**Intro 1** — A brief introduction to SAR models

Description Remarks and examples Also see

# Description

Sp can fit models with

1. spatial lags of dependent variables,
2. spatial lags of independent variables, and
3. spatially autoregressive errors.

The spatial features can be used in any combination.

This entry describes the above features and describes SAR models in general.

You may also be interested in introductions to other aspects of Sp. Below, we provide links to those other introductions.

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| Intro 2 | The **W** matrix |
| Intro 3 | Preparing data for analysis |
| Intro 4 | Preparing data: Data with shapefiles |
| Intro 5 | Preparing data: Data containing locations (no shapefiles) |
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| Intro 7 | Example from start to finish |
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# Remarks and examples

SAR models are fit using datasets that contain observations on spatial units such as countries, districts, or even nongeographical units such as social network nodes. For simplicity, we refer to these spatial units as areas. Datasets contain at a minimum a continuous outcome variable, such as incidence of disease, output of farms, or crime rates, along with the other variables assumed to predict the chosen outcome. The dataset could be used to fit a linear regression of the form

$$y_i = \beta_0 + x_{i,1}\beta_1 + x_{i,2}\beta_2 + \cdots + x_{i,k}\beta_k + \epsilon_i$$

This linear regression is provided as a starting point; it is not a SAR model. To give this starting point a spatial feel, we will call the observations areas. The variables contain characteristics of the areas. The notation we will use is

| | |
|---|---|
| $i$ | area (observation), numbered 1 to $N$ |
| $y_i$ | dependent (outcome) variable in area $i$ |
| $x_{i,1}$ | 1st independent variable in area $i$ |
| | ⋮ |
| $x_{i,j}$ | $j$th independent variable in area $i$ |
| | ⋮ |
| $x_{i,k}$ | last independent variable in area $i$ |
| $\epsilon_i$ | error (residual) in area $i$ |

The linear regression model can be written in column-vector notation:

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \cdots + \beta_k\mathbf{x}_k + \boldsymbol{\epsilon}$$

The boldfaced variables are each $N \times 1$ vectors.

You could fit the above model in Stata by typing

```
regress y x1 x2 ... xk
```

SAR models extend linear regression by allowing outcomes in one area to be affected by

1. outcomes in nearby areas,
2. covariates from nearby areas, and
3. errors from nearby areas.

Said in the spatial jargon, models can contain

1. spatial lags of the outcome variable,
2. spatial lags of covariates, and
3. spatially autoregressive errors.

These terms are borrowed from the time-series literature. In time series, an autoregressive AR(1) process is

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \epsilon_t$$

where $y_{t-1}$ is called the lag of $y$. In vector notation, L. is the lag operator, and the above equation could be written as

$$\mathbf{y} = \gamma_0 + \gamma_1 \mathrm{L}.\mathbf{y} + \boldsymbol{\epsilon}$$

Sometimes, AR(1) models also include autoregressive errors:

$$\mathbf{y} = \gamma_0 + \gamma_1 \mathrm{L}.\mathbf{y} + \mathbf{u}$$

where $\mathbf{u} = \rho\mathrm{L}.\mathbf{u} + \boldsymbol{\epsilon}$. In that case, the equation becomes

$$\mathbf{y} = \gamma_0 + \gamma_1 \mathrm{L}.\mathbf{y} + (\mathbf{I} - \rho\mathrm{L}.)^{-1}\boldsymbol{\epsilon}$$

The parameter $\rho$ measures the correlation in the errors and is a parameter to be estimated along with $\gamma_0$ and $\gamma_1$.

The time-series notation and jargon can be translated to the spatial domain. The lag operator becomes an $N \times N$ matrix $\mathbf{W}$. What was L.$\mathbf{y}$ becomes $\mathbf{Wy}$, which means matrix $\mathbf{W}$ multiplied by vector $\mathbf{y}$. The SAR model corresponding to the above time-series equation is

$$\mathbf{y} = \beta_0 + \beta_1 \mathbf{Wy} + \boldsymbol{\epsilon}$$

The SAR model corresponding to the time-series equation with autoregressive errors is

$$\mathbf{y} = \beta_0 + \beta_1 \mathbf{Wy} + (\mathbf{I} - \rho\mathbf{W})^{-1}\boldsymbol{\epsilon}$$

$\mathbf{W}$ is called the spatial weighting matrix. The values in the matrix characterize the spatial relationships between areas. $\mathbf{W}$ is the spatial analog of L.$\mathbf{y}$. Whereas L.$\mathbf{y}$ measures the potential spillover from time $t-1$ to $t$, elements $W_{i_1,i_2}$ specify how much potential spillover there is from area $i_2$ to $i_1$. $W_{i_1,i_2}$ is zero if area $i_2$ can have no effect on $i_1$. The more potential spillover there is, the larger $W_{i_1,i_2}$ is. The elements of $\mathbf{W}$ are specified before the model is fit.

In the mathematics of SAR models:

- $\mathbf{Wy}$ is the spatial equivalent of L.$\mathbf{y}$. Either way, it is the lag of the dependent variable.
- $\mathbf{Wx}_j$ is the spatial equivalent of L.$\mathbf{x}_j$. Either way, it is the lag of the $j$th independent variable.
- $(\mathbf{I} - \rho\mathbf{W})^{-1}\boldsymbol{\epsilon}$ is the spatial equivalent of $(1 - \rho\text{L.})^{-1}\boldsymbol{\epsilon}$. Either way, it is an autoregressive error.

Any of the above could be included in a SAR model.

Recall that the linear regression model we started with was

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \cdots + \beta_k\mathbf{x}_k + \boldsymbol{\epsilon}$$

We will keep the first two explanatory variables and drop the rest. The equation becomes

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \boldsymbol{\epsilon}$$

We could fit the shortened model by typing

```
regress y x1 x2
```

We could add $\mathbf{Wy}$ to the model:

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \beta_3\mathbf{Wy} + \boldsymbol{\epsilon}$$

We could fit this model by typing

```
spregress y x1 x2, gs2sls dvarlag(W)
```

The result would be that $\beta_3\mathbf{W}$ would measure the amount that outcomes are affected by nearby outcomes.

We could add $\mathbf{Wx}_1$ to the model:

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \beta_3\mathbf{Wy} + \beta_4\mathbf{Wx}_1 + \boldsymbol{\epsilon}$$

To fit this model, we would type

```
spregress y x1 x2, gs2sls dvarlag(W) ivarlag(W:x1)
```

The result would be that we would estimate an extra coefficient $\beta_4$ and that $\beta_4\mathbf{W}$ would measure the spillover of $\mathbf{x}_1$.

Spatial models can have more than one weighting matrix. If we had a second weighting matrix $\mathbf{V}$ in addition to $\mathbf{W}$, we could fit the model

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \beta_3\mathbf{W}\mathbf{y} + \beta_4\mathbf{W}\mathbf{x}_1 + \beta_5\mathbf{V}\mathbf{x}_1 + \boldsymbol{\epsilon}$$

by typing

```
spregress y x1 x2, gs2sls dvarlag(W) ivarlag(W:x1) ivarlag(V:x1)
```

We might do this if we were uncertain how spillover from nearby areas affects outcomes. We might be reasonably certain that there are spillover effects from adjacent areas and even from areas adjacent to adjacent areas. Let's call the adjacent areas "first-order neighbors" and the areas adjacent to adjacent areas "second-order neighbors". If we thought half the amount spilled over from second-order neighbors as from first-order neighbors, we would define $\mathbf{W}$ to constrain that by making $W_{i_1,i_2}$ for second-order neighbors half that of first-order neighbors. If we were uncertain about the one-half assumption, we could define $\mathbf{W}$ to allow spillovers only from first-order neighbors and $\mathbf{V}$ to allow spillovers only from second-order neighbors. The spillover effect from $\mathbf{x}_1$ would be $\beta_4\mathbf{W} + \beta_5\mathbf{V}$. The ratio of second- to first-order spillovers would then be $\beta_5/\beta_4$.

Fitting models that estimate instead of imposing such second-order effects is asking a lot of the data. But if you have a sufficient amount of data that support this model, the approach works well.

To keep our model simple, we will remove the second-order lag so that the model reverts to

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \beta_3\mathbf{W}\mathbf{y} + \beta_4\mathbf{W}\mathbf{x}_1 + \boldsymbol{\epsilon}$$

If we added the spatial lag of $\mathbf{x}_2$ to the model, it would become

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \beta_3\mathbf{W}\mathbf{y} + \beta_4\mathbf{W}\mathbf{x}_1 + \beta_5\mathbf{W}\mathbf{x}_2 + \boldsymbol{\epsilon}$$

We could fit this model by typing

```
spregress y x1 x2, gs2sls dvarlag(W) ivarlag(W:x1 x2)
```

Whatever other lags we include in the model, we could specify autoregressive errors. The model becomes

$$\mathbf{y} = \beta_0 + \beta_1\mathbf{x}_1 + \beta_2\mathbf{x}_2 + \beta_3\mathbf{W}\mathbf{y} + \beta_4\mathbf{W}\mathbf{x}_1 + \beta_5\mathbf{W}\mathbf{x}_2 + (\mathbf{I} - \rho\mathbf{W})^{-1}\boldsymbol{\epsilon}$$

To fit this model, we would type

```
spregress y x1 x2, gs2sls dvarlag(W) ivarlag(W:x1 x2) errorlag(W)
```

The parameters to be fit in the model are $\beta_0$ though $\beta_5$ and $\rho$, where $\rho$ is the correlation parameter of the residuals.

This is another model that is asking a lot of the data. Distinguishing correlated residuals from lagged dependent variables is especially tricky.

The machinery underlying `spregress` is complex. The `spregress` command with the `gs2sls` estimator uses a generalized method of moments estimator, which allows higher-order dependent variable lags and higher-order autoregressive error terms to be fit. `spregress` has an `ml` option for fitting a maximum likelihood estimator should you wish to fit a model under the assumption of normally distributed errors. You can read *Methods and formulas* in [SP] **spregress** for more details if you are curious.

You can fit SAR models for panel data with `spxtregress`, and you can fit SAR models with endogenous covariates using `spivregress`. These commands also incorporate spatial features like the ones described here. For details, see *Methods and formulas* in [SP] **spxtregress** and *Methods and formulas* in [SP] **spivregress**

See [SP] **Intro 8** for a brief tour of the Sp estimation commands.

## Also see

[SP] **Intro** — Introduction to spatial data and SAR models

[SP] **Intro 2** — The W matrix

[SP] **Intro 7** — Example from start to finish

[SP] **Intro 8** — The Sp estimation commands

[SP] **spivregress** — Spatial autoregressive models with endogenous covariates

[SP] **spmatrix** — Categorical guide to the spmatrix command

[SP] **spregress** — Spatial autoregressive models

[SP] **spxtregress** — Spatial autoregressive models for panel data

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