Description

tpoisson fits a truncated Poisson regression model when the number of occurrences of an event is restricted to be above a truncation point, below a truncation point, or between two truncation points. Truncated Poisson models are appropriate when neither the dependent variable nor the covariates are observed in the truncated part of the distribution. By default, tpoisson assumes left-truncation occurs at zero, but truncation may be specified at other fixed points or at values that vary across observations.

Quick start

Truncated Poisson regression of \( y \) on \( x_1 \) and \( x_2 \) with left-truncation at 0

\[
\text{tpoisson } y \ x_1 \ x_2
\]

Add categorical variable \( a \) using factor-variable syntax

\[
\text{tpoisson } y \ x_1 \ x_2 \ i.a
\]

As above, but report incidence-rate ratios and use a constant truncation point of 4

\[
\text{tpoisson } y \ x_1 \ x_2 \ i.a, \ irr \ ll(4)
\]

With offset variable \( \lnexp \)

\[
\text{tpoisson } y \ x_1 \ x_2 \ i.a, \ offset(\lnexp)
\]

As above, but with a variable truncation point stored in variable \( \text{min} \)

\[
\text{tpoisson } y \ x_1 \ x_2 \ i.a, \ offset(\lnexp) \ ll(\text{min})
\]

With variable left- and right-truncation points

\[
\text{tpoisson } y \ x_1 \ x_2, \ ll(\text{min}) \ ul(\text{max})
\]

With variable right-truncation points

\[
\text{tpoisson } y \ x_1 \ x_2, \ ul(\text{max})
\]

Constrain the coefficients for 2.\( a \) and 3.\( a \) to equality

\[
\text{constraint define 1 } 2.a = 3.a
\]

\[
\text{tpoisson } y \ x_1 \ x_2 \ i.a, \ constraints(1)
\]

Menu

Statistics > Count outcomes > Truncated Poisson regression
Syntax

tpoisson  depvar  [indepvars]  [if]  [in]  [weight]  [,  options]

options  Description

Model
noconstant  suppress constant term
ll(#)  |varname)  lower limit for truncation; default is ll(0) when neither
       ll() nor ul() is specified
ul(#)  |varname)  upper limit for truncation
exposure(varname_o)  include ln(varname_o) in model with coefficient constrained to 1
offset(varname_o)  include varname_o in model with coefficient constrained to 1
constraints(constraints)  apply specified linear constraints

SE/Robust
vce(vcetype)  vcetype may be oim, robust, cluster clustvar, opg, bootstrap,  
or jackknife

Reporting
level(#)  set confidence level; default is level(95)
irr  report incidence-rate ratios
noconsreport  do not display constraints
display_options  control columns and column formats, row spacing, line width,  
display of omitted variables and base and empty cells, and  
factor-variable labeling

Maximization
maximize_options  control the maximization process; seldom used

collinear  keep collinear variables
coefflegend  display legend instead of statistics

Options

noconstant; see [R] Estimation options.

ll(#)  |varname) and ul(#)  |varname) specify the lower and upper limits for truncation, respectively.
You may specify nonnegative integer values for one or both.
When neither ll() nor ul() is specified, the default is zero truncation, ll(0), equivalent to  
left-truncation at zero.
tpoisson — Truncated Poisson regression

exposure(varname), offset(varname), constraints(constraints); see [R] Estimation options.

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

level(#); see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, notvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

maximize_options: difficult, technique(algorithm_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), rtolerance(#), nonrtolerance, and from(init_specs); see [R] Maximize. These options are seldom used.

Setting the optimization type to technique(bhhh) resets the default vcetype to vce(opg).

The following options are available with tpoisson but are not shown in the dialog box: collinear, coeflegend; see [R] Estimation options.

Remarks and examples stata.com

tpoisson fits a truncated Poisson regression model by maximum likelihood estimation when the number of occurrences of an event is restricted to be above a truncation point, below a truncation point, or between two truncation points. If the dependent variable is not truncated, standard Poisson regression may be more appropriate; see [R] poisson.

When the data are truncated, we do not observe either the dependent variable or the covariates. For example, consider a study about the number of days that individuals with hyperglycemia are hospitalized after presenting to the hospital. If we select our sample only from admission records, then the sample is truncated at zero because we have data only on individuals who stayed at least one day. Now assume that we are relying on billing data and that hospitals may submit either a final bill when a patient is discharged or an interim bill every 30 days. In this case, we have no information about patients who are hospitalized fewer than 1 day or more than 30 days. Our data are left-truncated at 0 and right-truncated at 30.

A related phenomenon is censoring. For censored observations, we observe complete covariate information but only a censored value of the dependent variable. Different research designs can give rise to censored data or truncated data. See [R] cpoisson for information about censored Poisson regression.

Example 1: Left-truncation at zero

Consider the Simonoff (2003) dataset of running shoes for a sample of runners who registered an online running log. A running-shoe marketing executive is interested in knowing how the number of pairs of running shoes purchased relates to other factors such as gender, marital status, age, education, income, typical number of runs per week, average miles run per week, and the preferred type of running. These data are naturally truncated at zero. A truncated Poisson model is fit to the number of pairs of shoes owned on runs per week, miles run per week, gender, age, and marital status.

No options are needed because zero truncation is the default for tpoisson.

```
. use https://www.stata-press.com/data/r16/runshoes
. tpoisson shoes rpweek mpweek male age married
```

```
Iteration 0: log likelihood = -88.328151
Iteration 1: log likelihood = -86.272639
Iteration 2: log likelihood = -86.257999
Iteration 3: log likelihood = -86.257994
```

Truncated Poisson regression

| Coef. | Std. Err. | z  | P>|z| | [95% Conf. Interval] |
|-------|-----------|----|------|-------------------|
| rpweek | .1575811 | .1097893 | 1.44 | 0.151 | -.057602 | .3727641 |
| mpweek | .0210673 | .0091113 | 2.31 | 0.021 | .0032094 | .0389252 |
| male | .0446134 | .2444626 | 0.18 | 0.855 | -.4345246 | .5237513 |
| age | .0185565 | .0137786 | 1.35 | 0.178 | -.008449 | .045562 |
| married | -.1283912 | .2785044 | -0.46 | 0.645 | -.6742498 | .4174674 |
| _cons | -1.205844 | .6619774 | -1.82 | 0.069 | -2.503296 | .0916078 |

Using the zero-truncated Poisson regression with these data, only the coefficient on average miles per week is statistically significant at the 5% level.

Example 2: Left-truncation with a fixed-truncation point

Semiconductor manufacturing requires that silicon wafers be coated with a layer of metal oxide. The depth of this layer is strictly controlled. In this example, a critical oxide layer is designed for 300 ± 20 angstroms (Å).

After the oxide layer is coated onto a wafer, the wafer enters a photolithography step in which the lines representing the electrical connections are printed on the oxide and later etched and filled with metal. The widths of these lines are measured. In this example, they are controlled to 90 ± 5 micrometers (µm).

After these and other steps, each wafer is electrically tested at probe. If too many failures are discovered, the wafer is rejected and sent for engineering analysis. In this example, the maximum number of probe failures tolerated for this product is 10.
A major failure at probe has been encountered—88 wafers had more than 10 failures each. The 88 wafers that failed were tested using 4 probe machines. The engineer suspects that the failures were a result of faulty probe machines, poor depth control, or poor line widths. The line widths and depths in these data are the actual measurement minus its specification target, 300 Å for the oxide depths and 90 µm for the line widths.

The following table tabulates the average failure rate for each probe using Stata’s `mean` command; see [R] `mean`.

```
. use https://www.stata-press.com/data/r16/probe
. mean failures, over(probe)
```

<table>
<thead>
<tr>
<th>Number of obs = 88</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Mean estimation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. Err.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.875</td>
<td>1.186293 13.51711 18.23289</td>
</tr>
<tr>
<td>2</td>
<td>14.95833</td>
<td>.5912379  13.78318 16.13348</td>
</tr>
<tr>
<td>3</td>
<td>16.47059</td>
<td>.9279866  14.62611 18.31506</td>
</tr>
<tr>
<td>4</td>
<td>23.09677</td>
<td>.9451117  21.21826 24.97529</td>
</tr>
</tbody>
</table>

The 95% confidence intervals in this table suggest that there are about 5–11 additional failures per wafer on probe 4. These are unadjusted for varying line widths and oxide depths. Possibly, probe 4 received the wafers with larger line widths or extreme oxide depths.

Truncated Poisson regression more clearly identifies the root causes for the increased failures by estimating the differences between probes adjusted for the line widths and oxide depths. It also allows us to determine whether the deviations from specifications in line widths or oxide depths might be contributing to the problem.

```
. tpoisson failures i.probe depth width, ll(10) nolog
```

```
Truncated Poisson regression
Number of obs = 88
Limits: lower = 10 LR chi2(5) = 73.70
upper = +inf Prob > chi2 = 0.0000
Log likelihood = -239.35746 Pseudo R2 = 0.1334
```

| failures          | Coef.  | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|-------------------|--------|-----------|------|------|----------------------|
| probe             |        |           |      |      |                      |
| 2                 | -.1113037 | .1019786 | -1.09 | 0.275 | -.3111781 .0885707 |
| 3                 | .0114339  | .1036032  | 0.11 | 0.912 | -.1916245 .2144924 |
| 4                 | .4254115  | .0841277  | 5.06 | 0.000 | .2605242 .5902989 |
| depth             | -.0005034 | .0033375 | -0.15 | 0.880 | -.0070447 .006038 |
| width             | .0330225  | .015573   | 2.12 | 0.034 | .0025001 .063545  |
| _cons             | 2.714025  | .0752617  | 36.06 | 0.000 | 2.566515 2.861536 |

The coefficients listed for the probes are testing the null hypothesis: \( H_0: \beta_i = \beta_1 \), where \( i \) equals 2, 3, and 4. Because the only coefficient that is statistically significant is the one for testing for \( H_0: \beta_4 = \beta_1, p < 0.001 \), and because the \( p \)-values for the other probes are not statistically significant, that is, \( p \geq 0.275 \), the implication is that there is a difference between probe 4 and the other machines. Because the coefficient for this test is positive, 0.425, the conclusion is that the average failure rate for probe 4, after adjusting for line widths and oxide depths, is higher than the other probes. Possibly, probe 4 needs calibration or the head used with this machine is defective.

Line-width control is statistically significant, \( p = 0.034 \), but variation in oxide depths is not causing the increased failure rate. The engineer concluded that the sudden increase in failures is the result of
two problems. First, probe 4 is malfunctioning, and second, there is a possible lithography or etching problem.

**Stored results**

tpoisson stores the following in e():

Scalars

- `e(N)` number of observations
- `e(k)` number of parameters
- `e(k_eq)` number of equations in `e(b)`
- `e(k_eq_model)` number of equations in overall model test
- `e(k_dv)` number of dependent variables
- `e(df_m)` model degrees of freedom
- `e(r2_p)` pseudo-$R^2$
- `e(ll)` log likelihood
- `e(ll_0)` log likelihood, constant-only model
- `e(N_clust)` number of clusters
- `e(chi2)` $\chi^2$
- `e(p)` $p$-value for model test
- `e(rank)` rank of `e(V)`
- `e(ic)` number of iterations
- `e(rc)` return code
- `e(converged)` 1 if converged, 0 otherwise

Macros

- `e(cmd)` tpoisson
- `e(cmdline)` command as typed
- `e(depvar)` name of dependent variable
- `e(llopt)` contents of `ll()`, or 0 if neither `ll()` nor `ul()` is specified
- `e(ulopt)` contents of `ul()`, if specified
- `e(wtype)` weight type
- `e(wexp)` weight expression
- `e(title)` title in estimation output
- `e(clustvar)` name of cluster variable
- `e(offset)` linear offset variable
- `e(chi2type)` Wald or LR; type of model $\chi^2$ test
- `e(vcetype)` `vcetype` specified in `vce()`
- `e(vcetype)` title used to label Std. Err.
- `e(opt)` type of optimization
- `e(which)` max or min; whether optimizer is to perform maximization or minimization
- `e(ml_method)` type of ml method
- `e(user)` name of likelihood-evaluator program
- `e(technique)` maximization technique
- `e(properties)` b V
- `e(predict)` program used to implement predict
- `e(asbalanced)` factor variables fvset as asbalanced
- `e(asobserved)` factor variables fvset as asobserved

Matrices

- `e(b)` coefficient vector
- `e(Cns)` constraints matrix
- `e(ilog)` iteration log (up to 20 iterations)
- `e(gradient)` gradient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(V_modelbased)` model-based variance

Functions

- `e(sample)` marks estimation sample
In addition to the above, the following is stored in \( \textbf{r()} \):

Matrices

\[ \textbf{r(table)} \] matrix containing the coefficients with their standard errors, test statistics, \( p \)-values, and confidence intervals

Note that results stored in \( \textbf{r()} \) are updated when the command is replayed and will be replaced when any \( \text{r-class} \) command is run after the estimation command.

**Methods and formulas**

For a nonnegative count outcome \( Y \) with left-truncation point \( ll_{ij} \) and right-truncation point \( ul_{ij} \), we can write the truncated Poisson model as

\[
    f(y_j) = \frac{\exp(-\lambda_j)\lambda_j^{y_j}}{y_j! \Pr(ll_{ij} < Y < ul_{ij} | \xi_j)}
\]

where

\[
    \xi_j = x_j \beta + \text{offset}_j
\]

\[
    \lambda_j = \exp(\xi_j)
\]

and \( x_j \) is a vector of observed covariates. The conditional probability of observing \( y_j \) events, therefore given by \( ll_{ij} < y_j < ul_{ij} \), is

\[
    \Pr(Y = y_j | ll_{ij} < y_j < ul_{ij}, x_j) = \frac{\exp(-\lambda_j)\lambda_j^{y_j}}{y_j! \Pr(ll_{ij} < Y < ul_{ij} | x_j)}
\]

The log likelihood is given by

\[
    \ln L = \sum_{j=1}^{n} w_j \left[ -\lambda_j + \xi_j y_j - \ln(y_j!) - \ln \{ \Pr(ll_{ij} < Y < ul_{ij} | \xi_j) \} \right]
\]

If no weights are specified, \( w_j = 1 \).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using \texttt{vce(robust)} and \texttt{vce(cluster clustvar)}, respectively. See \([P]\) \texttt{robust}, particularly \textit{Maximum likelihood estimators} and \textit{Methods and formulas}.

tpoisson also supports estimation with survey data. For details on VCEs with survey data, see \([SVY] \textbf{Variance estimation}\).

**Acknowledgment**

We gratefully acknowledge the previous work by Joseph Hilbe (1944–2017) (1999), a former editor of the \textit{Stata Technical Bulletin} and coauthor of the Stata Press book \textit{Generalized Linear Models and Extensions}. 
References


Also see

[R] `tpoisson postestimation` — Postestimation tools for tpoisson

[R] `poisson` — Poisson regression

[R] `nbreg` — Negative binomial regression

[R] `tnbreg` — Truncated negative binomial regression

[R] `zip` — Zero-inflated Poisson regression

[BAYES] `bayes: tpoisson` — Bayesian truncated Poisson regression

[FMM] `fmm: tpoisson` — Finite mixtures of truncated Poisson regression models

[SVY] `svy estimation` — Estimation commands for survey data

[XT] `xtpoisson` — Fixed-effects, random-effects, and population-averaged Poisson models

[U] 20 Estimation and postestimation commands