spxtregress — Spatial autoregressive models for panel data

Description

spxtregress fits spatial autoregressive (SAR) models, also known as simultaneous autoregressive models, for panel data. The commands spxtregress, fe and spxtregress, re are extensions of xtreg, fe and xtreg, re for spatial data; see [XT] xtreg.

If you have not read [SP] Intro 1–[SP] Intro 8, you should do so before using spxtregress.

To use spxtregress, your data must be Sp data and xtset. See [SP] Intro 3 for instructions on how to prepare your data.

To specify spatial lags, you will need to have one or more spatial weighting matrices. See [SP] Intro 2 and [SP] spmatrix for an explanation of the types of weighting matrices and how to create them.

Quick start

SAR fixed-effects model of y on x1 and x2 with a spatial lag of y specified by the spatial weighting matrix W

spxtregress y x1 x2, fe dvarlag(W)

Add a spatially lagged error term also specified by W

spxtregress y x1 x2, fe dvarlag(W) errorlag(W)

Add spatial lags of covariates x1 and x2

spxtregress y x1 x2, fe dvarlag(W) errorlag(W) ivarlag(W: x1 x2)

Add an additional spatial lag of the covariates specified by the matrix M

spxtregress y x1 x2, fe dvarlag(W) errorlag(W) ivarlag(W: x1 x2) ivarlag(M: x1 x2)

SAR random-effects model

spxtregress y x1 x2, re dvarlag(W) errorlag(W) ivarlag(W: x1 x2) ivarlag(M: x1 x2)

An re model with panel effects that follow the same spatial process as the errors using sarpanel

spxtregress y x1 x2, re sarpanel dvarlag(W) errorlag(W) ivarlag(W: x1 x2) ivarlag(M: x1 x2)
Menu

Statistics > Spatial autoregressive models

Syntax

Fixed-effects maximum likelihood

\texttt{spxtregress \texttt{depvar} \[\texttt{indepvars}\] \[\texttt{if} \] \[\texttt{in}\], \texttt{fe} \[\texttt{fe_options}\]}

Random-effects maximum likelihood

\texttt{spxtregress \texttt{depvar} \[\texttt{indepvars}\] \[\texttt{if} \] \[\texttt{in}\], \texttt{re} \[\texttt{re_options}\]}

\texttt{fe_options} \quad \begin{tabular}{ll}
\hline
\texttt{fe} & use fixed-effects estimator \\
\texttt{dvarlag(\texttt{spmatname})} & spatially lagged dependent variable \\
\texttt{errorlag(\texttt{spmatname})} & spatially lagged errors \\
\texttt{ivarlag(\texttt{spmatname} : \texttt{varlist})} & spatially lagged independent variables; repeatable \\
\texttt{force} & allow estimation when estimation sample is a subset of the sample used to create the spatial weighting matrix \\
\texttt{gridsearch(#)} & resolution of the initial-value search grid; seldom used \\
\end{tabular}

\texttt{display_options} \quad \begin{tabular}{ll}
\hline
\texttt{level(#)} & set confidence level; default is \texttt{level(95)} \\
\end{tabular}

\begin{tabular}{ll}
\hline
\texttt{maximize_options} & control the maximization process; seldom used \\
\texttt{coeflegend} & display legend instead of statistics \\
\end{tabular}
**spxtregress** — Spatial autoregressive models for panel data

### Description

**Model**

- `*re` use random-effects estimator
- `dvarlag(spmatname)` spatially lagged dependent variable
- `errorlag(spmatname)` spatially lagged errors
- `ivarlag(spmatname : varlist)` spatially lagged independent variables; repeatable
- `sarpanel` alternative formulation of the estimator in which the panel effects follow the same spatial process as the errors
- `noconstant` suppress constant term
- `force` allow estimation when estimation sample is a subset of the sample used to create the spatial weighting matrix

### Reporting

- `level(#)` set confidence level; default is `level(95)`
- `display_options` control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

### Maximization

- `maximize_options` control the maximization process; seldom used
- `coeflegend` display legend instead of statistics

---

* You must specify either `fe` or `re`.

- `indepvars` and `varlist` specified in `ivarlag()` may contain factor variables; see [U] 11.4.3 Factor variables.
- `coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

---

### Options for spxtregress, fe

**fe** requests the fixed-effects regression estimator.

- `dvarlag(spmatname)` specifies a spatial weighting matrix that defines a spatial lag of the dependent variable. Only one `dvarlag()` option may be specified. By default, no spatial lags of the dependent variable are included.

- `errorlag(spmatname)` specifies a spatial weighting matrix that defines a spatially lagged error. Only one `errorlag()` option may be specified. By default, no spatially lagged errors are included.

- `ivarlag(spmatname : varlist)` specifies a spatial weighting matrix and a list of independent variables that define spatial lags of the variables. This option is repeatable to allow spatial lags created from different matrices. By default, no spatial lags of the independent variables are included.

- `force` requests that estimation be done when the estimation sample is a proper subset of the sample used to create the spatial weighting matrices. The default is to refuse to fit the model. Weighting matrices potentially connect all the spatial units. When the estimation sample is a subset of this space, the spatial connections differ and spillover effects can be altered. In addition, the normalization of the weighting matrix differs from what it would have been had the matrix been normalized over the estimation sample. The better alternative to `force` is first to understand the spatial space of the estimation sample and, if it is sensible, then create new weighting matrices for it. See [SP] spmatrix and Missing values, dropped observations, and the W matrix in [SP] Intro 2.
gridsearch(#) specifies the resolution of the initial-value search grid. The default is gridsearch(0.1). You may specify any number between 0.001 and 0.1 inclusive.

Reporting

level(#); see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize_options: difficult, technique(algorithm_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), and nonrtolerance; see [R] Maximize.

The following option is available with spxtregress, fe but is not shown in the dialog box: coeflegend; see [R] Estimation options.

Options for spxtregress, re

re requests the generalized least-squares random-effects estimator.

dvarlag(spmatname) specifies a spatial weighting matrix that defines a spatial lag of the dependent variable. Only one dvarlag() option may be specified. By default, no spatial lags of the dependent variable are included.

errorlag(spmatname) specifies a spatial weighting matrix that defines a spatially lagged error. Only one errorlag() option may be specified. By default, no spatially lagged errors are included.

ivarlag(spmatname : varlist) specifies a spatial weighting matrix and a list of independent variables that define spatial lags of the variables. This option is repeatable to allow spatial lags created from different matrices. By default, no spatial lags of the independent variables are included.

sarpanel requests an alternative formulation of the estimator in which the panel effects follow the same spatial process as the errors. By default, the panel effects are included in the estimation equation as an additive term, just as they are in the standard nonspatial random-effects model. When sarpanel and errorlag(spmatname) are specified, the panel effects also have a spatial autoregressive form based on spmatname. If errorlag() is not specified with sarpanel, the estimator is identical to the estimator when sarpanel is not specified. The sarpanel estimator was originally developed by Kapoor, Kelejian, and Prucha (2007); see Methods and formulas.

noconstant; see [R] Estimation options.

force requests that estimation be done when the estimation sample is a proper subset of the sample used to create the spatial weighting matrices. The default is to refuse to fit the model. This is the same force option described for use with spxtregress, fe.
Reporting level(#); see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvalabel, fwrap(#), fwrapon(style), cformat(%,fint), pformat(%,fint), sformat(%,fint), and nolstretch; see [R] Estimation options.

Maximization maximize_options: difficult, technique(algorithm_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), rtolerance(#), and nonrtolerance; see [R] Maximize.

The following option is available with spxtregress, re but is not shown in the dialog box: coeflegend; see [R] Estimation options.

Remarks and examples stata.com

See [SP] Intro for an overview of SAR models.

Datasets for Sp panel models contain observations on geographical areas or other units with multiple observations on each unit. See [SP] Intro 3 for an explanation of how to work with Sp panel data. The data must be xtset and must be strongly balanced. There must be a within-panel identifier, a variable indicating time or the equivalent, and the values of this identifier must be the same for every panel. The command spbalance will strongly balance datasets that are not strongly balanced. See [SP] Intro 3, [SP] Intro 7, and [SP] spbalance.

Remarks and examples are presented under the following headings:

- Sp panel models
- The fixed-effects model
- The random-effects model
- The random-effects model with autoregressive panel effects
- Differences among models
- Examples

Sp panel models

Both the fixed-effects and the random-effects models for spatial panel data can be written as

\[ y_{nt} = \lambda W y_{nt} + X_{nt} \beta + c_n + u_{nt} \]
\[ u_{nt} = \rho M u_{nt} + v_{nt} \quad t = 1, 2, \ldots, T \]  

where \( y_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})' \) is an \( n \times 1 \) vector of observations for the dependent variable for time period \( t \) with \( n \) number of panels; \( X_{nt} \) is a matrix of time-varying regressors; \( c_n \) is a vector of panel-level effects; \( u_{nt} \) is the spatially lagged error; \( v_{nt} \) is a vector of disturbances and is independent and identically distributed (i.i.d.) across panels and time with variance \( \sigma^2 \); and \( W \) and \( M \) are spatial weighting matrices.
The fixed-effects model

For fixed effects, \texttt{spxtregress}, \texttt{fe} implements the quasi–maximum likelihood (QML) estimator in Lee and Yu (2010a) to fit the model. A transformation is used to eliminate the fixed effects from the equations, yielding

\[
\begin{align*}
\tilde{y}_{nt} &= \lambda W \tilde{y}_{nt} + \tilde{X}_{nt} \beta + \tilde{u}_{nt} \\
\tilde{u}_{nt} &= \rho M \tilde{u}_{nt} + \tilde{v}_{nt} \quad t = 1, 2, \ldots, T - 1
\end{align*}
\]

Panel effects, which are effects that are constant within panels, are conditioned out of the likelihood. Only covariates that vary within panels can be fit with this estimator.

The random-effects model

For random effects, \texttt{spxtregress}, \texttt{re} assumes that \(c_n\) in (1) is normal i.i.d. across panels with mean 0 and variance \(\sigma^2_c\). The output of \texttt{spxtregress}, \texttt{re} displays estimates of \(\sigma_c\), labeled as /\sigma_u, and \(\sigma\), labeled as /\sigma_e, which is consistent with how \texttt{xtreg, re} labels the output.

The random-effects model with autoregressive panel effects

The \texttt{sarpanel} option for random-effects models fits a slightly different set of equations from (1):

\[
\begin{align*}
y_{nt} &= \lambda W y_{nt} + X_{nt} \beta + u_{nt} \\
u_{nt} &= \rho M u_{nt} + c_n + v_{nt}, \quad t = 1, 2, \ldots, T
\end{align*}
\]

In this variant due to Kapoor, Kelejian, and Prucha (2007), the panel-level effects \(c_n\) are considered a disturbance in the error equation. Because \(c_n\) enters the equation as an additive term next to \(v_{nt}\), the panel-level effects \(c_n\) have the same autoregressive form as the time-level errors \(v_{nt}\).

Differences among models

All three of the models—\texttt{fe}, \texttt{re}, and \texttt{re sarpanel}—are fit using maximum likelihood (ML) estimation. The differences are 1) \texttt{fe} removes the panel-level effects from the estimation and no distributional assumptions are made about them; 2) \texttt{re} models the panel-level effects as normal i.i.d.; and 3) \texttt{re sarpanel} assumes a normal distribution for panel-level effects but with the same autoregressive form as the time-level errors. The \texttt{fe} model allows the panel-level effects to be correlated with the observed covariates, whereas the \texttt{re} models require that the panel-level effects are independent of the observed covariates. See \textit{Methods and formulas} for details. Also see \textit{Choosing weighting matrices and their normalization} in [SP] \texttt{spregress}; the discussion there applies to these three estimation models.
Examples

➢ Example 1: spxtregress, re

We have data on the homicide rate in counties in southern states of the U.S. for the years 1960, 1970, 1980, and 1990. `homicide_1960_1990.dta` contains `hrate`, the county-level homicide rate per year per 100,000 persons for each of the four years. It also contains `ln_population`, the logarithm of the county population; `ln_density`, the logarithm of the population density; and `gini`, the Gini coefficient for the county, a measure of income inequality where larger values represent more inequality (Gini 1909). The data are an extract of the data originally used by Messner et al. (2000); see Britt (1994) for a literature review of the topic. The 1990 data are used in the examples in [SP] spregress.

We used `spshape2dta` to convert shapefiles into Stata .dta files, and then we merged the data file by county ID with our homicide-rate data. See [SP] Intro 4, [SP] Intro 7, [SP] spshape2dta, and [SP] spset.

Because the analysis dataset and the Stata-formatted shapefile must be in our working directory to `spset` the data, we first save both `homicide_1960_1990.dta` and `homicide_1960_1990.shp.dta` to our working directory by using the `copy` command. We then load the data and type `spset` to see the Sp settings.

```
. use homicide_1960_1990
. spset
Sp dataset homicide_1960_1990.dta
data: cross sectional
spatial-unit id: _ID
coordinates: _CX, _CY (planar)
linked shapefile: homicide_1960_1990_shp.dta
variable _ID does not uniquely identify the observations
r(459);
```

We get an error! The data have not been `xtset`, and spxtregress requires it. Our data consist of 1,412 counties, and for each county we have data for four years. Our data look like this:

```
. list _ID year in 1/8, sepby(_ID)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>_ID</td>
<td>year</td>
</tr>
<tr>
<td>1.</td>
<td>876</td>
</tr>
<tr>
<td>2.</td>
<td>876</td>
</tr>
<tr>
<td>3.</td>
<td>876</td>
</tr>
<tr>
<td>4.</td>
<td>876</td>
</tr>
<tr>
<td>5.</td>
<td>921</td>
</tr>
<tr>
<td>6.</td>
<td>921</td>
</tr>
<tr>
<td>7.</td>
<td>921</td>
</tr>
<tr>
<td>8.</td>
<td>921</td>
</tr>
</tbody>
</table>
```

We type

```
.xtset _ID year
panel variable: _ID (strongly balanced)
time variable: year, 1960 to 1990, but with gaps
delta: 1 unit
```

`xtset` reports that our data are strongly balanced. Each county has data for the same four years. `spxtr` requires the data to be strongly balanced. Missing values in our variables could cause the estimation sample to be unbalanced. The Sp panel estimators will complain, and we will have to make the data strongly balanced for the nonmissing values of the variables in our model. If you get a message that your data are not strongly balanced, see [SP] spbalance.

After having `xtset` our data, we type `spset` to check our Sp settings.

```
.spset
Sp dataset homicide_1960_1990.dta
data: panel
spatial-unit id: _ID
time id: year (see `xtset`)
coordinates: _CX, _CY (planar)
linked shapefile: homicide_1960_1990_shp.dta
```

We first run a nonspatial random-effects model by using `xtreg, re` and include dummies for the years by using the `i.year` factor-variable notation.

```
.xtreg hrate ln_population ln_pdensity gini i.year, re
Random-effects GLS regression Number of obs = 5,648
Group variable: _ID Number of groups = 1,412
R-sq: Obs per group:
    within = 0.0478 min = 4
    between = 0.1666 avg = 4.0
    overall = 0.0905 max = 4
Wald chi2(6) = 414.32
corr(u_i, X) = 0 (assumed) Prob > chi2 = 0.0000
```

Coef. Std. Err. z P>|z| [95% Conf. Interval]

```
hrate
ln_populat-n .4394103 .1830599 2.40 0.016 .0806194 .7982012
ln_pdensity .3220698 .1591778 2.02 0.043 .0100872 .6340525
gini 34.43792 2.905163 11.85 0.000 28.7439 40.13193
year
1970 1.411074 .2579218 5.47 0.000 .9055562 1.916591
1980 1.347822 .2499977 5.39 0.000 .8578352 1.837808
1990 .3668468 .2648395 1.39 0.166 -.1522291 .8859228
_cons -10.07267 1.800932 -5.59 0.000 -13.60243 -6.542908
```

We emphasize that you can ignore the spatial aspect of the data and use any of Stata’s estimation commands even though the data are spatial. Doing that is often a good idea. It provides a baseline against which you can compare subsequent spatial results.

We are now going to estimate a spatial random-effects model. To do that, we need a spatial weighting matrix. We will create one that puts the same positive weight on contiguous counties and a 0 weight on all other counties—a matrix known as a contiguity matrix. We will use the default
spectral normalization for this example. See [sp] spmatrix create. When we create the matrix, we must restrict \texttt{spmatrix create} to one observation per panel. That is easy to do using an \texttt{if} statement:

\begin{verbatim}
.spmatrix create contiguity W if year == 1990
\end{verbatim}

Do not misinterpret the purpose of \texttt{if year == 1990}. The matrix created will be appropriate for creating spatial lags for any year, because our map does not change. If two counties share a border in 1990, they share it in the other years too.

We can now fit our model. We include a spatial lag of the dependent variable and a spatially autoregressive error term.

\begin{verbatim}
.spxtregress hrate ln_population ln_pdensity gini i.year, re dvarlag(W) > errorlag(W)
\end{verbatim}

\begin{verbatim}
(5648 observations)
(5648 observations used)
(data contain 1412 panels (places))
(weighting matrix defines 1412 places)
\end{verbatim}

\textbf{Fitting starting values:}

\begin{verbatim}
Iteration 0:  log likelihood = -13299.332
Iteration 1:  log likelihood = -13298.431
Iteration 2:  log likelihood = -13298.43
Iteration 3:  log likelihood = -13298.43
\end{verbatim}

\textbf{Optimizing concentrated log likelihood:}

\begin{verbatim}
initial:  log likelihood = -18826.009
improve: log likelihood = -18826.009
rescale: log likelihood = -18826.009
rescale eq: log likelihood = -18500.374
Iteration 0:  log likelihood = -18500.374  (not concave)
Iteration 1:  log likelihood = -18473.617  (not concave)
Iteration 2:  log likelihood = -18465.333
Iteration 3:  log likelihood = -18434.609
Iteration 4:  log likelihood = -18356.316
Iteration 5:  log likelihood = -18354.863
Iteration 6:  log likelihood = -18354.84
Iteration 7:  log likelihood = -18354.84
\end{verbatim}

\textbf{Optimizing unconcentrated log likelihood:}

\begin{verbatim}
Iteration 0:  log likelihood = -18354.84
Iteration 1:  log likelihood = -18354.84 (backed up)
\end{verbatim}
Random-effects spatial regression

<table>
<thead>
<tr>
<th></th>
<th>Number of obs</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Group variable: _ID</td>
<td>Number of groups = 1,412</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Obs per group = 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wald chi2(7) = 1421.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Prob &gt; chi2 = 0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Log likelihood = -1.835e+04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pseudo R2 = 0.0911</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| hrate          | Coef. | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|----------------|-------|-----------|------|------|---------------------|
|ln_populat-n   |-.2988716 | .1622148 | -1.84 | 0.065 | -.6168068 to .0190637 |
|ln_density     | .7893219 | .1380612 | 5.72 | 0.000 | .518727 to 1.059917 |
|gini           | 22.77053 | 2.604624 | 8.74 | 0.000 | 17.66556 to 27.87555 |
|year 1970      | .3977166 | .1906034 | 2.09 | 0.037 | .0241408 to .7712924 |
|                | .4033441 | .1825721 | 2.21 | 0.027 | .0455094 to .7611789 |
|                | -.1284627| .1946898 | -0.66| 0.509 | -.5100478 to .2531224 |
|_cons          | -4.182034| 1.607561 | -2.60| 0.009 | -7.332796 to -1.031272 |

|                      | Coef. | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|----------------------|-------|-----------|------|------|---------------------|
|hrate                | .5740163 | .0249799 | 22.98| 0.000 | .5250565 to .622976 |
|e.hrate              |-.4626342| .0508732 | -9.09| 0.000 | -.5623438 to -.3629245 |
|/sigma_u            | 3.087658 | .1046893 | 2.88914 | 3.299816 |
|/sigma_e            | 5.40831 | .0661566 | 5.280188 | 5.539542 |

Wald test of spatial terms: chi2(2) = 713.88 Prob > chi2 = 0.0000

**spxtregress, re** first fits an **spxtregress, fe** model to get starting values. Then, it optimizes the concentrated log likelihood and then optimizes the unconcentrated log likelihood. The final log likelihood of the concentrated will always be equal to the optimized log likelihood of the unconcentrated. The unconcentrated starts at the right point, takes a step to check that it is the right point, backs up to this point, and declares convergence as it should.

We can compare estimates of /sigma_u, the standard deviation of the panel effects, and /sigma_e, the standard deviation of the errors, with those fit by **xtreg, re**. They are similar. We cannot, however, directly compare the coefficient estimates with those of **xtreg, re**. When a spatial lag of the dependent variable is included in the model, covariates have both direct and indirect effects, as explained in example 1 of [SP] **spregress**. To obtain the direct, indirect, and total effects of the covariates, we must use **estat impact**.
Here are the averages of the effects of `gini`:

```stata
. estat impact gini
progress : 100%
Average impacts

<table>
<thead>
<tr>
<th></th>
<th>Delta-Method</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
<td></td>
</tr>
<tr>
<td>direct</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>gini</code></td>
<td>24.1144</td>
<td>2.715901</td>
<td>8.88</td>
<td>0.000</td>
<td>18.79133</td>
<td>29.43747</td>
</tr>
<tr>
<td>indirect</td>
<td><code>gini</code></td>
<td>22.73746</td>
<td>2.787574</td>
<td>8.16</td>
<td>0.000</td>
<td>17.27391</td>
</tr>
<tr>
<td>total</td>
<td><code>gini</code></td>
<td>46.85185</td>
<td>5.126096</td>
<td>9.14</td>
<td>0.000</td>
<td>36.80489</td>
</tr>
</tbody>
</table>
```

The percentages at the top of the output indicate progress in the estimation process. For large datasets, calculating standard errors of the effects can be time consuming, so `estat impact` reports its progress as it does the computations.

`gini` has significant average direct and average indirect effects on `hrate`, with both being positive. An increase in inequality is associated with an increase in the homicide rate.

We used a contiguity weighting matrix `W` for the spatial lags. Alternatively, we can use a weighting matrix based on the inverse distance between counties. We create this matrix, using again the default spectral normalization:

```stata
. spmatrix create idistance M if year == 1990
. spmatrix dir
```

<table>
<thead>
<tr>
<th>Weighting matrix name</th>
<th>N x N</th>
<th>Type</th>
<th>Normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>M</code></td>
<td>1412 x 1412</td>
<td>idistance</td>
<td>spectral</td>
</tr>
<tr>
<td><code>W</code></td>
<td>1412 x 1412</td>
<td>contiguity</td>
<td>spectral</td>
</tr>
</tbody>
</table>
We would like to know if the effects of gini differ over time, so we include an interaction of gini and year in our model, and we use the weighting matrix $M$ that we just created.

```
.spxtregress hrate ln_population ln_pdensity c.gini##i.year, re
dvarlag(M) errorlag(M)
(5648 observations)
(5648 observations used)
(data contain 1412 panels (places))
(weighting matrix defines 1412 places)
(output omitted)
```

Random-effects spatial regression

| Coef. | Std. Err. | z | P>|z| | [95% Conf. Interval] |
|-------|-----------|---|------|------------------|
| hrate | .7908003  | .1764818 | 4.48 | 0.000 | .4449023 1.136698 |
| ln_populat'mn | -.1223671 | .166526  | -0.73 | 0.462 | -.448752 .2040178 |
| ln_pdensity | 17.82039  | 4.278775 | 4.16 | 0.000 | 9.434144 26.20663 |
| gini | 1970 | -2.456656  | 2.303069 | -1.07 | 0.286 | -6.970587  2.057275 |
|      | 1980 | -9.470622  | 2.501527 | -3.79 | 0.000 | -14.37353 -4.567718 |
|      | 1990 | -22.81817  | 2.528685 | -9.02 | 0.000 | -27.7743 -17.86204 |
| year#c.gini | 1970 | 6.664314  | 6.130443 | 1.09 | 0.277 | -5.351133 18.67976 |
|      | 1980 | 24.86122  | 6.715026 | 3.70 | 0.000 | 11.70001 38.02243 |
|      | 1990 | 57.40946  | 6.691086 | 8.58 | 0.000 | 44.29517 70.52374 |
| _cons | -11.17804 | 2.061044 | -5.42 | 0.000 | -15.21762 -7.138471 |

Using the `contrast` command, we test the significance of the gini and year interaction:

```
. contrasts c.gini#year
Contrasts of marginal linear predictions
Margins : asbalanced
```

| df | chi2 | P>|chi2| |
|----|------|------|
| hrate | 3 | 81.59 | 0.0000 |
| year#c.gini | | | |
The interaction is significant. We can explore the effect of \textit{gini} by year using \texttt{estat impact} with an \texttt{if} statement.

```stata
. estat impact gini if year == 1960
progress :100%
Average impacts

<table>
<thead>
<tr>
<th></th>
<th>Delta-Method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>direct</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>17.85376</td>
<td>4.285821</td>
<td>4.17</td>
<td>0.000</td>
<td>9.453709</td>
</tr>
<tr>
<td>indirect</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>37.06435</td>
<td>11.60646</td>
<td>3.19</td>
<td>0.001</td>
<td>14.31612</td>
</tr>
<tr>
<td>total</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>54.91812</td>
<td>14.85782</td>
<td>3.70</td>
<td>0.000</td>
<td>25.79732</td>
</tr>
</tbody>
</table>

. estat impact gini if year == 1970
progress :100%
Average impacts

<table>
<thead>
<tr>
<th></th>
<th>Delta-Method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>direct</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>24.53056</td>
<td>5.033537</td>
<td>4.87</td>
<td>0.000</td>
<td>14.66501</td>
</tr>
<tr>
<td>indirect</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>50.92536</td>
<td>15.21235</td>
<td>3.35</td>
<td>0.001</td>
<td>21.10971</td>
</tr>
<tr>
<td>total</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>75.45591</td>
<td>18.8175</td>
<td>4.01</td>
<td>0.000</td>
<td>38.57429</td>
</tr>
</tbody>
</table>

. estat impact gini if year == 1980
progress :100%
Average impacts

<table>
<thead>
<tr>
<th></th>
<th>Delta-Method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>direct</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>42.76155</td>
<td>5.683654</td>
<td>7.52</td>
<td>0.000</td>
<td>31.62179</td>
</tr>
<tr>
<td>indirect</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>88.77282</td>
<td>23.09515</td>
<td>3.84</td>
<td>0.000</td>
<td>43.50716</td>
</tr>
<tr>
<td>total</td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>gini</td>
<td>131.5344</td>
<td>26.20928</td>
<td>5.02</td>
<td>0.000</td>
<td>80.16512</td>
</tr>
</tbody>
</table>
```
The `if year == ...` statement used with `estat impact` allows us to estimate the average effects for each year. The direct, indirect, and total effects of `gini` trend upward.

Until now, we used the default form of the random-effects estimator. Let’s run the command again, specifying the `sarpanel` option to use the alternative form of the estimator, where the panel-level effects have the same autoregressive form as the time-level errors.
spxtregress — Spatial autoregressive models for panel data

```
. spxtregress hrate ln_population ln_pdensity c.gini##i.year, re sarpanel
> dvarlag(M) errorlag(M)
(5648 observations)
(5648 observations used)
(data contain 1412 panels (places) )
(weighting matrix defines 1412 places)
(output omitted)

Random-effects spatial regression
Group variable: _ID
Number of obs = 5,648
Number of groups = 1,412
Obs per group = 4
Wald chi2(10) = 1136.49
Prob > chi2 = 0.0000
Log likelihood = -1.824e+04 Pseudo R2 = 0.1177

|            | Coef. | Std. Err. | z    | P>|z|  | [95% Conf. Interval] |
|------------|-------|-----------|------|------|----------------------|
| hrate      |       |           |      |      |                      |
| ln_population | .4366742 | .1752502 | 2.49 | 0.013 | .0931901 .7801583    |
| ln_pdensity | .1896  | .1641334  | 1.16 | 0.248 | -.1320955 .5112956  |
| gini       | 18.92328 | 4.42621 | 4.28 | 0.000 | 10.24807 27.59849   |
| year       |       |           |      |      |                      |
| 1970       | -.9590229 | 2.362015 | -0.41 | 0.685 | -5.588488 3.670442  |
| 1980       | -8.19778  | 2.554504  | -3.21 | 0.001 | -13.20452 -3.191045 |
| 1990       | -22.4189  | 2.610152  | -8.59 | 0.000 | -27.53471 -17.3031  |
| year#c.gini|       |           |      |      |                      |
| 1970       | 5.865776  | 6.255297  | 0.94 | 0.348 | -6.39438 18.12593   |
| 1980       | 24.20335  | 6.834194  | 3.54 | 0.000 | 10.80858 37.59812   |
| 1990       | 58.38273  | 6.881893  | 8.48 | 0.000 | 44.89447 71.87099   |
| _cons      | -6.535916 | 2.257841 | -2.89 | 0.004 | -10.9612 -2.110629  |

M
|            | Coef. | Std. Err. | z    | P>|z|  | [95% Conf. Interval] |
|------------|-------|-----------|------|------|----------------------|
| hrate      |       |           |      |      |                      |
| e.hrate    | 2.860571 | 0.0558304 | 51.24 | 0.000 | 2.751145 2.969996   |

/sigma_u 2.686156 .1123355 2.474764 2.915605
/sigma_e 5.609948 .0612095 5.491253 5.731208

Wald test of spatial terms: chi2(2) = 2685.83 Prob > chi2 = 0.0000

The re and re sarpanel estimators give appreciably different estimates for the coefficient of the spatial lag of hrate and for the autoregressive error term. Estimates of other terms are similar. It appears that some of the spatial-lag effect of hrate is being accounted for by the autoregressive form of the panel effects in the sarpanel model.
Example 2: spxregress, fe

The random-effects estimator assumes that the panel-level effects are uncorrelated with the covariates in the model. We can relax that assumption using the fixed-effects estimator.

We will fit fixed-effects models for the same data we used in example 1. Here’s a nonspatial model fit with xtreg, fe.

```
. xtreg hrate ln_population ln_pdensity gini, fe

Fixed-effects (within) regression
Number of obs = 5,648
Group variable: _ID Number of groups = 1,412
R-sq: Obs per group:
within = 0.0356 min = 4
between = 0.0084 avg = 4.0
overall = 0.0131 max = 4
F(3,4233) = 52.04
corr(u_i, Xb) = -0.2819 Prob > F = 0.0000
```

```
hrate       Coef.  Std. Err.      t    P>|t|     [95% Conf. Interval]
------------------------------------------------------------------------------
ln_population  -2.16467   1.702073    -1.27   0.204    -5.501627   1.172286
ln_pdensity    1.007573   1.659751     0.61   0.544    -2.246409   4.261555
gini          35.12694   2.816652    12.47   0.000     29.60483   40.64906
_cons          13.90421   10.91007     1.27   0.203    -7.485242   35.29366
------------------------------------------------------------------------------
sigma_u       5.2469262
sigma_e       5.7428609
rho           0.45496484 (fraction of variance due to u_i)
```

F test that all u_i=0: F(1411, 4233) = 2.61 Prob > F = 0.0000

We now use spxregress, fe and include a spatial lag of the dependent variable hrate.

```
. spxregress hrate ln_population ln_pdensity gini, fe dvarlag(M)
(5648 observations)
(5648 observations used)
(data contain 1412 panels (places) )
(weighting matrix defines 1412 places)
Performing grid search ... finished
Optimizing concentrated log likelihood:
Iteration 0: log likelihood = -13321.27
Iteration 1: log likelihood = -13321.27  (backed up)
Iteration 2: log likelihood = -13321.269
Optimizing unconcentrated log likelihood:
Iteration 0: log likelihood = -13321.269
Iteration 1: log likelihood = -13321.269  (backed up)
```

stralregress — Spatial autoregressive models for panel data
spxtregress, fe does not give an estimate of /sigma_u because the spatial fixed-effects estimator does not give consistent estimates for the levels of the panel fixed effects nor for their standard deviation. See *Methods and formulas*.

We cannot fit a fixed-effects model with all the terms we included in example 1. The *i.year* dummies are not allowed because *spxtregress, fe* assumes individual fixed effects only, as specified in section 2 of Lee and Yu (2010a).
In example 1, we found that gini was an important regressor and that the effect of gini differed across time. We will use Stata’s factor-variable notation and add to the model \( c.gini\#i.year \), which is gini interacted by year without main effects.

```
. spxtregress hrate ln_population ln_pdensity c.gini#i.year, fe
   > dvarlag(M) errorlag(M)
   (5648 observations)
   (5648 observations used)
   (data contain 1412 panels (places) )
   (weighting matrix defines 1412 places)
   (output omitted)
Fixed-effects spatial regression
Number of obs = 5,648
Group variable: _ID
Number of groups = 1,412
Obs per group = 4
Wald chi2(7) = 128.16
Prob > chi2 = 0.0000
Log likelihood = -1.330e+04 Pseudo R2 = 0.0001

hrate       Coef.     Std. Err.     z  P>|z|     [95% Conf. Interval]
hrate      -2.169113   1.70931   -1.27  0.204    -5.519298   1.181073
ln_populat-  -.7395584   1.638919  -0.45  0.652   -3.95178   2.472663
ln_pdensity  1960     4.637191   4.648658   1.00  0.319    -4.474012   13.74839
           1970     11.15786   4.234693   2.63  0.008     2.858016   19.45771
           1980     11.92355   4.158854   2.87  0.004     3.77235   20.07476
           1990     11.13694   3.975612   2.80  0.005     3.344885   18.929
M
hrate      .1251126   .2552473   0.49  0.624   -.3751629   .625388
e.hrate    1.604259   .1898228   8.45  0.000     1.232213   1.976305
/\sigma_e   5.582721   .0606909  5.465027  5.702949
Wald test of spatial terms:  chi2(2) = 116.83  Prob > chi2 = 0.0000
```
We look at the effects:

```
. estat impact
progress : 33% 67% 100%
Average impacts Number of obs = 5,648

<table>
<thead>
<tr>
<th></th>
<th>Direct</th>
<th>Indirect</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>dy/dx</td>
</tr>
<tr>
<td>ln_populat-n</td>
<td>-2.169186</td>
<td>1.709375</td>
<td>-0.2894662</td>
</tr>
<tr>
<td>ln_pdensity</td>
<td>-0.7395835</td>
<td>1.638973</td>
<td>-0.0986934</td>
</tr>
<tr>
<td>gini</td>
<td>9.714218</td>
<td>4.112071</td>
<td>1.29631</td>
</tr>
</tbody>
</table>
```

The output shows the effects of `gini` across all the years. `estat impact` is smart enough to know that there are not `year` effects in the fixed-effects model. When it looks at the term `c.gini#i.year`, it only gives the effects for `gini`. If `year` were replaced by a variable that varied within time, `estat impact` would show the effects for that variable, too.

If we want to see how the effects of `gini` change across the years, we can use `if` with `estat impact` as we did in example 1.

```
. estat impact gini if year == 1960
progress :100%
Average impacts Number of obs = 1,412

<table>
<thead>
<tr>
<th></th>
<th>Direct</th>
<th>Indirect</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>dy/dx</td>
</tr>
<tr>
<td>ln_populat-n</td>
<td>4.637349</td>
<td>4.648981</td>
<td>4.637349</td>
</tr>
<tr>
<td>ln_pdensity</td>
<td>.6188292</td>
<td>1.70156</td>
<td>.6188292</td>
</tr>
<tr>
<td>gini</td>
<td>11.01053</td>
<td>5.357526</td>
<td>11.01053</td>
</tr>
</tbody>
</table>
```
There is no evidence of a trend in the average total effect of \textit{gini} from the \textit{fe} model.
Stored results

\texttt{spxtregress, fe} and \texttt{spxtregress, re} store the following in \texttt{e()}:  

Scalars
\begin{itemize}
  \item \texttt{e(N)} \quad \text{number of observations}
  \item \texttt{e(N_g)} \quad \text{number of groups (panels)}
  \item \texttt{e(g)} \quad \text{group size}
  \item \texttt{e(k)} \quad \text{number of parameters}
  \item \texttt{e(df_m)} \quad \text{model degrees of freedom}
  \item \texttt{e(df_c)} \quad \text{degrees of freedom for test of spatial terms}
  \item \texttt{e(ll)} \quad \text{log likelihood}
  \item \texttt{e(iterations)} \quad \text{number of maximum log-likelihood estimation iterations}
  \item \texttt{e(rank)} \quad \text{rank of } \texttt{e(V)}
  \item \texttt{e(r2_p)} \quad \text{pseudo-}R^2
  \item \texttt{e(chi2)} \quad \chi^2
  \item \texttt{e(chi2_c)} \quad \chi^2 \text{ for test of spatial terms}
  \item \texttt{e(p)} \quad \text{p-value for model test}
  \item \texttt{e(p_c)} \quad \text{p-value for test of spatial terms}
  \item \texttt{e(converged)} \quad 1 \text{ if converged, 0 otherwise}
\end{itemize}

Macros
\begin{itemize}
  \item \texttt{e(cmd)} \quad \texttt{spxtregress}
  \item \texttt{e(cmdline)} \quad \text{command as typed}
  \item \texttt{e(depvar)} \quad \text{name of dependent variable}
  \item \texttt{e(indeps)} \quad \text{names of independent variables}
  \item \texttt{e(idvar)} \quad \text{name of ID variable}
  \item \texttt{e(model)} \quad \texttt{fe, re, or re sarpanel}
  \item \texttt{e(title)} \quad \text{title in estimation output}
  \item \texttt{e(constant)} \quad \texttt{hasconstant or noconstant (re only)}
  \item \texttt{e(dlmat)} \quad \text{name of spatial weighting matrix applied to } \texttt{depvar}
  \item \texttt{e(elmat)} \quad \text{name of spatial weighting matrix applied to errors}
  \item \texttt{e(chi2type)} \quad \text{Wald; type of model } \chi^2 \text{ test}
  \item \texttt{e(vce)} \quad \text{\texttt{oim}}
  \item \texttt{e(ml_method)} \quad \text{type of \texttt{ml} method}
  \item \texttt{e(technique)} \quad \text{maximization technique}
  \item \texttt{e(properties)} \quad \texttt{b V}
  \item \texttt{e(estat_cmd)} \quad \text{program used to implement \texttt{estat}}
  \item \texttt{e(predict)} \quad \text{program used to implement \texttt{predict}}
  \item \texttt{e(marginsok)} \quad \text{predictions allowed by \texttt{margins}}
  \item \texttt{e(asbalanced)} \quad \text{factor variables \texttt{fvset} as \texttt{asbalanced}}
  \item \texttt{e(asobserved)} \quad \text{factor variables \texttt{fvset} as \texttt{asobserved}}
\end{itemize}

Matrices
\begin{itemize}
  \item \texttt{e(b)} \quad \text{coefficient vector}
  \item \texttt{e(ilog)} \quad \text{iteration log (up to 20 iterations)}
  \item \texttt{e(gradient)} \quad \text{gradient vector}
  \item \texttt{e(Hessian)} \quad \text{Hessian matrix}
  \item \texttt{e(V)} \quad \text{variance–covariance matrix of the estimators}
\end{itemize}

Functions
\begin{itemize}
  \item \texttt{e(sample)} \quad \text{marks estimation sample}
\end{itemize}

In addition to the above, the following is stored in \texttt{r()}:  

Matrices
\begin{itemize}
  \item \texttt{r(table)} \quad \text{matrix containing the coefficients with their standard errors, test statistics, } p \text{-values, and confidence intervals}
\end{itemize}

Note that results stored in \texttt{r()} are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.
Methods and formulas

spxtregress, fe estimates the parameters of the SAR model with spatially autoregressive errors and fixed effects using the QML estimator derived by Lee and Yu (2010a).

spxtregress, re estimates the parameters of two different SAR models with spatially autoregressive errors and random effects. In the default model, the random effects enter the equation for the dependent variable linearly. This model and the ML estimator for its parameters were derived by Lee and Yu (2010b). When the sarpanel option is specified, the random effects are subject to the same spatial autoregressive process as the idiosyncratic errors. This model and the ML estimator of its parameters were derived by Lee and Yu (2010b), which builds on the original formulation by Kapoor, Kelejian, and Prucha (2007). All of these papers build on theoretical work in Kelejian and Prucha (2001) and Lee (2004). We use the estimator derived by Baltagi and Liu (2011) to get initial values.

Methods and formulas are presented under the following headings:

Fixed-effects estimators
Random-effects estimators

Fixed-effects estimators

The Lee and Yu (2010a) SAR model for panel data with fixed effects is

\[ y_{nt} = \lambda W y_{nt} + X_{nt}\beta + c_n + u_{nt} \]

\[ u_{nt} = \rho M u_{nt} + v_{nt} \quad t = 1, 2, \ldots, T \]  \hspace{1cm} (2)

where

- \( y_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})' \) is an \( n \times 1 \) vector of observations on the dependent variable for time period \( t \);
- \( X_{nt} \) is an \( n \times k \) matrix of nonstochastic time-varying regressors for time period \( t \). \( X_{nt} \) may also contain spatial lag of exogenous covariates;
- \( c_n \) is an \( n \times 1 \) vector of individual effects;
- \( u_{nt} \) is an \( n \times 1 \) vector of spatially lagged error;
- \( v_{nt} = (v_{1t}, v_{2t}, \ldots, v_{nt})' \) is an \( n \times 1 \) vector of innovations, and \( v_{it} \) is i.i.d. across \( i \) and \( t \) with variance \( \sigma^2 \); and
- \( W \) and \( M \) are \( n \times n \) spatial weighting matrices.

spxtregress, fe estimates the parameters in this model by using the QML estimator derived by Lee and Yu (2010a). Lee and Yu (2010a) uses an orthogonal transformation to remove the fixed effects \( c_n \) without inducing dependence in the transformed errors. The transform \( F_{T,T-1} \) is part of \( [F_{T,T-1}, 1/\sqrt{T}I_T] \), which is the orthonormal eigenvector matrix of \( (I_T - 1/T I_T V_T) \), where \( I_T \) is the \( T \times T \) identity matrix and \( I_T \) is a \( T \times 1 \) vector of 1s. Kuersteiner and Prucha (2015) discuss this class of transforms.

For any \( n \times T \) matrix \( [z_{n1}, z_{n2}, \ldots, z_{nT}] \), the transformed \( n \times (T - 1) \) matrix is defined as

\[ \tilde{z}_{n1}, \tilde{z}_{n2}, \ldots, \tilde{z}_{n,T-1} = [z_{n1}, z_{n2}, \ldots, z_{nT}] F_{T,T-1} \]

Thus, the transformed model for (2) is

\[ \tilde{y}_{nt} = \lambda \tilde{W} \tilde{y}_{nt} + \tilde{X}_{nt}\beta + \tilde{u}_{nt} \]

\[ \tilde{u}_{nt} = \rho \tilde{M} \tilde{u}_{nt} + \tilde{v}_{nt} \quad t = 1, 2, \ldots, T - 1 \]

The transformed innovations \( \tilde{v}_{nt} \) are uncorrelated for all \( i \) and \( t \).
The log-likelihood function for the transformed model is
\[
\ln L_{n,T}(\theta) = -\frac{n(T - 1)}{2} \ln(2\pi \sigma^2) + (T - 1)[\ln|S_n(\lambda)| + \ln|R_n(\rho)|] - \frac{1}{2\sigma^2} \sum_{t=1}^{T-1} \tilde{v}'_t(\theta)\tilde{v}_t(\theta)
\]
where \( S_n(\lambda) = I_n - \lambda W \), \( R_n(\rho) = I_n - \rho M \), and \( \theta = (\beta', \lambda, \rho, \sigma^2)' \).

Random-effects estimators

\texttt{spxtregress, re} fits two different random-effects SAR models for panel data. In the default model, the random effects enter the equation for \( y_{nt} \) linearly.
\[
y_{nt} = \lambda Wy_{nt} + Z_{nt}\beta + c_n + u_{nt}
\]
\[
u_{nt} = \rho M u_{nt} + v_{nt} \quad t = 1, 2, \ldots, T
\] (3)
where
- \( Z_{nt} \) may contain time-variant and -invariant regressors;
- \( c_n \) is random effects with mean 0 and variance \( \sigma_c^2 \); and
- all the other terms are defined as in (2).

When the \texttt{sarpanel} option is specified, \texttt{xtsregress, re} fits a model in which the random effects \( c_n \) are subject to the same spatial autoregressive process as the errors.
\[
y_{nt} = \lambda Wy_{nt} + Z_{nt}\beta + u_{nt}
\]
\[
u_{nt} = \rho M u_{nt} + c_n + v_{nt} \quad t = 1, 2, \ldots, T
\] (4)
When the \( c_n \) are treated as fixed effects and transformed out of the model, the default model in (3) is equivalent to the \texttt{sarpanel} model in (4). When treating the \( c_n \) as random effects, these two models are different.

For (3) or (4), we can stack all the time periods and write the equations as an \( nT \times 1 \) vector form
\[
y_{nT} = \lambda (I_T \otimes W)y_{nT} + Z_{nT}\beta + \xi_{nT}
\] (5)
where
- \( y_{nT} = (y'_{n,1}, y'_{n,2}, \ldots, y'_{n,t})' \) is an \( nT \times 1 \) vector of observations of the dependent variable for \( i = 1, \ldots, n \) and \( t = 1, \ldots, T; \)
- \( v_{nT} = (v'_{n,1}, v'_{n,2}, \ldots, v'_{n,t})' \) is an \( nT \times 1 \) vector of innovations;
- \( Z_{nT} = \{Z'_{n,1}, Z'_{n,2}, \ldots, Z_{nT}(\rho)'\}' \) is an \( nT \times k \) matrix of \( k \) regressors for \( i = 1, \ldots, n \) and \( t = 1, \ldots, T; \) and
- \( \xi_{nT} \) is the overall disturbance \( nT \times 1 \) vector.

For (3), the overall disturbance vector \( \xi_{nT} \) is
\[
\xi_{nT} = I_n \otimes c_n + \{I_T \otimes R_n(\rho)^{-1}\}v_{nT}
\]
where \( R_n(\rho) = I_n - \rho M \). Its variance matrix is
\[
\Omega_{nT}(\theta) = \sigma_c^2 (I_T 1_T' \otimes I_T) + \sigma^2 \{I_T \otimes R_n(\rho)^{-1}R_n'(\rho)^{-1}\}
\]
For (4), the overall disturbance vector $\xi_{nT}$ is

$$
\xi_{nT} = I_T \otimes R_n(\rho)^{-1}c_n + \{I_T \otimes R_n(\rho)^{-1}\}v_{nT}
$$

Its variance matrix is

$$
\Omega_{nT}(\theta) = \sigma_e^2\{I_T \otimes R_n(\rho)^{-1}R_n'(\rho)^{-1}\} + \sigma^2\{I_T \otimes R_n(\rho)^{-1}R_n'(\rho)^{-1}\}
$$

The log-likelihood function for (5) is

$$
\ln L_{nT}(\theta) = -\frac{nT}{2} \ln(2\pi) - \frac{1}{2} \ln|\Omega_{nT}(\theta)| + T \ln|S_n(\lambda)| - \frac{1}{2} \xi_{nT}'(\theta)\Omega_{nT}(\theta)^{-1}\xi_{nT}(\theta)
$$

where $S_n(\lambda) = I_n - \lambda W$, and $\theta = (\beta', \lambda, \rho, \sigma_e^2, \sigma^2)'$.

References


Also see

*[SP] spxregress postestimation* — Postestimation tools for spxregress

*[SP] estat moran* — Moran’s test of residual correlation with nearby residuals

*[SP] Intro* — Introduction to spatial data and SAR models

*[SP] spbalance* — Make panel data strongly balanced

*[SP] spivregress* — Spatial autoregressive models with endogenous covariates

*[SP] spmatrix* — Categorical guide to the spmatrix command

*[SP] spregress* — Spatial autoregressive models

*[XT] xtreg* — Fixed-, between-, and random-effects and population-averaged linear models

*[U] 20 Estimation and postestimation commands*