**sts test — Test equality of survivor functions**

**Description**

sts test tests the equality of survivor functions across two or more groups. The log-rank, Cox, Wilcoxon–Breslow–Gehan, Tarone–Ware, Peto–Peto–Prentice, and Fleming–Harrington tests are provided, in both unstratified and stratified forms.

sts test also provides a test for trend.

sts test can be used with single- or multiple-record or single- or multiple-failure st data.

**Quick start**

Log-rank test for the equality of survivor functions across levels of v1 using stset data

```
sts test v1
```

Stratified log-rank test for equality of survivor functions across v1 with strata svar

```
sts test v1, strata(svar)
```

Same as above, and show tests for each stratum

```
sts test v1, strata(svar) detail
```

Log-rank test for equality, and test for a trend in survivor functions for v1

```
sts test v1, trend
```

Test equality of survivor functions using the Wilcoxon–Breslow–Gehan test

```
sts test v1, wilcoxon
```

Likelihood-ratio test for the equality of survivor functions based on the Cox model

```
sts test v1, cox
```

Stratified Cox test of equality of survivor functions with strata svar

```
sts test v1, cox strata(svar)
```

Test equality of survivor functions using the Tarone–Ware test

```
sts test v1, tware
```

Same as above, and test for a trend using the same weights as used in the Tarone–Ware test

```
sts test v1, tware trend
```

**Menu**

Statistics > Survival analysis > Summary statistics, tests, and tables > Test equality of survivor functions
Syntax

```
sts test varlist [if] [in] [, options]
```

**options**

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main</strong></td>
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<td><code>logrank</code></td>
</tr>
<tr>
<td><code>cox</code></td>
</tr>
<tr>
<td><code>wilcoxon</code></td>
</tr>
<tr>
<td><code>tware</code></td>
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<td><code>peto</code></td>
</tr>
<tr>
<td><code>fh(p q)</code></td>
</tr>
<tr>
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</tr>
<tr>
<td><code>strata(varlist)</code></td>
</tr>
<tr>
<td><code>detail</code></td>
</tr>
</tbody>
</table>

**Options**

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>mat(mname1 mname2)</code></td>
</tr>
<tr>
<td><code>noshow</code></td>
</tr>
<tr>
<td><code>notitle</code></td>
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</table>

You must `stset` your data before using `sts test`; see [ST] `stset`.

You must `stset` your data before using `sts test`; see [ST] `stset`.

Collect is allowed; see [U] 11.1.10 Prefix commands.

Note that `fweight`s, `iweight`s, and `pweight`s may be specified using `stset`; see [ST] `stset`.

Options

**Main**

`logrank`, `cox`, `wilcoxon`, `tware`, `peto`, and `fh(p q)` specify the test of equality desired. `logrank` is the default, unless the data are `pweighted`, in which case `cox` is the default and is the only possibility.

`wilcoxon` specifies the Wilcoxon–Breslow–Gehan test; `tware`, the Tarone–Ware test; `peto`, the Peto–Peto–Prentice test; and `fh()`, the generalized Fleming–Harrington test. The Fleming–Harrington test requires two arguments, `p` and `q`. When `p = 0` and `q = 0`, the Fleming–Harrington test reduces to the log-rank test; when `p = 1` and `q = 0`, the test reduces to the Mann–Whitney–Wilcoxon test.

`trend` specifies that a test for trend of the survivor function across three or more ordered groups be performed.

`strata(varlist)` requests that a stratified test be performed.

`detail` modifies `strata()`; it requests that, in addition to the overall stratified test, the tests for the individual strata be reported. `detail` is not allowed with `cox`.

**Options**

`mat(mname1 mname2)` requests that the vector `u` be stored in `mname1` and that matrix `V` be stored in `mname2`. The other tests are rank tests of the form `u'V^{-1}u`. This option may not be used with `cox`. 

You must `stset` your data before using `sts test`; see [ST] `stset`.
noshow prevents \
sts\ test\ from\ showing\ the\ key\ st\ variables.\ This\ option\ is\ seldom\ used\ because\ most\ people\ type\ \stset,\ \show\ or\ \stset,\ noshow\ to\ set\ whether\ they\ want\ to\ see\ these\ variables\ mentioned\ at\ the\ top\ of\ the\ output\ of\ every\ st\ command;\ see\ [ST] \stset.

notitle requests that the title printed above the test be suppressed.

Remarks and examples

Remarks are presented under the following headings:

- The log-rank test
- The Wilcoxon (Breslow–Gehan) test
- The Tarone–Ware test
- The Peto–Peto–Prentice test
- The generalized Fleming–Harrington tests
- The “Cox” test
- The trend test
- Video example

\sts test\ tests the equality of the survivor function across groups. With the exception of the Cox test, these tests are members of a family of statistical tests that are extensions to censored data of traditional nonparametric rank tests for comparing two or more distributions. A technical description of these tests can be found in the \textit{Methods and formulas} section of this entry. Simply, at each distinct failure time in the data, the contribution to the test statistic is obtained as a weighted standardized sum of the difference between the observed and expected number of deaths in each of the \(k\) groups. The expected number of deaths is obtained under the null hypothesis of no differences between the survival experience of the \(k\) groups.

The weights or weight function used determines the test statistic. For example, when the weight is 1 at all failure times, the log-rank test is computed, and when the weight is the number of subjects at risk of failure at each distinct failure time, the Wilcoxon–Breslow–Gehan test is computed.

The following table summarizes the weights used for each statistical test.

<table>
<thead>
<tr>
<th>Test</th>
<th>Weight at each distinct failure time ((t_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-rank</td>
<td>1</td>
</tr>
<tr>
<td>Wilcoxon–Breslow–Gehan</td>
<td>(n_i)</td>
</tr>
<tr>
<td>Tarone–Ware</td>
<td>(\sqrt{n_i})</td>
</tr>
<tr>
<td>Peto–Peto–Prentice</td>
<td>(\hat{S}(t_i))</td>
</tr>
<tr>
<td>Fleming–Harrington</td>
<td>(\hat{S}(t_{i-1})^p {1 - \hat{S}(t_{i-1})}^q)</td>
</tr>
</tbody>
</table>

where \(\hat{S}(t_i)\) is the estimated Kaplan–Meier survivor-function value for the combined sample at failure time \(t_i\), \(\hat{S}(t_i)\) is a modified estimate of the overall survivor function described in \textit{Methods and formulas}, and \(n_i\) is the number of subjects in the risk pool at failure time \(t_i\).

These tests are appropriate for testing the equality of survivor functions across two or more groups. Up to 800 groups are allowed.

The “Cox” test is related to the log-rank test but is performed as a likelihood-ratio test (or, alternatively, as a Wald test) on the results from a Cox proportional hazards regression. The log-rank test should be preferable to what we have labeled the Cox test, but with \texttt{pweight}ed data the log-rank test is not appropriate. Whether you perform the log-rank or Cox test makes little substantive difference with most datasets.
sts test, trend can be used to test against the alternative hypothesis that the failure rate increases or decreases as the level of the \( k \) groups increases or decreases. This test is appropriate only when there is a natural ordering of the comparison groups, for example, when each group represents an increasing or decreasing level of a therapeutic agent.

* trend* is not valid when *cox* is specified.

### The log-rank test

*sts test*, by default, performs the log-rank test, which is, to be clear, the exponential scores test (Savage 1956; Mantel and Haenszel 1959; Mantel 1963, 1966). This test is most appropriate when the hazard functions are thought to be proportional across the groups if they are not equal.

This test statistic is constructed by giving equal weights to the contribution of each failure time to the overall test statistic.

In *Testing equality of survivor functions* in [ST] sts, we demonstrated the use of this command with the heart transplant data, a multiple-record, single-failure st dataset.

```stata
. use https://www.stata-press.com/data/r18/stan3
(Heart transplant data)
. sts test posttran
Failure _d: died
Analysis time _t: t1
ID variable: id
Equality of survivor functions
Log-rank test

+-----------------+------------------+
|        | Observed | Expected |
|-----------------+------------------|
|     posttran    |        events    |       events |
|-----------------+------------------|
|     0           |           30     |        31.20 |
|     1           |           45     |        43.80 |
| Total           |          75      |        75.00 |
| chi2(1)        = 0.13          | Pr>chi2 = 0.7225 |

We cannot reject the hypothesis that the survivor functions are the same.

*sts test*, *logrank* can also perform the stratified log-rank test. Say that it is suggested that calendar year of acceptance also affects survival and that there are three important periods: 1967–1969, 1970–1972, and 1973–1974. Therefore, a stratified test should be performed:

```stata
. stset, noshow
. generate group = 1 if year <= 69
(117 missing values generated)
. replace group=2 if year>=70 & year<=72
(78 real changes made)
. replace group=3 if year>=73
(39 real changes made)
```
. sts test posttran, strata(group)

Equality of survivor functions
Stratified log-rank test
Strata: group

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.51</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.49</td>
</tr>
</tbody>
</table>

Total          75               75.00

* aggregated over stratum-specific results

chi2(1) = 0.20
Pr>chi2 = 0.6547

When a stratified test is requested, Stata computes the expected events under the null hypothesis separately for each stratum, and those results are aggregated over strata.

We now want to show the within-stratum tests, so we type

. sts test posttran, strata(group) detail

Equality of survivor functions
Stratified log-rank test
Strata: group

-> group = 1

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14</td>
<td>13.59</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>17.41</td>
</tr>
</tbody>
</table>

Total          31               31.00

chi2(1) = 0.03
Pr>chi2 = 0.8558

-> group = 2

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13</td>
<td>13.63</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>19.37</td>
</tr>
</tbody>
</table>

Total          33               33.00

chi2(1) = 0.09
Pr>chi2 = 0.7663

-> group = 3

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>4.29</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>6.71</td>
</tr>
</tbody>
</table>

Total          11               11.00

chi2(1) = 0.91
Pr>chi2 = 0.3410
The Wilcoxon (Breslow–Gehan) test

`sts test, wilcoxon` performs the generalized Wilcoxon test of Breslow (1970) and Gehan (1965). This test is appropriate when hazard functions are thought to vary in ways other than proportionally and when censoring patterns are similar across groups.

The Wilcoxon test statistic is constructed by weighting the contribution of each failure time to the overall test statistic by the number of subjects at risk. Thus it gives heavier weights to earlier failure times when the number at risk is higher. As a result, this test is susceptible to differences in the censoring pattern of the groups.

`sts test, wilcoxon` works the same way as `sts test, logrank`:

```
. sts test posttran, wilcoxon
Equality of survivor functions
Wilcoxon-Breslow-Gehan test

    posttran |  Observed  |  Expected  |    Sum of
   events  | events*    |   ranks    |
----------|-----------|-----------|-----------
    0       |    30     |    31.20  |     -85   |
    1       |    45     |    43.80  |      85   |
  Total     |    75     |    75.00  |       0   |

   chi2(1) =  0.14
   Pr>chi2 =  0.7083
```

With the `strata()` option, `sts test, wilcoxon` performs the stratified test:

```
. sts test posttran, wilcoxon strata(group)
Equality of survivor functions
Stratified Wilcoxon-Breslow-Gehan test
Strata: group

    posttran |  Observed  |  Expected  |    Sum of
   events  | events*    |   ranks*   |
----------|-----------|-----------|-----------
    0       |    30     |    31.51  |     -40   |
    1       |    45     |    43.49  |      40   |
  Total     |    75     |    75.00  |       0   |

   chi2(1) =  0.22
   Pr>chi2 =  0.6385
```

As with `sts test, logrank`, you can also specify the `detail` option to see the within-stratum tests.
The Tarone–Ware test

sts test, tware performs a test suggested by Tarone and Ware (1977), with weights equal to the square root of the number of subjects in the risk pool at time $t_i$.

Like Wilcoxon’s test, this test is appropriate when hazard functions are thought to vary in ways other than proportionally and when censoring patterns are similar across groups. The test statistic is constructed by weighting the contribution of each failure time to the overall test statistic by the square root of the number of subjects at risk. Thus, like the Wilcoxon test, it gives heavier weights, although not as large, to earlier failure times. Although less susceptible to the failure and censoring pattern in the data than Wilcoxon’s test, this could remain a problem if large differences in these patterns exist between groups.

sts test, tware works the same way as sts test, logrank:

```
. sts test posttran, tware
Equality of survivor functions
Tarone–Ware test

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
<th>Sum of ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.20</td>
<td>-9.3375685</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.80</td>
<td>9.3375685</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>0</td>
</tr>
</tbody>
</table>

chi2(1) = 0.12
Pr>chi2 = 0.7293
```

With the strata() option, sts test, tware performs the stratified test:

```
. sts test posttran, tware strata(group)
Equality of survivor functions
Stratified Tarone–Ware test
Strata: group

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events*</th>
<th>Sum of ranks*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.51</td>
<td>-7.4679345</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.49</td>
<td>7.4679345</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>0</td>
</tr>
</tbody>
</table>

* aggregated over stratum-specific results

chi2(1) = 0.21
Pr>chi2 = 0.6464
```

As with sts test, logrank, you can also specify the detail option to see the within-stratum tests.

The Peto–Peto–Prentice test

sts test, peto performs an alternative to the Wilcoxon test proposed by Peto and Peto (1972) and Prentice (1978). The test uses as the weight function an estimate of the overall survivor function, which is similar to that obtained using the Kaplan–Meier estimator. See Methods and formulas for details.
This test is appropriate when hazard functions are thought to vary in ways other than proportionally, but unlike the Wilcoxon–Breslow–Gehan test, it is not affected by differences in censoring patterns across groups.

\texttt{sts test, peto} works the same way as \texttt{sts test, logrank}:

\begin{verbatim}
. sts test posttran, peto
Equality of survivor functions
Peto-Peto-Prentice test

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
<th>Sum of ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.20</td>
<td>-.86708453</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.80</td>
<td>.86708453</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>0</td>
</tr>
</tbody>
</table>

\text{chi2(1) = 0.15}
\text{Pr>chi2 = 0.6979}
\end{verbatim}

With the \texttt{strata()} option, \texttt{sts test, peto} performs the stratified test:

\begin{verbatim}
. sts test posttran, peto strata(group)
Equality of survivor functions
Stratified Peto-Peto-Prentice test
Strata: group

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events*</th>
<th>Sum of ranks*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.51</td>
<td>-1.4212233</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.49</td>
<td>1.4212233</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>0</td>
</tr>
</tbody>
</table>

* aggregated over stratum-specific results
\text{chi2(1) = 0.43}
\text{Pr>chi2 = 0.5129}
\end{verbatim}

As with the previous tests, you can also specify the \texttt{detail} option to see the within-stratum tests.

\textbf{The generalized Fleming–Harrington tests}

\texttt{sts test, fh(p q)} performs the Harrington and Fleming (1982) class of test statistics. The weight function at each distinct failure time, \( t \), is the product of the Kaplan–Meier survivor estimate at time \( t - 1 \) raised to the \( p \) power and \( 1 - \) the Kaplan–Meier survivor estimate at time \( t - 1 \) raised to the \( q \) power. Thus, when specifying the Fleming and Harrington option, you must specify two nonnegative arguments, \( p \) and \( q \).

When \( p > q \), the test gives more weights to earlier failures than to later ones. When \( p < q \), the opposite is true, and more weight is given to later failures than to earlier ones. When \( p \) and \( q \) are both zero, the weight is 1 at all failure times and the test reduces to the log-rank test.
sts test, fh(p q) works the same way as sts test, logrank. If we specify \( p = 0 \) and \( q = 0 \) we will get the same results as the log-rank test:

```
. sts test posttran, fh(0 0)

Equality of survivor functions
Fleming-Harrington test
Powers: p = 0, q = 0

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
<th>Sum of ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.20</td>
<td>-1.1995511</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.80</td>
<td>1.1995511</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>0</td>
</tr>
</tbody>
</table>

chi2(1) = 0.13
Pr>chi2 = 0.7225
```

We could, for example, give more weight to later failures than to earlier ones.

```
. sts test posttran, fh(0 3)

Equality of survivor functions
Fleming-Harrington test
Powers: p = 0, q = 3

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
<th>Sum of ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.20</td>
<td>-0.10288411</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.80</td>
<td>0.10288411</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>0</td>
</tr>
</tbody>
</table>

chi2(1) = 0.01
Pr>chi2 = 0.9065
```

Similarly to the previous tests, with the `strata()` option, `sts test, fh()` performs the stratified test:

```
. sts test posttran, fh(0 3) strata(group)

Equality of survivor functions
Stratified Fleming-Harrington test
Powers: p = 0, q = 3
Strata: group

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events*</th>
<th>Sum of ranks*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.51</td>
<td>-0.05315105</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.49</td>
<td>0.05315105</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>0</td>
</tr>
</tbody>
</table>

* aggregated over stratum-specific results
chi2(1) = 0.00
Pr>chi2 = 0.9494
```

As with the other tests, you can also specify the `detail` option to see the within-stratum tests.
The “Cox” test

The term Cox test is our own, and this test is a variation on the log-rank test using Cox regression.

One way of thinking about the log-rank test is as a Cox proportional hazards model on indicator variables for each of the groups. The log-rank test is a test that the coefficients are zero or, if you prefer, that the hazard ratios are one. The log-rank test is, in fact, a score test of that hypothesis performed on a slightly different (partial) likelihood function that handles ties more accurately.

Many researchers think that a (less precise) score test on the precise likelihood function is preferable to a (more precise) likelihood-ratio test on the approximate likelihood function used in Cox regression estimation. In our experience, it makes little difference:

```
. sts test posttran, cox
Equality of survivor functions
Cox regression-based test

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
<th>Relative hazard</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>31.20</td>
<td>0.9401</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>43.80</td>
<td>1.0450</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>75.00</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
```

Note: Reporting LR model test from Cox regression on `posttran`.

By comparison, `sts test, logrank` also reported $\chi^2 = 0.13$, although the significance level was 0.7225, meaning that the $\chi^2$ values differed in the fourth digit. As mentioned by Kalbfleisch and Prentice (2002, 20), a primary advantage of the log-rank test is the ease with which it can be explained to nonstatisticians, because the test statistic is the difference between the observed and expected number of failures within groups.

Our purpose in offering `sts test, cox` is not to promote its use instead of the log-rank test but to provide a test for researchers with sample-weighted data.

If you have sample weights (if you specified `pweight` when you `stset` the data), you cannot run the log-rank or Wilcoxon tests. The Cox regression model, however, has been generalized to sample-weighted data, and Stata’s `stcox` can fit models with such data. In sample-weighted data, the likelihood-ratio statistic is no longer appropriate, but the Wald test based on the robust estimator of variance is.
Thus if we treated these data as sample-weighted data, we would obtain

```
. generate one = 1
. stset t1 [pw=one], id(id) time0(_t0) failure(died) noshow
```

**Survival-time data settings**

- **ID variable:** id
- **Failure event:** died!=0 & died<.
- **Observed time interval:** (_t0, t1]
- **Exit on or before:** failure
- **Weight:** [pweight=one]

---

**sts test test equality of survivor functions**

**Cox regression-based test**

<table>
<thead>
<tr>
<th>posttran</th>
<th>Observed events</th>
<th>Expected events</th>
<th>Relative hazard</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30.00</td>
<td>31.20</td>
<td>0.9401</td>
</tr>
<tr>
<td>1</td>
<td>45.00</td>
<td>43.80</td>
<td>1.0450</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>75.00</strong></td>
<td><strong>75.00</strong></td>
<td><strong>1.0000</strong></td>
</tr>
</tbody>
</table>

\[ \text{Wald chi2(1) = 0.13} \]
\[ \text{Pr>chi2 = 0.7181} \]

**Note:** Reporting Wald model test from Cox regression on `posttran`.

sts test, cox now reports the Wald statistic, which is, to two digits, 0.13, just like all the others.

**The trend test**

When the groups to be compared have a natural order, such as increasing or decreasing age groups or drug dosage, you may want to test the null hypothesis that there is no difference in failure rate among the groups versus the alternative hypothesis that the failure rate increases or decreases as you move from one group to the next.

We illustrate this test with a dataset from a carcinogenesis experiment reprinted in *Marubini and Valsecchi (1995, 126)*. Twenty-nine experimental animals were exposed to three levels (0, 1.5, 2.0) of a carcinogenic agent. The time in days to tumor formation was recorded. Here are a few of the observations:
. use https://www.stata-press.com/data/r18/marubini, clear
. list time event group dose in 1/9

<table>
<thead>
<tr>
<th>time</th>
<th>event</th>
<th>group</th>
<th>dose</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>1</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>150</td>
<td>1</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>47</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>75</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>58</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>136</td>
<td>1</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>58</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>150</td>
<td>1</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>43</td>
<td>0</td>
<td>2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

In these data, there are two variables that indicate exposure level. The group variable is coded 1, 2, and 3, indicating a one-unit separation between exposures. The dose variable records the actual exposure dosage. To test the null hypothesis of no difference among the survival experience of the three groups versus the alternative hypothesis that the survival experience of at least one of the groups is different, it does not matter if we use group or dose.

. stset time, fail(event) noshow
Survival-time data settings
  Failure event: event!=0 & event<.
  Observed time interval: (0, time]
  Exit on or before: failure

  29 total observations
  0 exclusions

  29 observations remaining, representing
    15 failures in single-record/single-failure data
  2,564 total analysis time at risk and under observation
    At risk from t = 0
    Earliest observed entry t = 0
    Last observed exit t = 246

. sts test group
Equality of survivor functions
Log-rank test

<table>
<thead>
<tr>
<th>group</th>
<th>Observed events</th>
<th>Expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>6.41</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6.80</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1.79</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>15.00</td>
</tr>
</tbody>
</table>

chi2(2) = 8.05
Pr>chi2 = 0.0179
. sts test dose

Equality of survivor functions
Log-rank test

<table>
<thead>
<tr>
<th>dose</th>
<th>Observed events</th>
<th>Expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>6.41</td>
</tr>
<tr>
<td>1.5</td>
<td>6</td>
<td>6.80</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1.79</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>15.00</td>
</tr>
</tbody>
</table>

\[ \chi^2(2) = 8.05 \]
\[ \text{Pr}>\chi^2 = 0.0179 \]

For the trend test, however, the distance between the values is important, so using `group` or `dose` will produce different results.

. sts test group, trend

Equality of survivor functions
Log-rank test

<table>
<thead>
<tr>
<th>group</th>
<th>Observed events</th>
<th>Expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>6.41</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6.80</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1.79</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>15.00</td>
</tr>
</tbody>
</table>

\[ \chi^2(2) = 8.05 \]
\[ \text{Pr}>\chi^2 = 0.0179 \]

Test for trend in survivor functions
\[ \chi^2(1) = 5.87 \]
\[ \text{Pr}>\chi^2 = 0.0154 \]

. sts test dose, trend

Equality of survivor functions
Log-rank test

<table>
<thead>
<tr>
<th>dose</th>
<th>Observed events</th>
<th>Expected events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>6.41</td>
</tr>
<tr>
<td>1.5</td>
<td>6</td>
<td>6.80</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1.79</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>15.00</td>
</tr>
</tbody>
</table>

\[ \chi^2(2) = 8.05 \]
\[ \text{Pr}>\chi^2 = 0.0179 \]

Test for trend in survivor functions
\[ \chi^2(1) = 3.66 \]
\[ \text{Pr}>\chi^2 = 0.0557 \]

Although the above trend test was constructed using the log-rank test, any of the previously mentioned weight functions can be used. For example, a trend test on the data can be performed using the same weights as the Peto–Peto–Prentice test by specifying the `peto` option.
14  \textit{sts test} — Test equality of survivor functions

\begin{verbatim}
. sts test dose, trend peto
Equality of survivor functions
Peto-Peto-Prentice test

<table>
<thead>
<tr>
<th>dose</th>
<th>Observed events</th>
<th>Expected events</th>
<th>Sum of ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>6.41</td>
<td>-1.2792221</td>
</tr>
<tr>
<td>1.5</td>
<td>6</td>
<td>6.80</td>
<td>-1.3150418</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1.79</td>
<td>2.5942639</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>15.00</td>
<td>0</td>
</tr>
</tbody>
</table>

\text{chi}^2(2) = 8.39
\text{Pr} > \text{chi}^2 = 0.0150

Test for trend in survivor functions
\text{chi}^2(1) = 2.85
\text{Pr} > \text{chi}^2 = 0.0914

\textbf{Video example}

How to test the equality of survivor functions using nonparametric tests

\textbf{Stored results}

\textit{sts test} stores the following in \texttt{r()}:  

\begin{verbatim}
Scalars
r(df)     degrees of freedom
r(df_tr)  degrees of freedom, trend test
r(chi2)   \chi^2
r(chi2_tr)\chi^2, trend test
\end{verbatim}

\textbf{Methods and formulas}

Let $t_1 < t_2 < \cdots < t_k$ denote the ordered failure times; let $d_j$ be the number of failures at $t_j$ and $n_j$ be the population at risk just before $t_j$; and let $d_{ij}$ and $n_{ij}$ denote the same things for group $i$, $i = 1, \ldots, r$.

We are interested in testing the null hypothesis

$$H_0: \lambda_1(t) = \lambda_2(t) = \cdots = \lambda_r(t)$$

where $\lambda(t)$ is the hazard function at time $t$, against the alternative hypothesis that at least one of the $\lambda_i(t)$ is different for some $t_j$.

As described in Klein and Moeschberger (2003, 205–216), Kalbfleisch and Prentice (2002, 20–22), and Collett (2015, 50–51), if the null hypothesis is true, the expected number of failures in group $i$ at time $t_j$ is $e_{ij} = n_{ij}d_j/n_j$, and the test statistic

$$u' = \sum_{j=1}^k W(t_j)(d_{1j} - e_{1j}, \ldots, d_{rj} - e_{rj})$$
is formed. \( W(t_j) \) is a positive weight function defined as zero when \( n_{ij} \) is zero. The various test statistics are obtained by selecting different weight functions, \( W(t_j) \). See the table in the Remarks and examples section of this entry for a list of these weight functions. For the Peto–Peto–Prentice test,

\[
W(t_j) = \tilde{S}(t_j) = \prod_{\ell: t_\ell \leq t_j} \left( 1 - \frac{d_\ell}{n_\ell + 1} \right)
\]

The variance matrix \( V \) for \( u \) has elements

\[
V_{il} = \sum_{j=1}^{k} \frac{W(t_j)^2 n_{ij} d_j (n_j - d_j)}{n_j (n_j - 1)} \left( \delta_{il} - \frac{n_{ij}}{n_j} \right)
\]

where \( \delta_{il} = 1 \) if \( i = l \) and \( \delta_{il} = 0 \), otherwise.

For the unstratified test, statistic \( u'V^{-1}u \) is distributed as \( \chi^2 \) with \( r - 1 \) degrees of freedom.

For the stratified test, let \( u_s \) and \( V_s \) be the results of performing the above calculation separately within stratum, and define \( u = \sum_s u_s \) and \( V = \sum_s V_s \). The \( \chi^2 \) test is given by \( u'V^{-1}u \) redefined in this way.

The “Cox” test is performed by fitting a (possibly stratified) Cox regression using \texttt{stcox} on \( r - 1 \) indicator variables, one for each group with one of the indicators omitted. The \( \chi^2 \) test reported is then the likelihood-ratio test (no \texttt{pweights}) or the Wald test (based on the robust estimate of variance); see [ST] \texttt{stcox}.

The reported relative hazards are the exponentiated coefficients from the Cox regression renormalized, and the renormalization plays no role in calculating the test statistic. The renormalization is chosen so that the expected-number-of-failures-within-group weighted average of the regression coefficients is 0 (meaning that the hazard is 1). Let \( b_i, i = 1, \ldots, r - 1 \), be the estimated coefficients, and define \( b_r = 0 \). The constant \( K \) is then calculated with

\[
K = \sum_{i=1}^{r} e_i b_i / d
\]

where \( e_i = \sum_j e_{ij} \) is the expected number of failures for group \( i \), \( d \) is the total number of failures across all groups, and \( r \) is the number of groups. The reported relative hazards are \( \exp(b_i - K) \).

The trend test assumes that there is natural ordering of the \( r \) groups, \( r > 2 \). Here we are interested in testing the null hypothesis

\[
H_0: \lambda_1(t) = \lambda_2(t) = \cdots = \lambda_r(t)
\]

against the alternative hypothesis

\[
H_a: \lambda_1(t) \leq \lambda_2(t) \leq \cdots \leq \lambda_r(t)
\]
The test uses $u$ as previously defined with any of the available weight functions. The test statistic is given by

$$\left(\sum_{i=1}^{r} a_i u_i\right)^2 \over a'V a$$

where $a_1 \leq a_2 \leq \cdots \leq a_r$ are scores defining the relationship of interest. A score is assigned to each comparison group, equal to the value of the grouping variable for that group. $a$ is the vector of these scores.

References


Also see

[ST] stcox — Cox proportional hazards model

[ST] sts — Generate, graph, list, and test the survivor and related functions

[ST] sts generate — Create variables containing survivor and related functions

[ST] sts graph — Graph the survivor or related function

[ST] sts list — List the survivor or related function

[ST] stset — Declare data to be survival-time data

[PSS-2] power exponential — Power analysis for a two-sample exponential test

[PSS-2] power logrank — Power analysis for the log-rank test