dfgls — DF-GLS unit-root test

Description

dfgls performs a modified Dickey–Fuller t test for a unit root in which the series has been transformed by a generalized least-squares regression.

Quick start

Modified Dickey–Fuller unit-root test for y1 using GLS-transformed series using tsset data

```
dfgls y1
```

As above, for series y2 that has no linear time trend

```
dfgls y2, notrend
```

As above, but with at most 2 lags

```
dfgls y2, notrend maxlag(2)
```

Menu

Statistics > Time series > Tests > DF-GLS test for a unit root

Syntax

`dfgls` *varname* [*if*] [*in*] [, *options*]

options	Description
Main	
`maxlag(#)`	use # as the highest lag order for Dickey–Fuller GLS regressions
`notrend`	series is stationary around a mean instead of around a linear time trend
`ers`	present interpolated critical values from Elliott, Rothenberg, and Stock (1996)

You must `tsset` your data before using `dfgls`; see [TS] **tsset**.
varname may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

Options

Main

`maxlag(#)` sets the value of k, the highest lag order for the first-differenced, detrended variable in the Dickey–Fuller regression. By default, `dfgls` sets k according to the method proposed by Schwert (1989); that is, `dfgls` sets $k_{\text{max}} = \text{floor}[12\{(T+1)/100\}^{0.25}]$.

`notrend` specifies that the alternative hypothesis be that the series is stationary around a mean instead of around a linear time trend. By default, a trend is included.

`ers` specifies that `dfgls` should present interpolated critical values from tables presented by Elliott, Rothenberg, and Stock (1996), which they obtained from simulations. See *Critical values* under *Methods and formulas* for details.

Remarks and examples

stata.com

`dfgls` tests for a unit root in a time series. It performs the modified Dickey–Fuller t test (known as the DF-GLS test) proposed by Elliott, Rothenberg, and Stock (1996). Essentially, the test is an augmented Dickey–Fuller test, similar to the test performed by Stata's `dfuller` command, except that the time series is transformed via a generalized least squares (GLS) regression before performing the test. Elliott, Rothenberg, and Stock and later studies have shown that this test has significantly greater power than the previous versions of the augmented Dickey–Fuller test.

`dfgls` performs the DF-GLS test for the series of models that include 1 to k lags of the first-differenced, detrended variable, where k can be set by the user or by the method described in Schwert (1989). Stock and Watson (2015, 651–655) provide an excellent discussion of the approach.

As discussed in [TS] **dfuller**, the augmented Dickey–Fuller test involves fitting a regression of the form

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta t + \zeta_1 \Delta y_{t-1} + \zeta_2 \Delta y_{t-2} + \cdots + \zeta_k \Delta y_{t-k} + \epsilon_t$$

and then testing the null hypothesis H_0: $\beta = 0$. The DF-GLS test is performed analogously but on GLS-detrended data. The null hypothesis of the test is that y_t is a random walk, possibly with drift. There are two possible alternative hypotheses: y_t is stationary about a linear time trend or y_t is stationary with a possibly nonzero mean but with no linear time trend. The default is to use the former. To specify the latter alternative, use the `notrend` option.

▷ Example 1

Here we use the German macroeconomic dataset and test whether the natural log of investment exhibits a unit root. We use the default options with `dfgls`.

```
. use http://www.stata-press.com/data/r15/lutkepohl2
(Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)

. dfgls ln_inv
DF-GLS for ln_inv                                        Number of obs =    80
Maxlag = 11 chosen by Schwert criterion
               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------

    11          -2.925           -3.610            -2.763            -2.489
    10          -2.671           -3.610            -2.798            -2.523
    9           -2.766           -3.610            -2.832            -2.555
    8           -3.259           -3.610            -2.865            -2.587
    7           -3.536           -3.610            -2.898            -2.617
    6           -3.115           -3.610            -2.929            -2.646
    5           -3.054           -3.610            -2.958            -2.674
    4           -3.016           -3.610            -2.986            -2.699
    3           -2.071           -3.610            -3.012            -2.723
    2           -1.675           -3.610            -3.035            -2.744
    1           -1.752           -3.610            -3.055            -2.762
Opt Lag (Ng-Perron seq t) =  7 with RMSE  .0388771
Min SC   = -6.169137 at lag  4 with RMSE  .0398949
Min MAIC = -6.136371 at lag  1 with RMSE  .0440319
```

The null hypothesis of a unit root is not rejected for lags 1–3, it is rejected at the 10% level for lags 9–10, and it is rejected at the 5% level for lags 4–8 and 11. For comparison, we also test for a unit root in log of investment by using `dfuller` with two different lag specifications. We need to use the `trend` option with `dfuller` because it is not included by default.

```
. dfuller ln_inv, lag(4) trend
Augmented Dickey-Fuller test for unit root         Number of obs   =        87
                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -3.133            -4.069            -3.463            -3.158
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0987
. dfuller ln_inv, lag(7) trend
Augmented Dickey-Fuller test for unit root         Number of obs   =        84
                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -3.994            -4.075            -3.466            -3.160
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0090
```

The critical values and the test statistic produced by `dfuller` with 4 lags do not support rejecting the null hypothesis, although the MacKinnon approximate p-value is less than 0.1. With 7 lags, the critical values and the test statistic reject the null hypothesis at the 5% level, and the MacKinnon approximate p-value is less than 0.01.

That the `dfuller` results are not as strong as those produced by `dfgls` is not surprising because the DF-GLS test with a trend has been shown to be more powerful than the standard augmented Dickey–Fuller test.

◁

Stored results

If `maxlag(0)` is specified, `dfgls` stores the following in `r()`:

Scalars

`r(rmse0)`	RMSE
`r(dft0)`	DF-GLS statistic

Otherwise, `dfgls` stores the following in `r()`:

Scalars

`r(maxlag)`	highest lag order k
`r(N)`	number of observations
`r(sclag)`	lag chosen by Schwarz criterion
`r(maiclag)`	lag chosen by modified AIC method
`r(optlag)`	lag chosen by sequential-t method

Matrices

`r(results)`	k, MAIC, SIC, RMSE, and DF-GLS statistics

Methods and formulas

`dfgls` tests for a unit root. There are two possible alternative hypotheses: y_t is stationary around a linear trend or y_t is stationary with no linear time trend. Under the first alternative hypothesis, the DF-GLS test is performed by first estimating the intercept and trend via GLS. The GLS estimation is performed by generating the new variables, $\widetilde{y}_t$, x_t, and z_t, where

$$
\begin{aligned}
\widetilde{y}_1 &= y_1 \\
\widetilde{y}_t &= y_t - \alpha^* y_{t-1}, \qquad t = 2, \ldots, T \\
x_1 &= 1 \\
x_t &= 1 - \alpha^*, \qquad\qquad t = 2, \ldots, T \\
z_1 &= 1 \\
z_t &= t - \alpha^*(t-1)
\end{aligned}
$$

and $\alpha^* = 1 - (13.5/T)$. An OLS regression is then estimated for the equation

$$\widetilde{y}_t = \delta_0 x_t + \delta_1 z_t + \epsilon_t$$

The OLS estimators $\widehat{\delta}_0$ and $\widehat{\delta}_1$ are then used to remove the trend from y_t; that is, we generate

$$y^* = y_t - (\widehat{\delta}_0 + \widehat{\delta}_1 t)$$

Finally, we perform an augmented Dickey–Fuller test on the transformed variable by fitting the OLS regression

$$\Delta y^*_t = \alpha + \beta y^*_{t-1} + \sum_{j=1}^{k} \zeta_j \Delta y^*_{t-j} + \epsilon_t \tag{1}$$

and then test the null hypothesis H_0: $\beta = 0$ by using tabulated critical values.

To perform the DF-GLS test under the second alternative hypothesis, we proceed as before but define $\alpha^* = 1-(7/T)$, eliminate z from the GLS regression, compute $y^* = y_t - \delta_0$, fit the augmented Dickey–Fuller regression by using the newly transformed variable, and perform a test of the null hypothesis that $\beta = 0$ by using the tabulated critical values.

`dfgls` reports the DF-GLS statistic and its critical values obtained from the regression in (1) for $k \in \{1, 2, \ldots, k_{\max}\}$. By default, `dfgls` sets $k_{\max} = \text{floor}[12\{(T+1)/100\}^{0.25}]$ as proposed by Schwert (1989), although you can override this choice with another value. The sample size available with $k_{\max}$ lags is used in all the regressions. Because there are $k_{\max}$ lags of the first-differenced series, $k_{\max}+1$ observations are lost, leaving $T - k_{\max}$ observations. `dfgls` requires that the sample of $T+1$ observations on $y_t = (y_0, y_1, \ldots, y_T)$ have no gaps.

`dfgls` reports the results of three different methods for choosing which value of k to use. These are method 1, the Ng–Perron sequential t; method 2, the minimum Schwarz information criterion (SIC); and method 3, the Ng–Perron modified Akaike information criterion (MAIC). Although the SIC has a long history in time-series modeling, the Ng–Perron sequential t was developed by Ng and Perron (1995), and the MAIC was developed by Ng and Perron (2000).

The SIC can be calculated using either the log likelihood or the sum-of-squared errors from a regression; `dfgls` uses the latter definition. Specifically, for each k

$$\text{SIC} = \ln(\widehat{\text{rmse}}^2) + (k+1)\frac{\ln(T - k_{\max})}{(T - k_{\max})}$$

where

$$\widehat{\text{rmse}} = \frac{1}{(T - k_{\max})} \sum_{t=k_{\max}+1}^{T} \widehat{e}_t^2$$

`dfgls` reports the value of the smallest SIC and the k that produced it.

Ng and Perron (1995) derived a sequential-t algorithm for choosing k:

i. Set $n = 0$ and run the regression in method 2 with all $k_{\max} - n$ lags. If the coefficient on $\beta_{k_{\max}}$ is significantly different from zero at level α, choose k to $k_{\max}$. Otherwise, continue to ii.

ii. If $n < k_{\max}$, set $n = n + 1$ and continue to iii. Otherwise, set $k = 0$ and stop.

iii. Run the regression in method 2 with $k_{\max} - n$ lags. If the coefficient on $\beta_{k_{\max}-n}$ is significantly different from zero at level α, choose k to $k_{\max} - n$. Otherwise, return to ii.

Per Ng and Perron (1995), `dfgls` uses $\alpha = 10\%$. `dfgls` reports the k selected by this sequential-t algorithm and the $\widehat{\text{rmse}}$ from the regression.

Method 3 is based on choosing k to minimize the MAIC. The MAIC is calculated as

$$\text{MAIC}(k) = \ln(\widehat{\text{rmse}}^2) + \frac{2\{\tau(k) + k\}}{T - k_{\max}}$$

where

$$\tau(k) = \frac{1}{\widehat{\text{rmse}}^2} \widehat{\beta}_0^2 \sum_{t=k_{\max}+1}^{T} \widetilde{y}_t^2$$

and $\widetilde{y}$ was defined previously.

Critical values

By default, `dfgls` uses the 5% and 10% critical values computed from the response surface analysis of Cheung and Lai (1995). Because Cheung and Lai (1995) did not present results for the 1% case, the 1% critical values are always interpolated from the critical values presented by ERS.

ERS presented critical values, obtained from simulations, for the DF-GLS test with a linear trend and showed that the critical values for the mean-only DF-GLS test were the same as those for the ADF test. If `dfgls` is run with the `ers` option, `dfgls` will present interpolated critical values from these tables. The method of interpolation is standard. For the trend case, below 50 observations and above 200 there is no interpolation; the values for 50 and ∞ are reported from the tables. For a value N that lies between two values in the table, say, N_1 and N_2, with corresponding critical values CV_1 and CV_2, the critical value

$$\text{CV} = \text{CV}_1 + \frac{N - N_1}{N_1}(\text{CV}_2 - \text{CV}_1)$$

is presented. The same method is used for the mean-only case, except that interpolation is possible for values between 50 and 500.

Acknowledgments

We thank Christopher F. Baum of the Department of Economics at Boston College and author of the Stata Press books *An Introduction to Modern Econometrics Using Stata* and *An Introduction to Stata Programming* and the late Richard Sperling for a previous version of `dfgls`.

References

Cheung, Y.-W., and K. S. Lai. 1995. Lag order and critical values of a modified Dickey–Fuller test. *Oxford Bulletin of Economics and Statistics* 57: 411–419.

del Barrio Castro, T., A. Bodnar, and A. Sansó. 2016. The lag-length selection and detrending methods for HEGY seasonal unit-root tests using Stata. *Stata Journal* 16: 740–760.

Dickey, D. A., and W. A. Fuller. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association* 74: 427–431.

Elliott, G. R., T. J. Rothenberg, and J. H. Stock. 1996. Efficient tests for an autoregressive unit root. *Econometrica* 64: 813–836.

Ng, S., and P. Perron. 1995. Unit root tests in ARMA models with data-dependent methods for the selection of the truncation lag. *Journal of the American Statistical Association* 90: 268–281.

———. 2000. Lag length selection and the construction of unit root tests with good size and power. *Econometrica* 69: 1519–1554.

Otero, J., and C. F. Baum. 2017. Response surface models for the Elliott, Rothenberg, and Stock unit-root test. *Stata Journal* 17: 985–1002.

Otero, J., and J. Smith. 2017. Response surface models for OLS and GLS detrending-based unit-root tests in nonlinear ESTAR models. *Stata Journal* 17: 704–722.

Rajbhandari, A. 2016. Unit-root tests in Stata. *The Stata Blog: Not Elsewhere Classified.* http://blog.stata.com/2016/06/21/unit-root-tests-in-stata/.

Schwert, G. W. 1989. Tests for unit roots: A Monte Carlo investigation. *Journal of Business and Economic Statistics* 2: 147–159.

Stock, J. H., and M. W. Watson. 2015. *Introduction to Econometrics*. Updated 3rd ed. Hoboken, NJ: Pearson.

Also see

[TS] **dfuller** — Augmented Dickey–Fuller unit-root test

[TS] **pperron** — Phillips–Perron unit-root test

[TS] **tsset** — Declare data to be time-series data

[XT] **xtunitroot** — Panel-data unit-root tests