#### tpoisson — Truncated Poisson regression

Description Options Acknowledgment Quick start Remarks and examples References Menu Stored results Also see Syntax Methods and formulas

## Description

tpoisson fits a truncated Poisson regression model when the number of occurrences of an event is restricted to be above a truncation point, below a truncation point, or between two truncation points. Truncated Poisson models are appropriate when neither the dependent variable nor the covariates are observed in the truncated part of the distribution. By default, tpoisson assumes left-truncation occurs at zero, but truncation may be specified at other fixed points or at values that vary across observations.

## **Quick start**

Truncated Poisson regression of y on x1 and x2 with left-truncation at 0 tpoisson y x1 x2

Add categorical variable a using factor-variable syntax

tpoisson y x1 x2 i.a

Same as above, but report incidence-rate ratios and use a constant truncation point of 4 tpoisson y x1 x2 i.a, irr ll(4)

With offset variable lnexp

tpoisson y x1 x2 i.a, offset(lnexp)

Same as above, but with a variable truncation point stored in variable min

```
tpoisson y x1 x2 i.a, offset(lnexp) ll(min)
```

With variable left- and right-truncation points tpoisson y x1 x2, ll(min) ul(max)

With variable right-truncation points tpoisson y x1 x2, ul(max)

Constrain the coefficients for 2.a and 3.a to equality constraint define 12.a = 3.a tpoisson y x1 x2 i.a, constraints(1)

## Menu

 $\label{eq:statistics} Statistics > \mbox{Count outcomes} > \mbox{Truncated Poisson regression}$ 

# **Syntax**

Description options Model noconstant suppress constant term 11(#|*varname*) lower limit for truncation; default is 11(0) when neither 11() nor u1() is specified ul(#|*varname*) upper limit for truncation exposure(varname\_) include ln(varname) in model with coefficient constrained to 1 offset(varname\_) include varname, in model with coefficient constrained to 1 constraints(constraints) apply specified linear constraints SE/Robust vce(vcetype) *vcetype* may be oim, robust, cluster *clustvar*, opg, bootstrap, or jackknife Reporting level(#) set confidence level; default is level(95) report incidence-rate ratios irr nocnsreport do not display constraints control columns and column formats, row spacing, line width, display\_options display of omitted variables and base and empty cells, and factor-variable labeling Maximization control the maximization process; seldom used maximize\_options collinear keep collinear variables display legend instead of statistics coeflegend indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

tpoisson depvar [indepvars] [if] [in] [weight] [, options]

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayes, bayesboot, bootstrap, by, collect, fmm, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: tpoisson and [FMM] fmm: tpoisson.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce() and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

# Options

Model

noconstant; see [R] Estimation options.

ll(#|varname) and ul(#|varname) specify the lower and upper limits for truncation, respectively. You may specify nonnegative integer values for one or both.

When neither 11() nor u1() is specified, the default is zero truncation, 11(0), equivalent to left-truncation at zero.

exposure(varname<sub>e</sub>), offset(varname<sub>o</sub>), constraints(constraints); see [R] Estimation options.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce\_option.

Reporting

level(#); see [R] Estimation options.

irr reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. irr may be specified at estimation or when replaying previously estimated results.

nocnsreport; see [R] Estimation options.

```
display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt),
sformat(%fmt), and nolstretch; see [R] Estimation options.
```

Maximization

maximize\_options: difficult, technique(algorithm\_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init\_specs); see [R] Maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default *vcetype* to vce(opg).

The following options are available with tpoisson but are not shown in the dialog box:

collinear, coeflegend; see [R] Estimation options.

### **Remarks and examples**

tpoisson fits a truncated Poisson regression model by maximum likelihood estimation when the number of occurrences of an event is restricted to be above a truncation point, below a truncation point, or between two truncation points. If the dependent variable is not truncated, standard Poisson regression may be more appropriate; see [R] poisson.

When the data are truncated, we do not observe either the dependent variable or the covariates. For example, consider a study about the number of days that individuals with hyperglycemia are hospitalized after presenting to the hospital. If we select our sample only from admission records, then the sample is truncated at zero because we have data only on individuals who stayed at least one day. Now assume that we are relying on billing data and that hospitals may submit either a final bill when a patient is discharged or an interim bill every 30 days. In this case, we have no information about patients who are hospitalized fewer than 1 day or more than 30 days. Our data are left-truncated at 0 and right-truncated at 30.

A related phenomenon is censoring. For censored observations, we observe complete covariate information but only a censored value of the dependent variable. Different research designs can give rise to censored data or truncated data. See [R] **cpoisson** for information about censored Poisson regression.

Truncated Poisson regression was first proposed by Grogger and Carson (1991). For an introduction to Poisson regression, see Cameron and Trivedi (2005, 2022) and Long and Freese (2014). For an introduction to truncated Poisson models, see Cameron and Trivedi (2013) and Long (1997, chap. 8).

#### Example 1: Left-truncation at zero

Consider the Simonoff (2003) dataset of running shoes for a sample of runners who registered an online running log. A running-shoe marketing executive is interested in knowing how the number of pairs of running shoes purchased relates to other factors such as gender, marital status, age, education, income, typical number of runs per week, average miles run per week, and the preferred type of running. These data are naturally truncated at zero. A truncated Poisson model is fit to the number of pairs of shoes owned on runs per week, miles run per week, gender, age, and marital status.

No options are needed because zero truncation is the default for tpoisson.

```
. use https://www.stata-press.com/data/r19/runshoes
(Running shoes)
. tpoisson shoes rpweek mpweek male age married
Iteration 0: Log likelihood = -88.328151
Iteration 1:
              Log likelihood = -86.272639
Iteration 2: Log likelihood = -86.257999
Iteration 3:
              Log likelihood = -86.257994
Truncated Poisson regression
                                                 Number of obs
                                                                    =
                                                                               60
                                                 LR chi2(5)
                                                                            22.75
Limits: lower =
                                                                    =
                          0
                                                 Prob > chi2
                                                                          0.0004
         upper =
                       +inf
                                                                    =
Log likelihood = -86.257994
                                                 Pseudo R2
                                                                          0.1165
                                                                    =
               Coefficient
                            Std. err.
                                                 P>|z|
                                                            [95% conf. interval]
       shoes
                                            z
      rpweek
                 .1575811
                             .1097893
                                          1.44
                                                 0.151
                                                            -.057602
                                                                         .3727641
                 .0210673
                             .0091113
                                          2.31
                                                 0.021
                                                            .0032094
                                                                         .0389252
      mpweek
                 .0446134
                             .2444626
                                          0.18
                                                 0.855
                                                           -.4345246
                                                                         .5237513
        male
                 .0185565
                             .0137786
                                          1.35
                                                 0.178
                                                            -.008449
                                                                          .045562
         age
     married
                -.1283912
                             .2785044
                                         -0.46
                                                 0.645
                                                           -.6742498
                                                                         .4174674
       _cons
                -1.205844
                             .6619774
                                         -1.82
                                                 0.069
                                                           -2.503296
                                                                         .0916078
```

Using the zero-truncated Poisson regression with these data, only the coefficient on average miles per week is statistically significant at the 5% level.

#### Example 2: Left-truncation with a fixed-truncation point

Semiconductor manufacturing requires that silicon wafers be coated with a layer of metal oxide. The depth of this layer is strictly controlled. In this example, a critical oxide layer is designed for  $300 \pm 20$  angstroms (Å).

After the oxide layer is coated onto a wafer, the wafer enters a photolithography step in which the lines representing the electrical connections are printed on the oxide and later etched and filled with metal. The widths of these lines are measured. In this example, they are controlled to  $90 \pm 5$  micrometers ( $\mu$ m).

After these and other steps, each wafer is electrically tested at probe. If too many failures are discovered, the wafer is rejected and sent for engineering analysis. In this example, the maximum number of probe failures tolerated for this product is 10.

A major failure at probe has been encountered—88 wafers had more than 10 failures each. The 88 wafers that failed were tested using 4 probe machines. The engineer suspects that the failures were a result of faulty probe machines, poor depth control, or poor line widths. The line widths and depths in these data are the actual measurement minus its specification target, 300 Å for the oxide depths and 90  $\mu$ m for the line widths.

The following table tabulates the average failure rate for each probe using Stata's mean command; see [R] mean.

. use https://www.stata-press.com/data/r19/probe (Silicon wafers)						
. mean failures, o	over(probe)					
Mean estimation			Number of obs = 88			
	Mean	Std. err.	[95% conf. interval]			
c.failures@probe						
- 1	15.875	1.186293	13.51711 18.23289			
2	14.95833	.5912379	13.78318 16.13348			
3	16.47059	.9279866	14.62611 18.31506			
4	23.09677	.9451117	21.21826 24.97529			

The 95% confidence intervals in this table suggest that there are about 5-11 additional failures per wafer on probe 4. These are unadjusted for varying line widths and oxide depths. Possibly, probe 4 received the wafers with larger line widths or extreme oxide depths.

Truncated Poisson regression more clearly identifies the root causes for the increased failures by estimating the differences between probes adjusted for the line widths and oxide depths. It also allows us to determine whether the deviations from specifications in line widths or oxide depths might be contributing to the problem.

. tpoisson fai	ilures i.probe	depth width	, 11(10)	nolog			
Truncated Pois	sson regressio	n		Number	of obs	=	88
Limits: lower	r = 10			LR chi2	(5)	=	73.70
upper	r = +inf			Prob >	chi2	=	0.0000
Log likelihood	1 = -239.35746			Pseudo	R2	=	0.1334
failures	Coefficient	Std. err.	Z	P> z	[95%	conf.	interval]
probe							
2	1113037	.1019786	-1.09	0.275	3111	L781	.0885707
3	.0114339	.1036032	0.11	0.912	1916	5245	.2144924
4	.4254115	.0841277	5.06	0.000	.2605	5242	.5902989
depth width	0005034 .0330225	.0033375	-0.15	0.880	0070		.006038
cons	2.714025	.0752617	36.06	0.000	2.566		2.861536

The coefficients listed for the probes are testing the null hypothesis:  $H_0$ : probe<sub>i</sub> = probe<sub>i</sub>, where i equals 2, 3, and 4. Because the only coefficient that is statistically significant is the one for testing for  $H_0$ : probe<sub>4</sub> = probe<sub>1</sub>, p < 0.001, and because the *p*-values for the other probes are not statistically significant, that is,  $p \ge 0.275$ , the implication is that there is a difference between probe 4 and the other machines. Because the coefficient for this test is positive, 0.425, the conclusion is that the average failure rate for probe 4, after adjusting for line widths and oxide depths, is higher than the other probes. Possibly, probe 4 needs calibration or the head used with this machine is defective.

Line-width control is statistically significant, p = 0.034, but variation in oxide depths is not causing the increased failure rate. The engineer concluded that the sudden increase in failures is the result of two problems. First, probe 4 is malfunctioning, and second, there is a possible lithography or etching problem.

4

# **Stored results**

tpoisson stores the following in e():

1	
Scalars	

Sealars			
e(N)	number of observations		
e(k)	number of parameters		
e(k_eq)	number of equations in e(b)		
e(k_eq_model)	number of equations in overall model test		
e(k_dv)	number of dependent variables		
e(df_m)	model degrees of freedom		
e(r2_p)	pseudo- $R^2$		
e(11)	log likelihood		
e(11_0)	log likelihood, constant-only model		
e(N_clust)	number of clusters		
e(chi2)	$\chi^2$		
e(p)	<i>p</i> -value for model test		
e(rank)	rank of e(V)		
e(ic)	number of iterations		
e(rc)	return code		
e(converged)	1 if converged, 0 otherwise		
Macros			
e(cmd)	tpoisson		
e(cmdline)	command as typed		
e(depvar)	name of dependent variable		
e(llopt)	contents of 11(), or 0 if neither 11() nor u1() is specified		
e(ulopt)	contents of ul(), if specified		
e(wtype)	weight type		
e(wexp)	weight expression		
e(title)	title in estimation output		
e(clustvar)	name of cluster variable		
e(offset)	linear offset variable		
e(chi2type)	Wald or LR; type of model $\chi^2$ test		
e(vce)	vcetype specified in vce()		
e(vcetype)	title used to label Std. err.		
e(opt)	type of optimization		
e(which)	max or min; whether optimizer is to perform maximization or minimization		
e(ml_method)	type of m1 method		
e(user)	name of likelihood-evaluator program		
e(technique)	maximization technique		
e(properties)	b V		
e(predict)	program used to implement predict		
e(asbalanced)	factor variables fvset as asbalanced		
e(asobserved)	factor variables fyset as asobserved		
Matrices			
e(b)	coefficient vector		
e(Cns)	constraints matrix		
e(ilog)	iteration log (up to 20 iterations)		
e(gradient)	gradient vector		
e(V)	variance-covariance matrix of the estimators		
e(V_modelbased)	model-based variance		
Functions			
e(sample)	marks estimation sample		

In addition to the above, the following is stored in r():

Matrices

r(table)

matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

## Methods and formulas

For a nonnegative count outcome Y with left-truncation point  $ll_j$  and right-truncation point  $ul_j$ , we can write the truncated Poisson model as

$$f(y_j) = \frac{\exp(-\lambda_j)\lambda_j^{y_j}}{y_j! \Pr(ll_j < Y < ul_j \,|\, \xi_j)}$$

where

$$\begin{aligned} \xi_j &= \mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j \\ \lambda_j &= \exp(\xi_j) \end{aligned}$$

and  $\mathbf{x}_j$  is a vector of observed covariates. The conditional probability of observing  $y_j$  events, therefore given by  $ll_j < y_j < ul_j$ , is

$$\Pr(Y = y_j \,|\, ll_j < y_j < ul_j, \mathbf{x}_j) = \frac{\exp(-\lambda_j)\lambda_j^{y_j}}{y_j! \Pr(ll_j < Y < ul_j \,|\, \mathbf{x}_j)}$$

The log likelihood is given by

$$\ln L = \sum_{j=1}^n w_j \left[ -\lambda_j + \xi_j y_j - \ln(y_j!) - \ln \left\{ \left. \Pr(ll_j < Y < ul_j \,|\, \xi_j) \right\} \right]$$

If no weights are specified,  $w_i = 1$ .

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster *clustvar*), respectively. See [P] **\_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*.

tpoisson also supports estimation with survey data. For details on VCEs with survey data, see [SVY] Variance estimation.

## Acknowledgment

We gratefully acknowledge the previous work by Joseph Hilbe (1944–2017) (1999), coauthor of the Stata Press book *Generalized Linear Models and Extensions*.

## References

- Cameron, A. C., and P. K. Trivedi. 2005. *Microeconometrics: Methods and Applications*. New York: Cambridge University Press.
- ------. 2022. Microeconometrics Using Stata. 2nd ed. College Station, TX: Stata Press.
- Farbmacher, H. 2011. Estimation of hurdle models for overdispersed count data. Stata Journal 11: 82-94.
- Grogger, J. T., and R. T. Carson. 1991. Models for truncated counts. Journal of Applied Econometrics 6: 225–238. https://doi.org/10.1002/jae.3950060302.
- Hardin, J. W., and J. M. Hilbe. 2015. Regression models for count data from truncated distributions. *Stata Journal* 15: 226–246.
- Hilbe, J. M. 1999. sg102: Zero-truncated Poisson and negative binomial regression. *Stata Technical Bulletin* 47: 37–40. Reprinted in *Stata Technical Bulletin Reprints*, vol. 8, pp. 233–236. College Station, TX: Stata Press.
- Long, J. S. 1997. Regression Models for Categorical and Limited Dependent Variables. Thousand Oaks, CA: Sage.
- Long, J. S., and J. Freese. 2014. Regression Models for Categorical Dependent Variables Using Stata. 3rd ed. College Station, TX: Stata Press.
- Simonoff, J. S. 2003. Analyzing Categorical Data. New York: Springer. https://doi.org/10.1007/978-0-387-21727-7.

## Also see

- [R] tpoisson postestimation Postestimation tools for tpoisson
- [R] poisson Poisson regression
- [R] **nbreg** Negative binomial regression
- [R] tnbreg Truncated negative binomial regression
- [R] zinb Zero-inflated negative binomial regression
- [R] zip Zero-inflated Poisson regression
- [BAYES] bayes: tpoisson Bayesian truncated Poisson regression
- [FMM] fmm: tpoisson Finite mixtures of truncated Poisson regression models
- [SVY] svy estimation Estimation commands for survey data
- [XT] xtpoisson Fixed-effects, random-effects, and population-averaged Poisson models
- [U] 20 Estimation and postestimation commands

Stata, Stata Press, Mata, NetCourse, and NetCourseNow are registered trademarks of StataCorp LLC. Stata and Stata Press are registered trademarks with the World Intellectual Property Organization of the United Nations. StataNow is a trademark of StataCorp LLC. Other brand and product names are registered trademarks or trademarks of their respective companies. Copyright © 1985–2025 StataCorp LLC, College Station, TX, USA. All rights reserved.



For suggested citations, see the FAQ on citing Stata documentation.