Example 12 — Seemingly unrelated regression

Description

`sem` can be used to estimate seemingly unrelated regression. We will use `auto.dta`, which surely needs no introduction:

```
. sysuse auto
   (1978 automobile data)
```

See *Structural models 11: Seemingly unrelated regression (SUR)* in [SEM] Intro 5.

Remarks and examples

Remarks are presented under the following headings:

* Fitting the seemingly unrelated regression model
* Fitting the model with the Builder

Fitting the seemingly unrelated regression model

We fit the following model:

```
      mpg
  +------------------+
     |                   |
  +------------------+
        price
  +------------------+
     |                   |
  +------------------+
    displacement
  +------------------+
        foreign
  +------------------+
     |                   |
  +------------------+
        weight
  +------------------+
         ε1

   . sem (price <- foreign mpg displacement)
     > (weight <- foreign length),
     >   cov(e.price*e.weight)

Endogenous variables
   Observed: price weight
Exogenous variables
   Observed: foreign mpg displacement length
```

Fitting target model:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2150.9983</td>
</tr>
<tr>
<td>1</td>
<td>-2138.5739</td>
</tr>
<tr>
<td>2</td>
<td>-2133.3461</td>
</tr>
<tr>
<td>3</td>
<td>-2133.1979</td>
</tr>
<tr>
<td>4</td>
<td>-2133.1956</td>
</tr>
<tr>
<td>5</td>
<td>-2133.1956</td>
</tr>
</tbody>
</table>
### Structural equation model

| OIM                  | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|----------------------|-------------|-----------|------|------|----------------------|
| **Structural**        |             |           |      |      |                      |
| price                |             |           |      |      |                      |
| foreign              | 2940.929    | 724.7311  | 4.06 | 0.000| 1520.482  4361.376   |
| mpg                  | -105.0163   | 57.93461  | -1.81| 0.070| -218.566  8.53347   |
| displace-t            | 17.22083    | 4.5941    | 3.75 | 0.000| 8.216558  26.2251    |
| _cons                | 4129.866    | 1984.253  | 2.08 | 0.037| 240.8022  8018.931   |
| **weight**            |             |           |      |      |                      |
| foreign              | -153.2515   | 76.21732  | -2.01| 0.044| -302.6347  -3.868275 |
| length               | 30.73507    | 1.584743  | 19.39| 0.000| 27.62903  33.84111  |
| _cons                | -2711.096   | 312.6813  | -8.67| 0.000| -3323.94  -2098.252 |
| **var(e.price)**     | 4732491     | 801783.1  |      |      | 3395302   6596312    |
| **var(e.weight)**    | 60253.09    | 9933.316  |      |      | 43616.45  83235.44  |
| **cov(e.price, e.weight)** | 209268 | 73909.54 | 2.83 | 0.005| 64407.92  354128 |

LR test of model vs. saturated: chi2(3) = 38.86  Prob > chi2 = 0.0000

**Notes:**

1. Point estimates are the same as reported by
   ```
   . sureg (price foreign mpg displ) (weight foreign length), isure
   ```
   sureg’s isure option is required to make sureg iterate to the maximum likelihood estimate.

2. If you wish to compare the estimated variances and covariances after estimation by sureg, type
   ```
   . matrix list e(Sigma)
   ```
   sureg does not estimate standard errors on variances and covariances.

3. Standard errors will be different between sem and sureg. In this case, there is no reason to
   prefer one set of standard errors over the other, and standard errors are asymptotically equivalent.
   This is a case of exogenous variables only on the right-hand side. When the model being fit is
   recursive, standard errors produced by sem are better than those from sureg, both asymptotically
   and in finite samples.

4. One reason you might want to use sem is that sem will provide robust standard errors whereas
   sureg does not.

5. Multivariate regression can be viewed as seemingly unrelated regression. You just need to specify
   the same regressors for each equation. In that case, standard errors reported by sem will be the same
   as those reported by mvreg if one applies the multiplicative $\sqrt{(N - p - 1)/N}$ degree-of-freedom
   adjustment.
Fitting the model with the Builder

Use the diagram above for reference.

1. Open the dataset.
   In the Command window, type
   
   . use https://www.stata-press.com/data/r17/auto

2. Open a new Builder diagram.
   Select menu item **Statistics > SEM (structural equation modeling) > Model building and estimation**.

3. Change the size of the observed variables’ rectangles.
   From the SEM Builder menu, select **Settings > Variables > All observed...**.
   In the resulting dialog box, change the first size to .85 and click on **OK**.

4. Create the four independent variables.
   Select the Add observed variables set tool, ⬤, and then click in the diagram about one-fourth of the way in from the left and one-fourth of the way up from the bottom.
   In the resulting dialog box,
   a. select the **Select variables** radio button (it may already be selected);
   b. use the **Variables** control to select the four variables in this order: mpg, displacement, foreign, and length;
   c. select **Vertical** in the **Orientation** control;
   d. click on **OK**.
   If you wish, move the set of variables by clicking on any variable and dragging it.

5. Create the two dependent variables.
   Select the Add observed variables set tool, ⬤, and then click about two-thirds of the way in from the left and vertically aligned with the top of the length rectangle.
   In the resulting dialog box,
   a. select the **Select variables** radio button (it may already be selected);
   b. use the **Variables** control to select the variables price and weight;
   c. select **Vertical** in the **Orientation** control;
   d. select the **Distances** tab;
   e. select .5 (inch) from the **Distance between variables** control;
   f. click on **OK**.
   If you wish, move the set of variables by clicking on any variable and dragging it.

6. Create paths from the independent variables to the dependent variables.
   a. Select the Add path tool, −.
   b. Click in the right side of the mpg rectangle (it will highlight when you hover over it), and drag a path to the left side of the price rectangle (it will highlight when you can release to connect the path).
c. Continuing with the tool, create the following paths by clicking first in the right side of the rectangle for the independent variable and dragging it to the left side of the rectangle for the dependent variable:

- displacement -> price
- foreign -> price
- foreign -> weight
- length -> weight

7. Correlate the error terms.
   a. Select the Add covariance tool, \( \epsilon \).
   b. Click in the \( \epsilon_1 \) circle (it will highlight when you hover over it), and drag a covariance to the \( \epsilon_2 \) circle (it will highlight when you can release to connect the covariance).

8. Clean up.
   If you do not like where a path has been connected to its variables, use the Select tool, to click on the path, and then simply click on where it connects to a rectangle and drag the endpoint. Similarly, you can change where the covariance connects to the error terms by clicking on the covariance and dragging the endpoint. You can also change the bow of the covariance by clicking on the covariance and dragging the control point that extends from one end of the selected covariance.

   Click on the Estimate button, in the Standard Toolbar, and then click on OK in the resulting SEM estimation options dialog box.

   You can open a completed diagram in the Builder by typing
   
   \[ . \text{webgetsem} \text{ sem_sureg} \]

Also see

[SEM] Example 13 — Equation-level Wald test

[SEM] sem — Structural equation model estimation command