stata/.
- Schwert, G. W. 1989. Tests for unit roots: A Monte Carlo investigation. *Journal of Business and Economic Statistics* 2: 147–159. https://doi.org/10.2307/1391432.

Stock, J. H., and M. W. Watson. 2019. Introduction to Econometrics. 4th ed. New York: Pearson.

### Also see

- [TS] dfuller Augmented Dickey–Fuller unit-root test
- [TS] **pperron** Phillips–Perron unit-root test
- [TS] tsset Declare data to be time-series data

[XT] **xtunitroot** — Panel-data unit-root tests

Stata, Stata Press, and Mata are registered trademarks of StataCorp LLC. Stata and Stata Press are registered trademarks with the World Intellectual Property Organization of the United Nations. StataNow and NetCourseNow are trademarks of StataCorp LLC. Other brand and product names are registered trademarks or trademarks of their respective companies. Copyright © 1985–2025 StataCorp LLC, College Station, TX, USA. All rights reserved.



For suggested citations, see the FAQ on citing Stata documentation.