eteffects — Endogenous treatment-effects estimation

Description

eteffects estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational data when treatment assignment is correlated with the potential outcomes. It allows for continuous, binary, count, fractional, and nonnegative outcomes and requires a binary treatment. To control for the endogeneity of the treatment assignment, the estimator includes residuals from the treatment model in the models for the potential outcomes, known as a control-function approach.

Quick start

ATE of binary treatment treat using a linear model for outcome y1 on x and the residuals from a probit model for treat on x and z

```
eteffects (y1 x) (treat x z)
```

As above, but estimate ATET

```
eteffects (y1 x) (treat x z), atet
```

As above, but estimate POMs

```
eteffects (y1 x) (treat x z), pomeans
```

As above, and show parameters from auxiliary equations

```
eteffects (y1 x) (treat x z), pomeans aequations
```

ATE of treat using an exponential-mean model for y1

```
eteffects (y1 x, exponential) (treat x z)
```

Same as above, but for count outcome y2

```
eteffects (y2 x, exponential) (treat x z)
```

As above, but use a probit model for binary outcome y3

```
eteffects (y3 x, probit) (treat x z)
```

As above, but use a fractional probit model for y4 ranging from 0 to 1

```
eteffects (y4 x, fractional) (treat x z)
```

Menu

Statistics > Treatment effects > Endogenous treatment > Control function estimator > Continuous outcomes

Statistics > Treatment effects > Endogenous treatment > Control function estimator > Binary outcomes

Statistics > Treatment effects > Endogenous treatment > Control function estimator > Count outcomes

Statistics > Treatment effects > Endogenous treatment > Control function estimator > Fractional outcomes

Statistics > Treatment effects > Endogenous treatment > Control function estimator > Nonnegative outcomes
**Syntax**

\[
\texttt{eteffects (ovar omvarlist [, omodel noconstant])}
\]

\[
\texttt{ (tvar tmvarlist [, noconstant]) [if] [in] [weight] [, stat options]}
\]

*ovar* is the *depvar* of the outcome model.

*omvarlist* is the list of exogenous *indepvars* in the outcome model.

*tvar* is the binary treatment variable.

*tmvarlist* is the list of covariates that predict treatment assignment.

<table>
<thead>
<tr>
<th>omodel</th>
<th>Description</th>
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<tbody>
<tr>
<td>linear</td>
<td>linear outcome model; the default</td>
</tr>
<tr>
<td>fractional</td>
<td>fractional probit outcome model</td>
</tr>
<tr>
<td>probit</td>
<td>probit outcome model</td>
</tr>
<tr>
<td>exponential</td>
<td>exponential-mean outcome model</td>
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<tr>
<td>ate</td>
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<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
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<tr>
<td>pommeans</td>
<td>estimate potential-outcome means</td>
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<table>
<thead>
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<tr>
<td>noconstant</td>
<td>suppress constant term</td>
</tr>
<tr>
<td>vce(vctype)</td>
<td>vctype may be robust, cluster clustvar, bootstrap, or jackknife</td>
</tr>
</tbody>
</table>

**Model**

- **Model**
  - **linear**: linear outcome model; the default
  - **fractional**: fractional probit outcome model
  - **probit**: probit outcome model
  - **exponential**: exponential-mean outcome model

**Stat**

- **ate**: estimate average treatment effect in population; the default
- **atet**: estimate average treatment effect on the treated
- **pommeans**: estimate potential-outcome means

- **options**
  - **noconstant**: suppress constant term
  - **vce(vctype)**: vctype may be robust, cluster clustvar, bootstrap, or jackknife

- **Reporting**
  - **level(#)**: set confidence level; default is level(95)
  - **aequations**: display auxiliary-equation results
  - **display_options**: control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

- **Maximization**
  - **maximize_options**: control the maximization process; seldom used

- **Advanced**
  - **pstolerance(#)**: set tolerance for overlap assumption
  - **osample(newvar)**: generate newvar to mark observations that violate the overlap assumption
  - **coeflegend**: display legend instead of statistics
omvarlist and tmvarlist may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

noconstant; see [R] Estimation options.

stat is one of three statistics: ate, atet, or pomeans. ate is the default.

ate specifies that the average treatment effect be estimated.

atet specifies that the average treatment effect on the treated be estimated.

pomeans specifies that the potential-outcome means for each treatment level be estimated.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are robust to
some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar),
and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting

level(#); see [R] Estimation options.

eaequations specifies that the results for the outcome-model or the treatment-model parameters be
displayed. By default, the results for these auxiliary parameters are not displayed.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
allbaselevels, nofvlabel, fwrap(#), fwraption(style), cformat(%,fmt), pformat(%,fmt),
sformat(%,fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize_options: iterate(#), [no]log, and from(init_specs); see [R] Maximize. These options
are seldom used.

init_specs is one of

matname [, skip copy]

# [, # ...], copy

Advanced

pstolerance(#) specifies the tolerance used to check the overlap assumption. The default value
is pstolerance(1e-5). eteffects will exit with an error if an observation has an estimated
propensity score smaller than that specified by pstolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that
violate the overlap assumption.

The following option is available with eteffects but is not shown in the dialog box:
coeflegend; see [R] Estimation options.
Remarks and examples

If you are unfamiliar with treatment-effects estimators for observational data or the teffects commands, we recommend that you look at [TE] teffects intro. For the intuition behind some of the concepts discussed below, we recommend that you read Defining treatment effects in [TE] teffects intro advanced.

The estimators implemented in teffects extend the regression adjustment (RA) estimators implemented in teffects ra to allow for endogenous treatments, that is, when treatment assignment is not independent of outcomes. This endogeneity is a violation of the conditional mean independence assumption used by teffects ra, as discussed in The potential-outcome model in [TE] teffects intro advanced.

eteffects estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs). It uses a linear, a probit, a fractional probit, or an exponential-mean model for the potential outcomes and a probit model for treatment assignment. After conditioning on the observable covariates, eteffects allows some remaining unobservable components to affect both treatment assignment and the potential outcomes. The treatment assignment process is endogenous because these unobservable components affect both treatment assignment and the potential outcomes.

To control for the endogeneity of the treatment assignment, eteffects uses a control-function approach. This method controls for endogeneity by including the residuals from the treatment-assignment model as a regressor in the models for the potential outcome. The implementation in eteffects follows Wooldridge (2010), who provides an excellent discussion of the control-function approach that addresses endogeneity problems in a treatment-effects context.

The control-function approach estimates the parameters of the conditional means of the potential outcomes. Sample averages of the conditional means are used to estimate the unconditional ATE, ATET, or POMs. This method is known as RA.

Taken collectively, the estimators implemented in eteffects are control-function RA estimators. See Methods and formulas below for details about the estimation procedure.

Example 1: Linear outcome estimates for ATE

Suppose we want to know the effect of a mother smoking while pregnant on the birthweight of her infant. We use an extract from Cattaneo (2010) in which bweight records the baby’s birthweight and mbsmoke is the variable (0 or 1) indicating whether a mother smoked while pregnant.

We may believe that birthweight (the potential outcome) is influenced by whether the mother had a prenatal exam in the first trimester, whether the mother is married, the mother’s age, whether this is the first birth, and the education level of the father. We may also believe that the smoking decision (the treatment) is influenced by the mother’s marital status, the education level of the mother, her age, whether she had a prenatal exam in the first trimester, and whether this baby is her first baby.

Thus we condition on different sets of covariates in the models for treatment assignment and the potential outcomes. In the probit model for smoking status (mbsmoke), we condition on marital status (mmarried), age (mage), mother’s education level (medu), father’s education level (fedu), and whether it was the mother’s first baby (fbaby). We model birthweight (bweight) as a linear function of whether the mother had a first-trimester prenatal exam (prenatal1), mmarried, mage, and fbaby. We can estimate the ATE of smoking status using one of the teffects estimators if we believe that there are no unobservable components that affect both the decision to smoke while pregnant and the potential birthweights.
If we believe there is some unobservable factor that affects both assignment to treatment and the potential outcome, we must select another estimator. For example, we do not observe a mother’s health consciousness, which affects both the smoking decision and each potential birthweight through other behaviors such as intake of prenatal vitamins. Under these assumptions, the estimators in eteffects consistently estimate the ATE, but the estimators in \( \text{TE} \) \teffects\ yield inconsistent estimates.

\[
\text{. use https://www.stata-press.com/data/r16/cattaneo2}
\]
\[
\text{. eteffects (bweight i.prenatal1 i.married mage i.fbaby)}
\>
\text{> (mbsmoke i.married mage i.fbaby medu fedu)}
\]
\[\text{Iteration 0: EE criterion = 4.704e-24}
\]
\[\text{Iteration 1: EE criterion = 1.223e-25}
\]

Endogenous treatment-effects estimation

\[\begin{array}{l}
\text{Outcome model : linear}
\end{array}\]

\[\begin{array}{l}
\text{Treatment model: probit}
\end{array}\]

\[\begin{array}{l|lllll}
\text{ATE} & \text{mbsmoke (smoker vs nonsmoker)} & -455.9119 & 212.4393 & -2.15 & 0.032 & -872.2853 & -39.53852
\end{array}\]

\[\begin{array}{l|lllll}
\text{POmean mbsmoke nonsmoker} & 3437.964 & 31.21145 & 110.15 & 0.000 & 3376.791 & 3499.138
\end{array}\]

When no mother smokes, the average birthweight is 3,438 grams. The average birthweight is 456 grams less when all mothers smoke than when no mother smokes.

We can compare these results with those obtained if we ignore the endogeneity of the smoking decision. Below we estimate the ATE using the inverse-probability-weighted regression-adjustment estimator in \[\text{TE} \] \teffects\ ipwra.

\[
\text{. teffects ipwra (bweight i.prenatal1 i.married mage i.fbaby)}
\>
\text{> (mbsmoke i.married mage i.fbaby medu fedu)}
\]
\[\text{Iteration 0: EE criterion = 3.036e-22}
\]
\[\text{Iteration 1: EE criterion = 3.755e-26}
\]

Treatment-effects estimation

\[\begin{array}{l}
\text{Number of obs = 4,642}
\end{array}\]

\[\begin{array}{l}
\text{Outcome model : linear}
\end{array}\]

\[\begin{array}{l}
\text{Treatment model: logit}
\end{array}\]

\[\begin{array}{l|lllll}
\text{ATE} & \text{mbsmoke (smoker vs nonsmoker)} & -233.6835 & 25.07695 & -9.32 & 0.000 & -282.8335 & -184.5336
\end{array}\]

\[\begin{array}{l|lllll}
\text{POmean mbsmoke nonsmoker} & 3403.191 & 9.529709 & 357.11 & 0.000 & 3384.513 & 3421.869
\end{array}\]
In magnitude, the estimated ATE is more than half the estimate that allows for endogenous treatment assignment. If there is endogeneity, disregarding it underestimates the effect of smoking on birthweight. We show how to test for endogeneity in example 1 of \texttt{eteffects postestimation}.

\textbf{Example 2: Estimating the ATET}

Continuing example 1, we can use the \texttt{atet} option to estimate the ATET.

\begin{verbatim}
. eteffects (bweight i.prenatal i.mmarried mage i.fbaby) > (mbsmoke i.mmarried mage i.fbaby medu fedu), atet
Iteration 0: EE criterion = 4.688e-24
Iteration 1: EE criterion = 8.479e-26
Endogenous treatment-effects estimation Number of obs = 4,642
Outcome model : linear
Treatment model: probit

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
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<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
</tr>
<tr>
<td>bweight</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATET mbsmoke</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(smoker vs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nonsmoker)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-409.8527</td>
<td>161.4816</td>
<td>-2.54</td>
<td>0.011</td>
<td>-726.3507</td>
</tr>
<tr>
<td>POmean mbsmoke</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nonsmoker</td>
<td>3547.512</td>
<td>160.0595</td>
<td>22.16</td>
<td>0.000</td>
</tr>
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</table>

In the population of mothers who smoke, the average infant birthweight would be 3,548 grams if none of these mothers smoked. For the mothers who smoke, the average infant birthweight is 410 grams less than if none of these mothers smoked.

\textbf{Example 3: Exponential-mean outcomes}

We estimate the ATE of living in an urban area on monthly earnings (\texttt{wage}), using a subset of the National Longitudinal Survey in 1980 found in \textit{Wooldridge (2010)}. We assume that once we condition on work experience (\texttt{exper}), whether education level attained is college or higher (\texttt{college}), and IQ (\texttt{iq}), individual wages follow an exponential mean. The variables used to predict residence in an urban area (\texttt{urban}) are \texttt{college} and whether the respondent’s father attained a bachelor’s degree or higher (\texttt{fcollege}).
. use https://www.stata-press.com/data/r16/nlsy80
. eteffects (wage exper iq i.college, exponential nocons)
> (urban i.college fcollege)
Iteration 0: EE criterion = 2.903e-25
Iteration 1: EE criterion = 2.903e-25 (backed up)

Endogenous treatment-effects estimation
Number of obs = 935
Outcome model : exponential
treatment model: probit

|            | Robust        | z  | P>|z| | [95% Conf. Interval] |
|------------|---------------|----|------|----------------------|
| wage       |               |    |      |                      |
| ATE        |               |    |      |                      |
| urban      |               |    |      |                      |
| (1 vs 0)   | 481.0465      | 31.74882 | 15.15  | 0.000 | 418.82  | 543.2731 |
| POmean     |               |    |      |                      |
| urban      |               |    |      |                      |
| 0          | 233.8083      | 13.51028 | 17.31  | 0.000 | 207.3286 | 260.288  |

When everyone lives outside urban areas, wages are $234 a month on average. Wages are $481 a month greater, on average, when everyone lives in urban areas.

Stored results

eteffects stores the following in \texttt{e}():

Scalars
- \texttt{e(N)}: number of observations
- \texttt{e(nj)}: number of observations for treatment level \texttt{j}
- \texttt{e(k_eq)}: number of equations in \texttt{e(b)}
- \texttt{e(k_levels)}: number of levels in treatment variable
- \texttt{e(rank)}: rank of \texttt{e(V)}
- \texttt{e(converged)}: 1 if converged, 0 otherwise

Macros
- \texttt{e(cmd)}: eteffects
- \texttt{e(cmdline)}: command as typed
- \texttt{e(depvar)}: name of outcome variable
- \texttt{e(tvar)}: name of treatment variable
- \texttt{e(omodel)}: fractional, linear, probit, or exponential
- \texttt{e(stat)}: statistic estimated, \texttt{ate}, \texttt{atet}, or \texttt{pomeans}
- \texttt{e(wtype)}: weight type
- \texttt{e(wexp)}: weight expression
- \texttt{e(title)}: title in estimation output
- \texttt{e(tlevels)}: levels of treatment variable
- \texttt{e(vce)}: \texttt{vcetype} specified in \texttt{vce()}
- \texttt{e(vcetype)}: title used to label Std. Err.
- \texttt{e(properties)}: \texttt{b V}
- \texttt{e(estat_cmd)}: program used to implement \texttt{estat}
- \texttt{e(predict)}: program used to implement \texttt{predict}
- \texttt{e(marginsnotok)}: predictions disallowed by \texttt{margins}
- \texttt{e(asbalanced)}: factor variables \texttt{fvset} as \texttt{asbalanced}
- \texttt{e(asobserved)}: factor variables \texttt{fvset} as \texttt{asobserved}

Matrices
- \texttt{e(b)}: coefficient vector
- \texttt{e(V)}: variance–covariance matrix of the estimators

Functions
- \texttt{e(sample)}: marks estimation sample
Methods and formulas

The treatment-effects models considered in eteffects are given by

\[ y_{i0} = E(y_{i0}|x_i) + \epsilon_{i0} \]  
\[ y_{i1} = E(y_{i1}|x_i) + \epsilon_{i1} \]  
\[ t_i = E(t_i|z_i) + \nu_i \]  
\[ y_i = t_iy_{i1} + (1 - t_i)y_{i0} \]  
\[ E(\epsilon_{ij}|x_i, z_i) = E(\epsilon_{ij}|z_i) = E(\epsilon_{ij}|x_i) = 0 \text{ for } j \in \{0, 1\} \]  
\[ E(\epsilon_{ij}|t) \neq 0 \text{ for } j \in \{0, 1\} \]  

where the subscript \( i \) denotes individual level observations, \( y_{i1} \) is the potential outcome of receiving the treatment, \( y_{i0} \) is the potential outcome when the treatment is not received, \( t_i \) is the observed binary treatment, and \( y_i \) is the observed outcome. Each one of the potential outcomes is determined by its expected value conditional on a set of regressors \( x_i \) and an unobserved random component \( \epsilon_{ij} \), for \( j \in \{0, 1\} \). Similarly, the treatment is given by its expectation conditional on a set of regressors \( z_i \), which does not need to differ from \( x_i \), and an unobserved component \( \nu_i \).

Equations (1)–(5) describe the parametric treatment-effects models in [TE] eteffects. Equation (6) adds endogeneity to the framework. It states that the unobservables in the potential-outcome equations are correlated to treatment status. For our birthweight example, this would happen if mothers who do not smoke are more health conscious than those who smoke and if we do not observe health awareness in our data. If we do not observe health awareness, the decision to smoke or not to smoke is not independent of the infant’s birthweight.

Equations (3), (5), and (6) are the basis of the control-function estimator implemented by eteffects. Equation (5) states that the unobserved components in the potential outcome are independent of \( z_i \). Therefore, the correlation between \( t_i \) and the unobserved components must be equivalent to the correlation between \( \epsilon_{ij} \) and \( \nu_i \). Another way of stating this is

from (3) \[ E(\epsilon_{ij}|t_i) = E(\epsilon_{ij}|E(t_i|z_i) + \nu_i) \]  
from (5) \[ = E(\epsilon_{ij}|\nu_i) \]  
\[ = \nu_i\beta_{2j} \]

We fit (3) using a probit estimator. We then obtain \( \hat{\nu}_i \) as the difference between the treatment and our estimate of \( E(t_i|z_i) \) and use this statistic to compute an estimate of \( E(y_{ij}|x_i, \nu_i, t_i) \) for \( j \in \{0, 1\} \). If the outcome is linear, for instance,

\[ E(y_{ij}|x_i, \nu_i, t_i = j) = x_i'\beta_{1j} + \nu_i\beta_{2j} \text{ for } j \in \{0, 1\} \]  

(7)
For the probit and exponential-mean cases, respectively, we have the following:

\[
E(y_{ij} | x_i, \nu_i, t_i = j) = \Phi (x_i' \beta_{1j} + \nu_i \beta_{2j})
\] (8)

\[
E(y_{ij} | x_i, \nu_i, t_i = j) = \exp (x_i' \beta_{1j} + \nu_i \beta_{2j})
\] (9)

The parameters of (3) and (7)–(9), and the ATE, ATET, and POMs are estimated using the generalized method of moments (GMM). The moment equations used in GMM are the sample analogs of \( E \{ w_i' \epsilon_i(\theta) \} = 0 \), where \( w_i \) are the instruments, \( \epsilon_i(\theta) \) are residuals, and \( \theta \) are the parameters of the model (see [R] gmm). The moment conditions in the GMM estimation for the linear model are given by

\[
\frac{1}{n} \sum_{i=1}^{n} x_i'(y_i - x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j}) t_i = 0
\] (10)

\[
\frac{1}{n} \sum_{i=1}^{n} x_i'(y_i - x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})(1 - t_i) = 0
\] (11)

\[
\frac{1}{n} \sum_{i=1}^{n} z_i' \left\{ t_i \frac{\phi (z_i' \hat{\pi})}{\Phi (z_i' \hat{\pi})} - (1 - t_i) \frac{\phi (z_i' \hat{\pi})}{1 - \Phi (z_i' \hat{\pi})} \right\} = 0
\] (12)

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ (x_i' \hat{\beta}_{10} + \hat{\nu}_i \hat{\beta}_{20}) - \hat{\text{POM0}} \right\} = 0
\] (13)

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ (x_i' \hat{\beta}_{11} + \hat{\nu}_i \hat{\beta}_{21}) - \hat{\text{POM0}} - \hat{\text{ATE}} \right\} = 0
\] (14)

where \( \hat{\nu}_i = t_i - \Phi (z_i' \hat{\pi}) \), \( n \) is the number of observations, and \( \hat{\beta}_{11}, \hat{\beta}_{10}, \hat{\beta}_{21}, \hat{\beta}_{20}, \hat{\pi}, \hat{\text{ATE}}, \) and \( \hat{\text{POM0}} \) are the parameters. If we want to estimate the ATET, we replace (14) with

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ (x_i' \hat{\beta}_{11} + \hat{\nu}_i \hat{\beta}_{21}) \frac{n}{n_t} - \hat{\text{POM0}} \frac{n}{n_t} - \hat{\text{ATE}} \right\} = 0
\] (15)

and if we want to estimate the potential-outcome means, we replace (14) with

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ (x_i' \hat{\beta}_{11} + \hat{\nu}_i \hat{\beta}_{21}) - \hat{\text{POM1}} \right\} = 0
\] (16)

where \( \hat{\text{ATE}} \) and \( \hat{\text{POM1}} \) are the parameters of the model, and \( n_t \) is the number of treated units.
For the exponential-mean outcome model, we replace \( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \) with \( \exp(x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j}) \) to obtain the residual equations in (10)–(16). For the probit outcome model, we replace (10) and (11) with the following:

\[
\frac{1}{n} \sum_{i=1}^{n} t_i x_i' \begin{cases} 
\phi \left( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \right) y_i - (1 - y_i) \frac{\phi \left( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \right)}{1 - \Phi \left( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \right)} \end{cases} = 0 \\
\frac{1}{n} \sum_{i=1}^{n} (1 - t_i) x_i' \begin{cases} 
\phi \left( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \right) y_i - (1 - y_i) \frac{\phi \left( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \right)}{1 - \Phi \left( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \right)} \end{cases} = 0 
\]

For the remaining equations, \( x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j} \) is replaced with \( \Phi(x_i' \hat{\beta}_{1j} + \nu_i \hat{\beta}_{2j}) \). The fractional probit model uses the same moment conditions as the probit model.

Acknowledgment

We thank Jeffrey Wooldridge of Michigan State University for helpful conversations and advice about the literature and methods in this area.

References


