lrtest — Likelihood-ratio test after estimation

Description

`lrtest` performs a likelihood-ratio test of the null hypothesis that the parameter vector of a statistical model satisfies some smooth constraint. To conduct the test, both the unrestricted and the restricted models must be fit using the maximum likelihood method (or some equivalent method), and the results of at least one must be stored using `estimates store`.

`lrtest` also supports composite models. In a composite model, we assume that the log likelihood and dimension (number of free parameters) of the full model are obtained as the sum of the log-likelihood values and dimensions of the constituting models.

Quick start

Likelihood-ratio test that the coefficients for `x2` and `x3` are equal to 0

```
logit y x1 x2 x3
estimates store full
logit y x1 if e(sample)
estimates store restricted
lrtest full restricted
```

Display additional information, including AIC and BIC

```
lrtest full restricted, stats
```

Likelihood-ratio test that the coefficients for `x1` and `x3` are equal

```
constraint 1 x1=x3
logit y x1 x2 x3, constraints(1)
estimates store constrained
lrtest full constrained
```

Compare stored estimates `full` with the last model run

```
lrtest full
```

Menu

Statistics > Postestimation

Syntax

`lrtest` *modelspec*$_1$ [*modelspec*$_2$] [, *options*]

modelspec$_1$ and *modelspec*$_2$ specify the restricted and unrestricted model in any order. *modelspec#* is

name | `.` | `(`*namelist*`)`

name is the name under which estimation results were stored using `estimates store` (see [R] **estimates store**), and "`.`" refers to the last estimation results, whether or not these were already stored. If *modelspec*$_2$ is not specified, the last estimation result is used; this is equivalent to specifying *modelspec*$_2$ as "`.`".

If *namelist* is specified for a composite model, *modelspec*$_1$ and *modelspec*$_2$ cannot have names in common; for example, `lrtest (A B C) (C D E)` is not allowed because both model specifications include `C`.

options	Description
`stats`	display statistical information about the two models
`dir`	display descriptive information about the two models
`df(`*#*`)`	override the automatic degrees-of-freedom calculation; seldom used
`force`	force testing even when apparently invalid

`collect` is allowed; see [U] **11.1.10 Prefix commands**.

Options

`stats` displays statistical information about the unrestricted and restricted models, including the information indices of Akaike and Schwarz.

`dir` displays descriptive information about the unrestricted and restricted models; see `estimates dir` in [R] **estimates store**.

`df(`*#*`)` is seldom specified; it overrides the automatic degrees-of-freedom calculation.

`force` forces the likelihood-ratio test calculations to take place in situations where `lrtest` would normally refuse to do so and issue an error. Such situations arise when one or more assumptions of the test are violated, for example, if the models were fit with `vce(robust)`, `vce(cluster` *clustvar*`)`, or `pweight`s; when the dependent variables in the two models differ; when the null log likelihoods differ; when the samples differ; or when the estimation commands differ. If you use the `force` option, there is no guarantee as to the validity or interpretability of the resulting test.

Remarks and examples

stata.com

The standard way to use `lrtest` is to do the following:

1. Fit either the restricted model or the unrestricted model by using one of Stata's estimation commands and then store the results using `estimates store` *name*.
2. Fit the alternative model (the unrestricted or restricted model) and then type '`lrtest` *name* `.`'. `lrtest` determines for itself which of the two models is the restricted model by comparing the degrees of freedom.

Often, you may want to store the alternative model with `estimates store` *name*$_2$, for instance, if you plan additional tests against models yet to be fit. The likelihood-ratio test is then obtained as `lrtest` *name* *name*$_2$.

Remarks are presented under the following headings:

Nested models

`lrtest` may be used with any estimation command that reports a log likelihood, including `heckman`, `logit`, `poisson`, `stcox`, and `streg`. You must check that one of the model specifications implies a statistical model that is *nested within* the model implied by the other specification. Usually, this means that both models are fit with the same estimation command (for example, both are fit by `logit`, with the same dependent variables) and that the set of covariates of one model is a subset of the covariates of the other model. Second, `lrtest` is valid only for models that are fit by maximum likelihood or by some equivalent method, so it does not apply to models that were fit with probability weights or clusters. Specifying the `vce(robust)` option similarly would indicate that you are worried about the valid specification of the model, so you would not use `lrtest`. Third, `lrtest` assumes that under the null hypothesis, the test statistic is (approximately) distributed as χ^2. This assumption is not true for likelihood-ratio tests of "boundary conditions", such as tests for the presence of overdispersion or random effects (Gutierrez, Carter, and Drukker 2001).

▷ Example 1

We have data on infants born with low birthweights along with the characteristics of the mother (Hosmer, Lemeshow, and Sturdivant 2013; see also [R] **logistic**). We fit the following model:

```
. use https://www.stata-press.com/data/r17/lbw
(Hosmer & Lemeshow data)

. logistic low age lwt i.race smoke ptl ht ui

Logistic regression                                     Number of obs =    189
                                                        LR chi2(8)    =  33.22
                                                        Prob > chi2   = 0.0001
Log likelihood = -100.724                               Pseudo R2     = 0.1416

------------------------------------------------------------------------------
         low | Odds ratio   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         age |   .9732636   .0354759    -0.74   0.457     .9061578    1.045339
         lwt |   .9849634   .0068217    -2.19   0.029     .9716834    .9984249
             |
        race |
      Black  |   3.534767   1.860737     2.40   0.016     1.259736    9.918406
      Other  |   2.368079   1.039949     1.96   0.050     1.001356    5.600207
             |
       smoke |   2.517698    1.00916     2.30   0.021     1.147676    5.523162
         ptl |   1.719161   .5952579     1.56   0.118     .8721455    3.388787
          ht |   6.249602   4.322408     2.65   0.008     1.611152    24.24199
          ui |     2.1351   .9808153     1.65   0.099     .8677528      5.2534
       _cons |   1.586014   1.910496     0.38   0.702     .1496092     16.8134
------------------------------------------------------------------------------
Note: _cons estimates baseline odds.
```

We now wish to test the constraint that the coefficients on `age`, `lwt`, `ptl`, and `ht` are all zero or, equivalently here, that the odds ratios are all 1. One solution is to type

```
. test age lwt ptl ht
 ( 1)  [low]age = 0
 ( 2)  [low]lwt = 0
 ( 3)  [low]ptl = 0
 ( 4)  [low]ht = 0
           chi2(  4) =   12.38
         Prob > chi2 =    0.0147
```

This test is based on the inverse of the information matrix and is therefore based on a quadratic approximation to the likelihood function; see [R] **test**. A more precise test would be to refit the model, applying the proposed constraints, and then calculate the likelihood-ratio test.

We first save the current model:

```
. estimates store full
```

We then fit the constrained model, which here is the model omitting `age`, `lwt`, `ptl`, and `ht`:

```
. logistic low i.race smoke ui
Logistic regression                                     Number of obs =    189
                                                        LR chi2(4)    =  18.80
                                                        Prob > chi2   = 0.0009
Log likelihood = -107.93404                             Pseudo R2     = 0.0801
```

low	Odds ratio	Std. err.	z	P>|z|	[95% conf.	interval]
race						
Black	3.052746	1.498087	2.27	0.023	1.166747	7.987382
Other	2.922593	1.189229	2.64	0.008	1.316457	6.488285
smoke	2.945742	1.101838	2.89	0.004	1.415167	6.131715
ui	2.419131	1.047359	2.04	0.041	1.035459	5.651788
_cons	.1402209	.0512295	-5.38	0.000	.0685216	.2869447

```
Note: _cons estimates baseline odds.
```

That done, `lrtest` compares this model with the model we previously stored:

```
. lrtest full .
Likelihood-ratio test
Assumption: . nested within full
 LR chi2(4) =  14.42
Prob > chi2 = 0.0061
```

Let's compare results. `test` reported that `age`, `lwt`, `ptl`, and `ht` were jointly significant at the 1.5% level; `lrtest` reports that they are significant at the 0.6% level. Given the quadratic approximation made by `test`, we could argue that `lrtest`'s results are more accurate.

`lrtest` explicates the assumption that, from a comparison of the degrees of freedom, it has assessed that the last fit model (`.`) is nested within the model stored as `full`. In other words, `full` is the unconstrained model and `.` is the constrained model.

The names in "`(Assumption: . nested in full)`" are actually links. Click on a name, and the results for that model are replayed.

◁

Aside: The `nestreg` command provides a simple syntax for performing likelihood-ratio tests for nested model specifications; see [R] **nestreg**. In the previous example, we fit a full `logistic` model, used `estimates store` to store the `full` model, fit a constrained `logistic` model, and used `lrtest` to report a likelihood-ratio test between two models. To do this with one call to `nestreg`, use the `lrtable` option.

❑ Technical note

`lrtest` determines the degrees of freedom of a model as the rank of the (co)variance matrix `e(V)`. There are two issues here. First, the *numerical* determination of the rank of a matrix is a subtle problem that can, for instance, be affected by the scaling of the variables in the model. The rank of a matrix depends on the number of (independent) linear combinations of coefficients that sum exactly to zero. In the world of numerical mathematics, it is hard to tell whether a very small number is really nonzero or is a real zero that happens to be slightly off because of roundoff error from the finite precision with which computers make floating-point calculations. Whether a small number is being classified as one or the other, typically on the basis of a threshold, affects the determined degrees of freedom. Although Stata generally makes sensible choices, it is bound to make mistakes occasionally. The moral of this story is to make sure that the calculated degrees of freedom is as you expect before interpreting the results.

❑

❑ Technical note

A second issue involves `regress` and related commands such as `anova`. Mainly for historical reasons, `regress` does not treat the residual variance, σ^2, the same way that it treats the regression coefficients. Type `estat vce` after `regress`, and you will see the regression coefficients, not $\widehat{\sigma}^2$. Most estimation commands for models with ancillary parameters (for example, `streg` and `heckman`) treat all parameters as equals. There is nothing technically wrong with `regress` here; we are usually focused on the regression coefficients, and their estimators are uncorrelated with $\widehat{\sigma}^2$. But, formally, σ^2 adds a degree of freedom to the model, which does not matter if you are comparing two regression models by a likelihood-ratio test. This test depends on the difference in the degrees of freedom, and hence being "off by 1" in each does not matter. But, if you are comparing a regression model with a larger model—for example, a heteroskedastic regression model fit by `arch`—the automatic determination of the degrees of freedom is incorrect, and you must specify the `df(#)` option.

❑

▷ Example 2

Returning to the low-birthweight data in example 1, we now wish to test that the coefficient on `2.race` (black) is equal to that on `3.race` (other). The base model is still stored under the name `full`, so we need only fit the constrained model and perform the test. With z as the index of the logit model, the base model is

$$z = \beta_0 + \beta_1 \texttt{age} + \beta_2 \texttt{lwt} + \beta_3 \texttt{2.race} + \beta_4 \texttt{3.race} + \cdots$$

If $\beta_3 = \beta_4$, this can be written as

$$z = \beta_0 + \beta_1 \texttt{age} + \beta_2 \texttt{lwt} + \beta_3 (\texttt{2.race} + \texttt{3.race}) + \cdots$$

We can fit the constrained model as follows:

```
. constraint 1 2.race = 3.race
. logistic low age lwt i.race smoke ptl ht ui, constraints(1)
Logistic regression                                 Number of obs =    189
                                                    Wald chi2(7)  =  25.17
Log likelihood = -100.9997                          Prob > chi2   = 0.0007
 ( 1)  [low]2.race - [low]3.race = 0
```

low	Odds ratio	Std. err.	z	P>|z|	[95% conf. interval]	
age	.9716799	.0352638	-0.79	0.429	.9049649	1.043313
lwt	.9864971	.0064627	-2.08	0.038	.9739114	.9992453
race						
Black	2.728186	1.080207	2.53	0.011	1.255586	5.927907
Other	2.728186	1.080207	2.53	0.011	1.255586	5.927907
smoke	2.664498	1.052379	2.48	0.013	1.228633	5.778414
ptl	1.709129	.5924776	1.55	0.122	.8663666	3.371691
ht	6.116391	4.215585	2.63	0.009	1.58425	23.61385
ui	2.09936	.9699702	1.61	0.108	.8487997	5.192407
_cons	1.309371	1.527398	0.23	0.817	.1330839	12.8825

Note: **_cons** estimates baseline odds.

Comparing this model with our original model, we obtain

```
. lrtest full .
Likelihood-ratio test
Assumption: . nested within full
 LR chi2(1) =   0.55
Prob > chi2 = 0.4577
```

By comparison, typing `test 2.race=3.race` after fitting our base model results in a significance level of 0.4572. Alternatively, we can first store the restricted model, here using the name `equal`. Next, `lrtest` is invoked specifying the names of the restricted and unrestricted models (we do not care about the order). This time, we also add the option `stats` requesting a table of model statistics, including the model selection indices AIC and BIC.

```
. estimates store equal
. lrtest equal full, stats
Likelihood-ratio test
Assumption: equal nested within full
 LR chi2(1) =   0.55
Prob > chi2 = 0.4577
Akaike's information criterion and Bayesian information criterion
```

Model	N	ll(null)	ll(model)	df	AIC	BIC
equal	189	.	-100.9997	8	217.9994	243.9334
full	189	-117.336	-100.724	9	219.448	248.6237

Note: BIC uses N = number of observations. See **[R] BIC note**.

◁

Composite models

lrtest supports composite models; that is, models that can be fit by fitting a series of simpler models or by fitting models on subsets of the data. Theoretically, a composite model is one in which the likelihood function, $L(\theta)$, of the parameter vector, θ, can be written as the product

$$L(\theta) = L_1(\theta_1) \times L_2(\theta_2) \times \cdots \times L_k(\theta_k)$$

of likelihood terms with $\theta = (\theta_1, \ldots, \theta_k)$ a partitioning of the full parameter vector. In such a case, the full-model likelihood $L(\theta)$ is maximized by maximizing the likelihood terms $L_j(\theta_j)$ in turn. Obviously, $\log L(\widehat{\theta}) = \sum_{j=1}^k \log L_j(\widehat{\theta}_j)$. The degrees of freedom for the composite model is obtained as the sum of the degrees of freedom of the constituting models.

▷ Example 3

As an example of the application of composite models, we consider a test of the hypothesis that the coefficients of a statistical model do not differ between different portions ("regimes") of the covariate space. Economists call a test for such a hypothesis a *Chow test*.

We continue the analysis of the data on children of low birthweight by using logistic regression modeling and study whether the regression coefficients are the same among the three races: white, black, and other. A likelihood-ratio Chow test can be obtained by fitting the logistic regression model for each of the races and then comparing the combined results with those of the model previously stored as full. Because the full model included dummies for the three races, this version of the Chow test allows the intercept of the logistic regression model to vary between the regimes (races).

```
. logistic low age lwt smoke ptl ht ui if 1.race, nolog
Logistic regression                                     Number of obs =     96
                                                        LR chi2(6)    =  13.86
                                                        Prob > chi2   = 0.0312
Log likelihood = -45.927061                             Pseudo R2     = 0.1311

------------------------------------------------------------------------------
         low | Odds ratio   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         age |   .9869674   .0527757    -0.25   0.806     .8887649    1.096021
         lwt |   .9900874   .0106101    -0.93   0.353     .9695089    1.011103
       smoke |   4.208697   2.680133     2.26   0.024      1.20808    14.66222
         ptl |   1.592145   .7474264     0.99   0.322     .6344379    3.995544
          ht |   2.900166   3.193537     0.97   0.334     .3350554     25.1032
          ui |   1.229523   .9474768     0.27   0.789     .2715165    5.567715
       _cons |   .4891008    .993785    -0.35   0.725     .0091175    26.23746
------------------------------------------------------------------------------
Note: _cons estimates baseline odds.
. estimates store white
```

```
. logistic low age lwt smoke ptl ht ui if 2.race, nolog
Logistic regression                                     Number of obs =     26
                                                        LR chi2(6)    =  10.12
                                                        Prob > chi2   = 0.1198
Log likelihood = -12.654157                             Pseudo R2     = 0.2856

------------------------------------------------------------------------------
         low | Odds ratio   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         age |   .8735313   .1377846    -0.86   0.391     .6412332    1.189983
         lwt |   .9747736    .016689    -1.49   0.136     .9426065    1.008038
       smoke |   16.50373   24.37044     1.90   0.058     .9133647    298.2083
         ptl |   4.866916    9.33151     0.83   0.409     .1135573    208.5895
          ht |   85.05605   214.6382     1.76   0.078     .6049308    11959.27
          ui |   67.61338   133.3313     2.14   0.033     1.417399    3225.322
       _cons |    48.7249   169.9216     1.11   0.265     .0523961    45310.94
------------------------------------------------------------------------------
Note: _cons estimates baseline odds.
. estimates store black
. logistic low age lwt smoke ptl ht ui if 3.race, nolog
Logistic regression                                     Number of obs =     67
                                                        LR chi2(6)    =  14.06
                                                        Prob > chi2   = 0.0289
Log likelihood = -37.228444                             Pseudo R2     = 0.1589

------------------------------------------------------------------------------
         low | Odds ratio   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         age |   .9263905   .0665386    -1.06   0.287     .8047407     1.06643
         lwt |   .9724499    .015762    -1.72   0.085     .9420424    1.003839
       smoke |   .7979034   .6340585    -0.28   0.776     .1680885    3.787586
         ptl |   2.845675   1.777944     1.67   0.094     .8363053    9.682908
          ht |   7.767503   10.00537     1.59   0.112     .6220764    96.98826
          ui |   2.925006   2.046473     1.53   0.125     .7423107    11.52571
       _cons |   49.09444   113.9165     1.68   0.093     .5199275    4635.769
------------------------------------------------------------------------------
Note: _cons estimates baseline odds.
. estimates store other
```

We are now ready to perform the likelihood-ratio Chow test:

```
. lrtest (full) (white black other), stats
Likelihood-ratio test
Assumption: full nested within (white, black, other)
LR chi2(12) =   9.83
Prob > chi2 = 0.6310
Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |          N   ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
        full |        189   -117.336   -100.724       9     219.448   248.6237
       white |         96  -52.85752  -45.92706       7    105.8541   123.8046
       black |         26  -17.71291  -12.65416       7    39.30831   48.11499
       other |         67  -44.26039  -37.22844       7    88.45689   103.8897
-----------------------------------------------------------------------------
Note: BIC uses N = number of observations. See [R] BIC note.
```

We cannot reject the hypothesis that the logistic regression model applies to each of the races at any reasonable significance level. By specifying the `stats` option, we can verify the degrees of freedom of the test: $12 = 7 + 7 + 7 - 9$. We can obtain the same test by fitting an expanded model with interactions between all covariates and `race`.

```
. logistic low race##c.(age lwt smoke ptl ht ui)

Logistic regression                                     Number of obs =    189
                                                        LR chi2(20)   =  43.05
                                                        Prob > chi2   = 0.0020
Log likelihood = -95.809661                             Pseudo R2     = 0.1835

------------------------------------------------------------------------------
         low | Odds ratio   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        race |
      Black  |   99.62137   402.0829     1.14   0.254     .0365434    271578.9
      Other  |   100.3769    309.586     1.49   0.135     .2378638    42358.38
             |
         age |   .9869674   .0527757    -0.25   0.806     .8887649    1.096021
         lwt |   .9900874   .0106101    -0.93   0.353     .9695089    1.011103
       smoke |   4.208697   2.680133     2.26   0.024      1.20808    14.66222
         ptl |   1.592145   .7474264     0.99   0.322     .6344379    3.995544
          ht |   2.900166   3.193537     0.97   0.334     .3350554     25.1032
          ui |   1.229523   .9474768     0.27   0.789     .2715165    5.567715
             |
  race#c.age |
      Black  |    .885066   .1474079    -0.73   0.464      .638569    1.226714
      Other  |   .9386232   .0840486    -0.71   0.479     .7875366    1.118695
             |
  race#c.lwt |
      Black  |   .9845329   .0198857    -0.77   0.440     .9463191     1.02429
      Other  |   .9821859   .0190847    -0.93   0.355     .9454839    1.020313
             |
race#c.smoke |
      Black  |   3.921338   6.305992     0.85   0.395      .167725    91.67917
      Other  |   .1895844   .1930601    -1.63   0.102      .025763    1.395113
             |
  race#c.ptl |
      Black  |    3.05683   6.034089     0.57   0.571     .0638301    146.3918
      Other  |   1.787322   1.396789     0.74   0.457     .3863582    8.268285
             |
   race#c.ht |
      Black  |     29.328    80.7482     1.23   0.220     .1329492    6469.623
      Other  |   2.678295   4.538712     0.58   0.561     .0966916    74.18702
             |
   race#c.ui |
      Black  |   54.99155   116.4274     1.89   0.058     .8672471    3486.977
      Other  |   2.378976   2.476124     0.83   0.405      .309335    18.29579
             |
       _cons |   .4891008    .993785    -0.35   0.725     .0091175    26.23746
------------------------------------------------------------------------------
Note: _cons estimates baseline odds.

. lrtest full .

Likelihood-ratio test
Assumption: full nested within .

LR chi2(12) =   9.83
Prob > chi2 = 0.6310
```

Applying `lrtest` for the full model against the model with all interactions yields the same test statistic and p-value as for the full model against the composite model for the three regimes. Here the specification of the model with interactions was convenient, and `logistic` had no problem computing the estimates for the expanded model. In models with more complicated likelihoods, such as Heckman's selection model (see [R] **heckman**) or complicated survival-time models (see [ST] **streg**), fitting the models with all interactions may be numerically demanding and may be much more time consuming than fitting a series of models separately for each regime.

Given the model with all interactions, we could also test the hypothesis of no differences among the regions (races) by a Wald version of the Chow test by using the `testparm` command; see [R] **test**.

```
. testparm race#c.(age lwt smoke ptl ht ui)
 ( 1)  [low]2.race#c.age = 0
 ( 2)  [low]3.race#c.age = 0
 ( 3)  [low]2.race#c.lwt = 0
 ( 4)  [low]3.race#c.lwt = 0
 ( 5)  [low]2.race#c.smoke = 0
 ( 6)  [low]3.race#c.smoke = 0
 ( 7)  [low]2.race#c.ptl = 0
 ( 8)  [low]3.race#c.ptl = 0
 ( 9)  [low]2.race#c.ht = 0
 (10)  [low]3.race#c.ht = 0
 (11)  [low]2.race#c.ui = 0
 (12)  [low]3.race#c.ui = 0
           chi2( 12) =    8.24
         Prob > chi2 =    0.7663
```

We conclude that, here, the Wald version of the Chow test is similar to the likelihood-ratio version of the Chow test.

◁

Stored results

`lrtest` stores the following in `r()`:

Scalars

`r(p)`	p-value for likelihood-ratio test
`r(df)`	degrees of freedom
`r(chi2)`	LR test statistic

Programmers wishing their estimation commands to be compatible with `lrtest` should note that `lrtest` requires that the following results be returned:

`e(cmd)`	name of estimation command
`e(ll)`	log likelihood
`e(V)`	variance–covariance matrix of the estimators
`e(N)`	number of observations

`lrtest` also verifies that `e(N)`, `e(ll_0)`, and `e(depvar)` are consistent between two noncomposite models.

Methods and formulas

Let L_0 and L_1 be the log-likelihood values associated with the full and constrained models, respectively. The test statistic of the likelihood-ratio test is $\text{LR} = -2(L_1 - L_0)$. If the constrained model is true, LR is approximately χ^2 distributed with $d_0 - d_1$ degrees of freedom, where d_0 and d_1 are the model degrees of freedom associated with the full and constrained models, respectively (Greene 2018, 554–555).

`lrtest` determines the degrees of freedom of a model as the rank of `e(V)`, computed as the number of nonzero diagonal elements of `invsym(e(V))`.

References

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Also see

[R] **test** — Test linear hypotheses after estimation

[R] **testnl** — Test nonlinear hypotheses after estimation

[R] **nestreg** — Nested model statistics