Example 23 — Specifying parameter constraints across groups

Description

Below we demonstrate how to constrain the parameters we want constrained to be equal across groups when using \texttt{sem} with the \texttt{group()} option.

We pick up where \texttt{SEM Example 22} left off:

\begin{verbatim}
.use https://www.stata-press.com/data/r16/sem_2fmmby
.sem (Peer -> peerrel1 peerrel2 peerrel3 peerrel4) ///
(Par -> parrel1 parrel2 parrel3 parrel4), group(grade)
.estat ginvariant
\end{verbatim}

The \texttt{estat ginvariant} command implied that perhaps we could constrain all the variances and covariances to be equal across groups except for the variances of \texttt{e.parrel2} and \texttt{Peer}.

Remarks and examples

Remarks are presented under the following headings:

\textit{Background}

\textit{Fitting the constrained model}

Background

We can specify which parameters we wish to allow to vary. Remember that \texttt{sem}'s \texttt{group()} option classifies the parameters of the model as follows:

<table>
<thead>
<tr>
<th>Class description</th>
<th>Class name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. structural coefficients</td>
<td>scoef</td>
</tr>
<tr>
<td>2. structural intercepts</td>
<td>scons</td>
</tr>
<tr>
<td>3. measurement coefficients</td>
<td>mcoef</td>
</tr>
<tr>
<td>4. measurement intercepts</td>
<td>mcons</td>
</tr>
<tr>
<td>5. covariances of structural errors</td>
<td>serrvar</td>
</tr>
<tr>
<td>6. covariances of measurement errors</td>
<td>merrvar</td>
</tr>
<tr>
<td>7. covariances between structural and measurement errors</td>
<td>smerrcov</td>
</tr>
<tr>
<td>8. means of exogenous variables</td>
<td>meanex (*)</td>
</tr>
<tr>
<td>9. covariances of exogenous variables</td>
<td>covex (*)</td>
</tr>
<tr>
<td>10. all the above</td>
<td>all (*)</td>
</tr>
<tr>
<td>11. none of the above</td>
<td>none</td>
</tr>
</tbody>
</table>

(*) Exogenous variables means just the latent exogenous variables unless you specify \texttt{sem option noxconditional} or you specify option \texttt{method(mlmv)} (which implies option \texttt{noxconditional}); see \texttt{SEM sem option noxconditional}.
When fitting a model with the *group()* option,
\[
\text{. sem ..., ... group(varname)}
\]
you may also specify the *ginvariant()* option:
\[
\text{. sem ..., ... group(varname) ginvariant(class names)}
\]
You may specify any of the class names as being *ginvariant()*.
You may specify as many class names as you wish.
When you specify *ginvariant()*,
*sem* cancels its default actions on which parameters vary and which do not,
and uses the information you specify. All classes that you do not
mention as being *ginvariant()* are allowed to vary across groups.

By using *ginvariant()*
, you can constrain, or free by your silence, whole classes of parameters.
For instance, you could type
\[
\text{. sem ..., group(mygroup) ginvariant(mcoef mcons serrvar)}
\]
and you are constraining those parameters to be equal across groups
and leaving unconstrained scoef, scons, merrvar, smerrcov, meanex, and covex.

In addition, if a class is constrained, you can still unconstrain individual coefficients.
Consider the model
\[
\text{. sem ... (x1<-L) ...}
\]
If you typed
\[
\text{. sem ... (1: x1<-L@a1) (2: x1<-L@a2) ..., group(mygroup) ginvariant(all)}
\]
then all estimated parameters would be the same across groups except for the path \( x1\leftarrow L \),
and it would be free to vary in groups 1 and 2.

By the same token, if a class is unconstrained,
you can still constrain individual coefficients.
If you typed
\[
\text{. sem ... (1: x1<-L@a) (2: x1<-L@a) ..., group(mygroup) ginvariant(none)}
\]
then you would leave unconstrained all parameters except the path \( x1\leftarrow L \),
and it would be constrained to be equal in groups 1 and 2.

This is all discussed in [SEM] Intro 6, including how to constrain and free variance and covariance
parameters.

Fitting the constrained model

In our case, we wish to fit our model:
\[
\begin{align*}
\text{. sem (Peer } & \text{-} \text{ peerrel1 peerrel2 peerrel3 peerrel4) } \\
\text{ (Par } & \text{-} \text{ parrel1 parrel2 parrel3 parrel4), } \\
\text{ group(grade)}
\end{align*}
\]
We impose constraints on all parameters except the variances of e.parrel2 and Peer. We can do that by typing

```
.semin (Peer -> peerrel1 peerrel2 peerrel3 peerrel4)
>   (Par -> parrel1 parrel2 parrel3 parrel4),
>     group(grade)
>     ginvariant(all)
>     bypvarm
>     var(1: e.parrel2@v1)
>     var(2: e.parrel2@v2)
>     var(1: Peer@v3)
>     var(2: Peer@v4)
```

Endogenous variables
Measurement:  peerrel1 peerrel2 peerrel3 peerrel4 parrel1 parrel2 parrel3 parrel4

Exogenous variables
Latent:  Peer Par

Fitting target model:
Iteration 0:  log likelihood = -5560.9934
Iteration 1:  log likelihood = -5552.3122
Iteration 2:  log likelihood = -5549.5391
Iteration 3:  log likelihood = -5549.3528
Iteration 4:  log likelihood = -5549.3501
Iteration 5:  log likelihood = -5549.3501

Structural equation model

Number of obs = 385
Grouping variable = grade
Number of groups = 2
Estimation method = ml
Log likelihood = -5549.3501

( 1) [peerrel1]1bn.grade#c.Peer = 1
( 2) [peerrel2]1bn.grade#c.Peer - [peerrel2]2.grade#c.Peer = 0
( 3) [peerrel3]1bn.grade#c.Peer - [peerrel3]2.grade#c.Peer = 0
( 4) [peerrel4]1bn.grade#c.Peer - [peerrel4]2.grade#c.Peer = 0
( 5) [parrel1]1bn.grade#c.Par = 1
( 6) [parrel2]1bn.grade#c.Par - [parrel2]2.grade#c.Par = 0
( 7) [parrel3]1bn.grade#c.Par - [parrel3]2.grade#c.Par = 0
( 8) [parrel4]1bn.grade#c.Par - [parrel4]2.grade#c.Par = 0
( 9) [/var(e.peerrel1)#1bn.grade - [/var(e.peerrel1)#2.grade = 0
(10) [/var(e.peerrel2)#1bn.grade - [/var(e.peerrel2)#2.grade = 0
(11) [/var(e.peerrel3)#1bn.grade - [/var(e.peerrel3)#2.grade = 0
(12) [/var(e.peerrel4)#1bn.grade - [/var(e.peerrel4)#2.grade = 0
(13) [/var(e.parrel1)#1bn.grade - [/var(e.parrel1)#2.grade = 0
(14) [/var(e.parrel3)#1bn.grade - [/var(e.parrel3)#2.grade = 0
(15) [/var(e.parrel4)#1bn.grade - [/var(e.parrel4)#2.grade = 0
(16) [/cov(Peer,Par)#1bn.grade - [/cov(Peer,Par)#2.grade = 0
(17) [/var(Par)#1bn.grade - [/var(Par)#2.grade = 0
(18) [peerrel1]1bn.grade - [peerrel1]2.grade = 0
(19) [peerrel2]1bn.grade - [peerrel2]2.grade = 0
(20) [peerrel3]1bn.grade - [peerrel3]2.grade = 0
(21) [peerrel4]1bn.grade - [peerrel4]2.grade = 0
(22) [parrel1]1bn.grade - [parrel1]2.grade = 0
(23) [parrel2]1bn.grade - [parrel2]2.grade = 0
(24) [parrel3]1bn.grade - [parrel3]2.grade = 0
(25) [parrel4]1bn.grade - [parrel4]2.grade = 0
(26) [peerrel1]2.grade#c.Peer = 1
(27) [parrel1]2.grade#c.Par = 1
| Measurement | Coef. | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|-------------|-------|-----------|------|------|----------------------|
| peerrel1    |       |           |      |      |                      |
| Peer        | 1 (constrained) |     |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 8.708274 | .0935844 | 93.05| 0.000| 8.524852 8.891696    |
| peerrel2    |       |           |      |      |                      |
| Peer        |       |           |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 1.112225 | .0973506 | 11.42| 0.000| .9214217 1.303029    |
| peerrel3    |       |           |      |      |                      |
| Peer        |       |           |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 1.416486 | .113489  | 12.48| 0.000| 1.194052 1.638921    |
| peerrel4    |       |           |      |      |                      |
| Peer        |       |           |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 1.196494 | .0976052 | 12.26| 0.000| 1.005191 1.387796    |
| parrel1     |       |           |      |      |                      |
| Par         | 1 (constrained) |     |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 9.339558 | .0648742 | 143.96| 0.000| 9.212407 9.46671    |
| parrel2     |       |           |      |      |                      |
| Par         |       |           |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 1.100315 | .1362999 | 8.07 | 0.000| .8331722 1.367458    |
| parrel3     |       |           |      |      |                      |
| Par         |       |           |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 2.051278 | .2066714 | 9.93 | 0.000| 1.64621 2.456347    |
| parrel4     |       |           |      |      |                      |
| Par         |       |           |      |      |                      |
| _cons       |       |           |      |      |                      |
|             | 1.529938 | .154971  | 9.87 | 0.000| 1.2262 1.833675     |
|             | 9.045247 | .0722358 | 125.22| 0.000| 8.903667 9.186826   |
Example 23 — Specifying parameter constraints across groups

| var(e.peer~1) [*] | 1.799133   .159059   1.512898   2.139523 |
| var(e.peer~2) [*] | 2.186953   .193911   1.54056   2.382094 |
| var(e.peer~3) [*] | 1.915661   .2129913   1.45622   2.144965 |
| var(e.peer~4) [*] | 1.767354   .1746104   1.45622   2.144965 |
| var(e.parr~1)[*] | 1.125082   .0901338   .9615942   1.316366 |
| var(e.parr~2) [*] | 1.9603043   .13383   1.261927   1.848084 |
| var(e.parr~3) [*] | .9606889   .1420406   1.283617   1.848084 |
| var(e.parr~4) [*] | .8496935   .0933448   .6850966   1.053835 |
| var(Peer) 1 | 1.951555   .3387796   1.388727   2.742489 |
| var(Peer) 2 | 1.361431   .2122853   1.002927   1.848084 |
| var(Par) [*] | .4952527   .0927994   .3430288   .7150281 |
| cov(Peer,Par) [*] | .4096197   .0708726   5.78   0.000   .2707118   .5485275 |

Note: [*] identifies parameter estimates constrained to be equal across groups.

LR test of model vs. saturated: chi2(61) = 75.25, Prob > chi2 = 0.1037

Notes:

1. In [SEM] Example 20, we previously fit this model by typing

```
   . sem (...) (...), group(grade)
```

   This time, we typed

```
   . sem (...) (...), group(grade) ///
       ginvariant(all) ///
       byparm ///
       var(1: e.parrel2@v1) ///
       var(2: e.parrel2@v2) ///
       var(1: Peer@v3) ///
       var(2: Peer@v4)
```

2. We specified the `byparm` option so that the results are sorted by parameters rather than groups. With this option, `sem` will report a single value for a parameter estimate if it is constrained to be equal across the groups. Because most of the parameters are constrained to be equal across groups, the output is much shorter than the default output. The default output produces a separate table for each group.

3. Previously, `sem, group()` mentioned 20 constraints that it imposed because of normalization or because of assumed `ginvariant(mcoef mcons)`.

   This time, `sem, group()` mentioned 27 constraints. It applied more constraints because we specified `ginvariant(all)`.
4. After the `ginvariant(all)` option, we relaxed the following constraints:

\[
\begin{align*}
\text{var(1: e.parrel2@v1)} \\
\text{var(2: e.parrel2@v2)} \\
\text{var(1: Peer@v3)} \\
\text{var(2: Peer@v4)}
\end{align*}
\]

`ginvariant(all)` specified, among other constraints, that

\[
\begin{align*}
\text{var(1: e.parrel2)} == \text{var(2: e.parrel2)} \\
\text{var(1: Peer)} == \text{var(2: Peer)}
\end{align*}
\]

`ginvariant(all)` did that by secretly issuing the options

\[
\begin{align*}
\text{var(1: e.parrel2@secretname1)} \\
\text{var(2: e.parrel2@secretname1)} \\
\text{var(1: Peer@secretname2)} \\
\text{var(2: Peer@secretname2)}
\end{align*}
\]

because that is how you impose equality constraints with the path notation. When we specified

\[
\begin{align*}
\text{var(1: e.parrel2@v1)} \\
\text{var(2: e.parrel2@v2)} \\
\text{var(1: Peer@v3)} \\
\text{var(2: Peer@v4)}
\end{align*}
\]

our new constraints overrode the secretly issued constraints. It would not have worked to leave off the symbolic names; see [SEM] sem path notation extensions. We specified the symbolic names `v1`, `v2`, `v3`, and `v4`. `v1` and `v2` overrode `secretname1`, and thus the constraint that \(\text{var(e.parrel2)}\) be equal across the two groups was relaxed. `v3` and `v4` overrode `secretname2`, and thus the constraint that \(\text{var(Peer)}\) be equal across groups was relaxed.

Also see

[SEM] Example 20 — Two-factor measurement model by group

[SEM] Example 22 — Testing parameter equality across groups

[SEM] Intro 6 — Comparing groups

[SEM] sem group options — Fitting models on different groups