

## Example 31g — Two-factor measurement model (generalized response)

Description

Remarks and examples

Also see

### Description

We demonstrate a two-factor generalized linear measurement model with the same data used in [SEM] Example 29g:

```
. use https://www.stata-press.com/data/r19/gsem_cfa
(Fictional math abilities data)

. describe

Contains data from https://www.stata-press.com/data/r19/gsem_cfa.dta
Observations:      500      Fictional math abilities data
Variables:         19      21 Mar 2024 10:38
                        (_dta has notes)
```

Variable name	Storage type	Display format	Value label	Variable label
school	byte	%9.0g		School ID
id	long	%9.0g		Student ID
q1	byte	%9.0g	result	q1 correct
q2	byte	%9.0g	result	q2 correct
q3	byte	%9.0g	result	q3 correct
q4	byte	%9.0g	result	q4 correct
q5	byte	%9.0g	result	q5 correct
q6	byte	%9.0g	result	q6 correct
q7	byte	%9.0g	result	q7 correct
q8	byte	%9.0g	result	q8 correct
att1	byte	%26.0g	agree	Skills taught in math class will help me get a better job.
att2	byte	%26.0g	agree	Math is important in everyday life
att3	byte	%26.0g	agree	Working math problems makes me anxious.
att4	byte	%26.0g	agree	Math has always been my worst subject.
att5	byte	%26.0g	agree	I am able to learn new math concepts easily.
test1	byte	%9.0g		Score, math test 1
test2	byte	%9.0g		Score, math test 2
test3	byte	%9.0g		Score, math test 3
test4	byte	%9.0g		Score, math test 4

Sorted by:

- ```
. notes
_dta:
1. Fictional data on math ability and attitudes of 500 students from 20 schools.
2. Variables q1-q8 are incorrect/correct (0/1) on individual math questions.
3. Variables att1-att5 are items from a Likert scale measuring each student's attitude toward math.
4. Variables test1-test4 are test scores from tests of four different aspects of mathematical abilities. Range of scores: 0-100.
```

These data record results from a fictional instrument measuring mathematical ability. Variables q1 through q8 are the items from the instrument.

In this example, we will also be using variables att1 through att5. These are five Likert-scale questions measuring each student's attitude toward math.

See [Multiple-factor measurement models](#) in [SEM] [Intro 5](#) for background.

## Remarks and examples

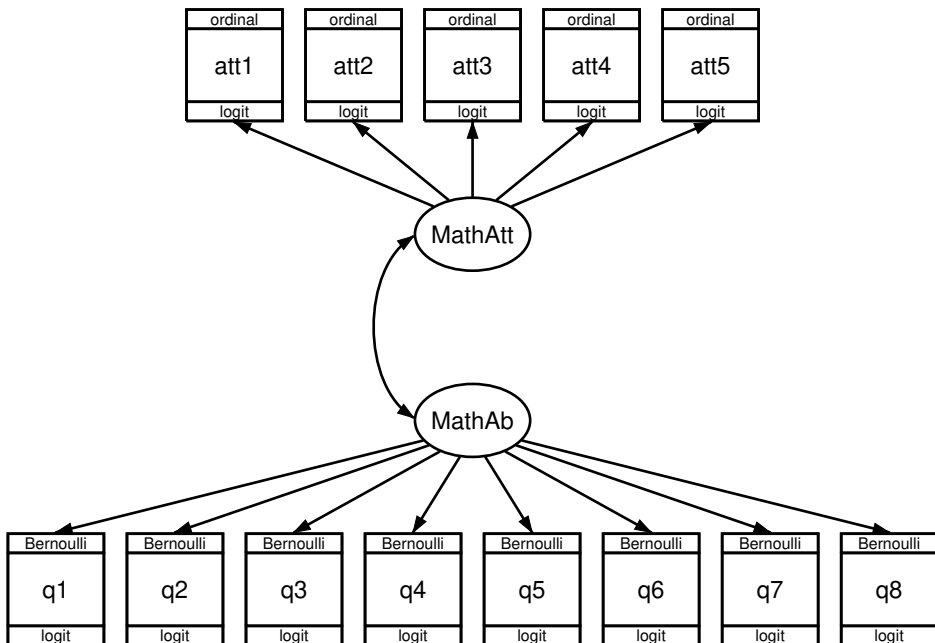
Remarks are presented under the following headings:

[Fitting the two-factor model](#)

[Fitting the model with the Builder](#)

## Fitting the two-factor model

We extend the measurement model fit in [\[SEM\] Example 29g](#) from one factor, math ability, to two factors, math ability and attitude. The model we wish to fit is



In this model, mathematical ability affects the correctness of the answers to the items just as [previously](#). The new component, attitude toward mathematics, is correlated with math ability. We expect this correlation to be positive, but that is yet to be determined.

What is important about the attitudinal questions is that the responses are ordinal, that is, the ordering of the possible answers is significant. In other cases, we might have a categorical variable taking on, say, five values; even if the values are 1, 2, 3, 4, and 5, there is no case in which answer 5 is greater than answer 4, answer 4 is greater than answer 3, and so on.

For our attitude measures, however, response 5 signifies strong agreement with a statement and 1 signifies strong disagreement. We handle the ordinal property by specifying that the attitudinal responses are family ordinal, link logit, also known as ordered logit or ordinal logistic regression, and also known in Stata circles as `ologit`.

In the command language, to fit a one-factor measurement model with math ability, we would type

```
gsem (MathAb -> q1-q8), logit
```

To include the second factor, attitude correlated with math ability, we would type

```
gsem (MathAb -> q1-q8,      logit)    ///  
      (MathAtt -> att1-att5, ologit)
```

The covariance between MathAtt and MathAb does not even appear in the command! That is because latent exogenous variables are assumed to be correlated in the command language unless you specify otherwise; in path diagrams, such variables are correlated only if a curved path is drawn between them.

There is another, minor difference in syntax between the one-factor and two-factor models that is worth your attention. Notice that the `logit` was outside the parentheses in the command to fit the one-factor model, but it is inside the parentheses in the command to fit the two-factor model. Actually, `logit` could have appeared inside the parentheses to fit the one-factor model. When options appear inside parentheses, they affect only what is specified inside the parentheses. When they appear outside parentheses, they affect all parenthetical specifications.

To obtain the estimates of the two-factor model, we type

```
. gsem (MathAb -> q1-q8, logit)  
>      (MathAtt -> att1-att5, ologit)  
  
Fitting fixed-effects model:  
Iteration 0:  Log likelihood = -6629.7253  
Iteration 1:  Log likelihood = -6628.7848  
Iteration 2:  Log likelihood = -6628.7848  
  
Refining starting values:  
Grid node 0:  Log likelihood = -6457.4584  
  
Fitting full model:  
Iteration 0:  Log likelihood = -6457.4584  
Iteration 1:  Log likelihood = -6437.9594  
Iteration 2:  Log likelihood = -6400.2731  
Iteration 3:  Log likelihood = -6396.3795  
Iteration 4:  Log likelihood = -6394.5787  
Iteration 5:  Log likelihood = -6394.4019  
Iteration 6:  Log likelihood = -6394.3923  
Iteration 7:  Log likelihood = -6394.3923  
  
Generalized structural equation model  
Number of obs = 500  
  
Response: q1  
Family:   Bernoulli  
Link:     Logit  
  
Response: q2  
Family:   Bernoulli  
Link:     Logit  
  
Response: q3  
Family:   Bernoulli  
Link:     Logit  
  
Response: q4  
Family:   Bernoulli  
Link:     Logit  
  
Response: q5  
Family:   Bernoulli  
Link:     Logit
```

Response: q6  
Family: Bernoulli  
Link: Logit

Response: q7  
Family: Bernoulli  
Link: Logit

Response: q8  
Family: Bernoulli  
Link: Logit

Response: att1  
Family: Ordinal  
Link: Logit

Response: att2  
Family: Ordinal  
Link: Logit

Response: att3  
Family: Ordinal  
Link: Logit

Response: att4  
Family: Ordinal  
Link: Logit

Response: att5  
Family: Ordinal  
Link: Logit

Log likelihood = -6394.3923

( 1) [q1]MathAb = 1

( 2) [att1]MathAtt = 1

|    |        | Coefficient     | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----|--------|-----------------|-----------|-------|-------|----------------------|-----------|
| q1 | MathAb | 1 (constrained) |           |       |       |                      |           |
|    | _cons  | .0446118        | .1272964  | 0.35  | 0.726 | -.2048845            | .2941082  |
| q2 | MathAb | .3446081        | .1050264  | 3.28  | 0.001 | .1387601             | .5504562  |
|    | _cons  | -.4572215       | .0979965  | -4.67 | 0.000 | -.6492911            | -.265152  |
| q3 | MathAb | .5445245        | .1386993  | 3.93  | 0.000 | .272679              | .8163701  |
|    | _cons  | .1591406        | .1033116  | 1.54  | 0.123 | -.0433464            | .3616276  |
| q4 | MathAb | .2858874        | .0948553  | 3.01  | 0.003 | .0999743             | .4718004  |
|    | _cons  | -.3196648       | .0947684  | -3.37 | 0.001 | -.5054075            | -.1339222 |
| q5 | MathAb | .8174803        | .1867024  | 4.38  | 0.000 | .4515504             | 1.18341   |
|    | _cons  | -.04543         | .116575   | -0.39 | 0.697 | -.2739127            | .1830527  |
| q6 | MathAb | .6030448        | .1471951  | 4.10  | 0.000 | .3145478             | .8915419  |
|    | _cons  | -.309992        | .1070853  | -2.89 | 0.004 | -.5198753            | -.1001086 |

|                         |           |               |       |       |           |           |  |
|-------------------------|-----------|---------------|-------|-------|-----------|-----------|--|
| q7                      |           |               |       |       |           |           |  |
| MathAb                  | .72084    | .1713095      | 4.21  | 0.000 | .3850796  | 1.056601  |  |
| _cons                   | .1047265  | .1116494      | 0.94  | 0.348 | -.1141023 | .3235552  |  |
| q8                      |           |               |       |       |           |           |  |
| MathAb                  | .5814761  | .1426727      | 4.08  | 0.000 | .3018428  | .8611094  |  |
| _cons                   | -.0250442 | .1045134      | -0.24 | 0.811 | -.2298868 | .1797983  |  |
| att1                    |           |               |       |       |           |           |  |
| MathAtt                 | 1         | (constrained) |       |       |           |           |  |
| att2                    |           |               |       |       |           |           |  |
| MathAtt                 | .3788714  | .0971223      | 3.90  | 0.000 | .1885152  | .5692276  |  |
| att3                    |           |               |       |       |           |           |  |
| MathAtt                 | -1.592717 | .3614859      | -4.41 | 0.000 | -2.301216 | -.8842173 |  |
| att4                    |           |               |       |       |           |           |  |
| MathAtt                 | -.8100107 | .153064       | -5.29 | 0.000 | -1.11001  | -.5100108 |  |
| att5                    |           |               |       |       |           |           |  |
| MathAtt                 | .5225423  | .1170141      | 4.47  | 0.000 | .2931988  | .7518858  |  |
| /att1                   |           |               |       |       |           |           |  |
| cut1                    | -1.10254  | .1312272      |       |       | -1.359741 | -.8453396 |  |
| cut2                    | -.2495339 | .1160385      |       |       | -.4769651 | -.0221027 |  |
| cut3                    | .2983261  | .1164414      |       |       | .0701052  | .5265471  |  |
| cut4                    | 1.333053  | .1391907      |       |       | 1.060244  | 1.605861  |  |
| /att2                   |           |               |       |       |           |           |  |
| cut1                    | -1.055791 | .1062977      |       |       | -1.264131 | -.8474513 |  |
| cut2                    | -.1941211 | .0941435      |       |       | -.378639  | -.0096032 |  |
| cut3                    | .3598488  | .0952038      |       |       | .1732528  | .5464448  |  |
| cut4                    | 1.132624  | .1082204      |       |       | .9205156  | 1.344732  |  |
| /att3                   |           |               |       |       |           |           |  |
| cut1                    | -1.053519 | .1733999      |       |       | -1.393377 | -.7136614 |  |
| cut2                    | -.0491073 | .1442846      |       |       | -.3318999 | .2336853  |  |
| cut3                    | .5570671  | .1538702      |       |       | .2554871  | .8586471  |  |
| cut4                    | 1.666859  | .2135554      |       |       | 1.248298  | 2.08542   |  |
| /att4                   |           |               |       |       |           |           |  |
| cut1                    | -1.07378  | .1214071      |       |       | -1.311734 | -.8358264 |  |
| cut2                    | -.2112462 | .1076501      |       |       | -.4222366 | -.0002559 |  |
| cut3                    | .406347   | .1094847      |       |       | .191761   | .620933   |  |
| cut4                    | 1.398185  | .1313327      |       |       | 1.140778  | 1.655593  |  |
| /att5                   |           |               |       |       |           |           |  |
| cut1                    | -1.244051 | .1148443      |       |       | -1.469142 | -1.018961 |  |
| cut2                    | -.336135  | .0986678      |       |       | -.5295203 | -.1427498 |  |
| cut3                    | .2137776  | .0978943      |       |       | .0219084  | .4056468  |  |
| cut4                    | .9286849  | .107172       |       |       | .7186316  | 1.138738  |  |
| var(MathAb)             | 2.300652  | .7479513      |       |       | 1.216527  | 4.350909  |  |
| var(MathAtt)            | 1.520854  | .4077674      |       |       | .8992196  | 2.572228  |  |
| cov(MathAb,<br>MathAtt) | .8837681  | .2204606      | 4.01  | 0.000 | .4516733  | 1.315863  |  |

Notes:

1. The estimated covariance between math attitude and ability is 0.88.
2. There is something new in the output, namely, things labeled cut1, ..., cut4. These appear for each of the five attitudinal measures. These are the ordered logit's cutpoints, the values on the logit's distribution that separate attitude 1 from attitude 2, attitude 2 from attitude 3, and so on. The four cutpoints map the continuous distribution into five ordered, categorical groups.
3. There's something interesting hiding in the MathAtt coefficients: the coefficients for two of the paths, att3 att4 <- MathAtt, are negative! If you look back to the description of the data, you will find that the sense of these two questions was reversed from those of the other questions. Strong agreement on these two questions was agreement with a negative feeling about mathematics.

estat sd displays the fitted variance components as standard deviations and correlations. From the following, we see that the estimated correlation between attitude and ability is 0.4725.

```
. estat sd
```

|                          | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |
|--------------------------|-------------|-----------|------|-------|----------------------|
| sd(MathAb)               | 1.51679     | .2465573  |      |       | 1.102963 2.085883    |
| sd(MathAtt)              | 1.233229    | .1653251  |      |       | .9482719 1.603817    |
| corr(MathAb,<br>MathAtt) | .4724644    | .0649541  | 7.27 | 0.000 | .3357955 .5896506    |

## Fitting the model with the Builder

Use the diagram in *Fitting the two-factor model* above for reference.


1. Open the dataset.


In the Command window, type

```
. use https://www.stata-press.com/data/r19/gsem_cfa
```

2. Open a new Builder diagram.

Select menu item **Statistics > SEM (structural equation modeling) > Model building and estimation**.

3. Put the Builder in gsem mode by clicking on the  button.
4. Create the measurement component for MathAb.


Select the Add measurement component tool, , and then click in the diagram about one-third of the way up from the bottom and slightly left of the center.

In the resulting dialog box,

- a. change the *Latent variable name* to MathAb;
- b. select q1, q2, q3, q4, q5, q6, q7, and q8 by using the *Measurement variables* control;
- c. check *Make measurements generalized*;
- d. select Bernoulli, Logit in the *Family/Link* control;
- e. select Down in the *Measurement direction* control;
- f. click on **OK**.

If you wish, move the component by clicking on any variable and dragging it.

5. Create the measurement component for MathAtt.


Select the Add measurement component tool, , and then click in the diagram about one-third of the way down from the top and slightly left of the center.

In the resulting dialog box,


- change the *Latent variable name* to MathAtt;
- select att1, att2, att3, att4, and att5 by using the *Measurement variables* control;
- check *Make measurements generalized*;
- select Ordinal, Logit in the *Family/Link* control;
- select Up in the *Measurement direction* control;
- click on **OK**.

If you wish, move the component by clicking on any variable and dragging it.


6. Create the covariance between MathAtt and MathAb.

- Select the Add covariance tool, .
- Click in the top-left quadrant of the MathAb oval, and drag a covariance to the bottom left of the MathAtt oval.

7. Clean up.

If you do not like where a covariance has been connected to its variable, use the Select tool, , to simply click on the covariance, and then click on where it connects to an oval and drag the endpoint. You can also change the bow of the covariance by dragging the control point that extends from one end of the selected covariance.

8. Estimate.

Click on the **Estimate** button, , in the Standard Toolbar, and then click on **OK** in the resulting *GSEM estimation options* dialog box.

You can open a completed diagram in the Builder by typing

```
. webgetsem gsem_2fmm
```

## Also see

[SEM] [Example 27g](#) — Single-factor measurement model (generalized response)

[SEM] [Example 29g](#) — Two-parameter logistic IRT model

[SEM] [Example 32g](#) — Full structural equation model (generalized response)

[SEM] [Intro 5](#) — Tour of models

[SEM] [gsem](#) — Generalized structural equation model estimation command

[SEM] [estat sd](#) — Display variance components as standard deviations and correlations

Stata, Stata Press, and Mata are registered trademarks of StataCorp LLC. Stata and Stata Press are registered trademarks with the World Intellectual Property Organization of the United Nations. StataNow and NetCourseNow are trademarks of StataCorp LLC. Other brand and product names are registered trademarks or trademarks of their respective companies. Copyright © 1985–2025 StataCorp LLC, College Station, TX, USA. All rights reserved.

For suggested citations, see the FAQ on [citing Stata documentation](#).

