### loneway - Large one-way ANOVA, random effects, and reliability

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# Description

loneway fits one-way analysis-of-variance (ANOVA) models. loneway relaxes the restriction imposed by oneway that factors must have fewer than 376 levels. The command additionally reports the intraclass correlation, its standard error, and confidence interval; the estimated reliability of the group-averaged mean; the standard deviation of the group effect; and the standard deviation of the within-group effect.

# **Quick start**

One-way ANOVA model of y for factor a

loneway y a

Report an exact 95% confidence interval for the intraclass correlation

loneway y a, exact

Same as above, but report a 90% confidence interval

loneway y a, exact level(90)

# Menu

 ${\it Statistics} > {\it Linear models and related} > {\it ANOVA/MANOVA} > {\it Large one-way ANOVA}$ 

## Syntax

loneway response_var group_var [	if		in		weight		[, options]
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options	Description
Main	
mean	expected value of $F$ distribution; default is 1
median	median of $F$ distribution; default is 1
<u>ex</u> act	exact confidence intervals (groups must be equal with no weights)
<u>l</u> evel(#)	set confidence level; default is level(95)

by and collect are allowed; see [U] 11.1.10 Prefix commands. aweights are allowed; see [U] 11.1.6 weight.

# Options

∫Main

- mean specifies that the expected value of the  $F_{k-1,N-k}$  distribution be used as the reference point  $F_m$  in the estimation of  $\rho$  instead of the default value of 1.
- median specifies that the median of the  $F_{k-1,N-k}$  distribution be used as the reference point  $F_m$  in the estimation of  $\rho$  instead of the default value of 1.
- exact requests that exact confidence intervals be computed, as opposed to the default asymptotic confidence intervals. This option is allowed only if the groups are equal in size and weights are not used.
- level(#) specifies the confidence level, as a percentage, for confidence intervals of the coefficients. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

## **Remarks and examples**

loneway and oneway both fit ANOVA models, but loneway presents different ancillary statistics from oneway:

oneway	loneway
х	x
х	х
	х
х	
х	
	х
	х
	х
	х
	х
	x x x

Remarks are presented under the following headings:

The one-way ANOVA model  $R^2$ The random-effects ANOVA model Intraclass correlation Estimated reliability of the group-averaged score

### The one-way ANOVA model

#### Example 1

loneway's output looks like that of oneway, except that loneway presents more information at the end. Using our automobile dataset, we have created a (numeric) variable called manufacturer\_grp identifying the manufacturer of each car, and within each manufacturer we have retained a maximum of four models, selecting those with the lowest mpg. We can compute the intraclass correlation of mpg for all manufacturers with at least four models as follows:

```
. use https://www.stata-press.com/data/r19/auto7 (1978 automobile data)
```

. loneway mpg manufacturer\_grp if nummake == 4

	10	_0 I					
	One-way	analysis of	variance	for m	pg: Mi	leage (mj	pg)
				Num	ber of	obs =	36
						ared =	0.5228
_							
Sour	ce	SS	df	MS		F	Prob > F
Between	manufactur~p	621.88889	8	77.73	6111	3.70	0.0049
Within m	anufactur~p	567.75	27	21.02	7778		
Total		1189.6389	35	33.98	9683		
	Intraclass	Asy.					
	correlation	S.E.	[95%	conf.	interv	al]	
	0.40270	0.18770	0.03	481	0.77	060	
	Estimated SD	of manufact	ur~p effe	ct	3.765	247	
	Estimated SD	within manu	factur~p		4.585	605	
	Est. reliabi (evalua	lity of a main ted at n=4.00		p mean	0.72	950	

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In addition to the standard one-way ANOVA output, loneway produces the  $R^2$ , the estimated standard deviation of the group effect, the estimated standard deviation within group, the intragroup correlation, the estimated reliability of the group-averaged mean, and, for unweighted data, the asymptotic standard error and confidence interval for the intragroup correlation.

 $\mathbb{R}^2$ 

The  $R^2$  is, of course, simply the underlying  $R^2$  for a regression of *response\_var* on the levels of *group\_var*, or mpg on the various manufacturers here.

### The random-effects ANOVA model

loneway assumes that we observe a variable,  $y_{ij}$ , measured for  $n_i$  elements within k groups or classes such that

 $y_{ij} = \mu + \alpha_i + \epsilon_{ij}, \quad i = 1, 2, \dots, k, \quad j = 1, 2, \dots, n_i$ 

and  $\alpha_i$  and  $\epsilon_{ij}$  are independent zero-mean random variables with variance  $\sigma_{\alpha}^2$  and  $\sigma_{\epsilon}^2$ , respectively. This is the random-effects ANOVA model, also known as the components-of-variance model, in which it is typically assumed that the  $y_{ij}$  are normally distributed.

The interpretation with respect to our example is that the observed value of our response variable, mpg, is created in two steps. First, the *i*th manufacturer is chosen, and a value,  $\alpha_i$ , is determined—the typical mpg for that manufacturer less the overall mpg  $\mu$ . Then a deviation,  $\epsilon_{ij}$ , is chosen for the *j*th model within this manufacturer. This is how much that particular automobile differs from the typical mpg value for models from this manufacturer.

For our sample of 36 car models, the estimated standard deviations are  $\sigma_{\alpha} = 3.8$  and  $\sigma_{\epsilon} = 4.6$ . Thus, a little more than half of the variation in mpg between cars is attributable to the car model, with the rest attributable to differences between manufacturers. These standard deviations differ from those that would be produced by a (standard) fixed-effects regression in that the regression would require the sum within each manufacturer of the  $\epsilon_{ij}$ ,  $\epsilon_{i}$ . for the *i*th manufacturer, to be zero, whereas these estimates merely impose the constraint that the sum is expected to be zero.

#### Intraclass correlation

There are various estimators of the intraclass correlation, such as the pairwise estimator, which is defined as the Pearson product-moment correlation computed over all possible pairs of observations that can be constructed within groups. For a discussion of various estimators, see Donner (1986). Loneway computes what is termed the analysis of variance, or ANOVA, estimator. This intraclass correlation is the theoretical upper bound on the variation in *response\_var* that is explainable by *group\_var*, of which  $R^2$  is an overestimate because of the serendipity of fitting. This correlation is comparable to an  $R^2$ —you do not have to square it.

In our example, the intra-manu correlation, the correlation of mpg within manufacturer, is 0.40. Because aweights were not used and the default correlation was computed (that is, the mean and median options were not specified), loneway also provided the asymptotic confidence interval and standard error of the intraclass correlation estimate.

#### Estimated reliability of the group-averaged score

The estimated reliability of the group-averaged score or mean has an interpretation similar to that of the intragroup correlation; it is a comparable number if we average *response\_var* by *group\_var*, or mpg by manu in our example. It is the theoretical upper bound of a regression of manufacturer-averaged mpg on characteristics of manufacturers. Why would we want to collapse our 36-observation dataset into a 9-observation dataset of manufacturer averages? Because the 36 observations might be a mirage. When General Motors builds cars, do they sometimes put a Pontiac label and sometimes a Chevrolet label on them, so that it appears in our data as if we have two cars when we really have only one, replicated? If that were the case, and if it were the case for many other manufacturers, then we would be forced to admit that we do not have data on 36 cars; we instead have data on nine manufacturer-averaged characteristics.

## **Stored results**

loneway stores the following in r():

Scalars

r(N)	number of observations	r(rho_t)	estimated reliability
r(rho)	intraclass correlation	r(se)	asymp. SE of intraclass correlation
r(1b)	lower bound of 95% CI for rho	r(sd_w)	estimated SD within group
r(ub)	upper bound of 95% CI for rho	r(sd_b)	estimated SD of group effect

## Methods and formulas

The mean squares in the loneway's ANOVA table are computed as

$$\mathrm{MS}_{\alpha} = \sum_i w_{i\cdot} (\overline{y}_{i\cdot} - \overline{y}_{..})^2 / (k-1)$$

and

$$\mathrm{MS}_{\epsilon} = \sum_i \sum_j w_{ij} (y_{ij} - \overline{y}_{i.})^2 / (N-k)$$

in which

$$w_{i\cdot} = \sum_j w_{ij} \quad w_{\cdot \cdot} = \sum_i w_{i\cdot} \quad \overline{y}_{i\cdot} = \sum_j w_{ij} y_{ij} / w_{i\cdot} \quad \text{and} \quad \overline{y}_{\cdot \cdot} = \sum_i w_{i\cdot} \overline{y}_{i\cdot} / w_{\cdot}$$

The corresponding expected values of these mean squares are

$$E(\mathrm{MS}_\alpha) = \sigma_\epsilon^2 + g \sigma_\alpha^2 \qquad \text{and} \qquad E(\mathrm{MS}_\epsilon) = \sigma_\epsilon^2$$

in which

$$g = \frac{w_{\cdot\cdot} - \sum_i w_{i\cdot}^2 / w_{\cdot\cdot}}{k-1}$$

In the unweighted case, we get

$$g = \frac{N - \sum_i n_i^2 / N}{k - 1}$$

As expected, g = m for the case of no weights and equal group sizes in the data, that is,  $n_i = m$  for all *i*. Replacing the expected values with the observed values and solving yields the ANOVA estimates of  $\sigma_{\alpha}^2$  and  $\sigma_{\epsilon}^2$ . Substituting these into the definition of the intraclass correlation

$$\rho = \frac{\sigma_{\alpha}^2}{\sigma_{\alpha}^2 + \sigma_{\epsilon}^2}$$

yields the ANOVA estimator of the intraclass correlation:

$$\rho_A = \frac{F_{\rm obs}-1}{F_{\rm obs}-1+g}$$

 $F_{\rm obs}$  is the observed value of the F statistic from the ANOVA table. For no weights and equal  $n_i$ ,  $\rho_A =$  roh, which is the intragroup correlation defined by Kish (1965). Two slightly different estimators are available through the mean and median options (Gleason 1997). If either of these options is specified, the estimate of  $\rho$  becomes

$$\rho = \frac{F_{\rm obs} - F_m}{F_{\rm obs} + (g-1)F_m}$$

For the mean option,  $F_m = E(F_{k-1,N-K}) = (N-k)/(N-k-2)$ , that is, the expected value of the ANOVA table's *F* statistic. For the median option,  $F_m$  is simply the median of the *F* statistic. Setting  $F_m$  to 1 gives  $\rho_A$ , so for large samples, these different point estimators are essentially the same. Also, because the intraclass correlation of the random-effects model is by definition nonnegative, for any of the three possible point estimators,  $\rho$  is truncated to zero if  $F_{obs}$  is less than  $F_m$ .

For no weighting, interval estimators for  $\rho_A$  are computed. If the groups are equal sized (all  $n_i$  equal) and the exact option is specified, the following exact (assuming that the  $y_{ij}$  are normally distributed)  $100(1 - \alpha)\%$  confidence interval is computed:

$$\left\{\frac{F_{\mathrm{obs}}-F_mF_u}{F_{\mathrm{obs}}+(g-1)F_mF_u},\frac{F_{\mathrm{obs}}-F_mF_l}{F_{\mathrm{obs}}+(g-1)F_mF_l}\right\}$$

with  $F_m = 1$ ,  $F_l = F_{\alpha/2,k-1,N-k}$ , and  $F_u = F_{1-\alpha/2,k-1,N-k}$ ,  $F_{\cdot,k-1,N-k}$  being the cumulative distribution function for the *F* distribution with k-1 and N-k degrees of freedom. If mean or median is specified,  $F_m$  is defined as above. If the groups are equal sized and exact is not specified, the following asymptotic  $100(1-\alpha)\%$  confidence interval for  $\rho_A$  is computed,

$$\left[\rho_A-z_{\alpha/2}\sqrt{V(\rho_A)},\rho_A+z_{\alpha/2}\sqrt{V(\rho_A)}\right]$$

where  $z_{\alpha/2}$  is the  $100(1-\alpha/2)$  percentile of the standard normal distribution and  $\sqrt{V(\rho_A)}$  is the asymptotic standard error of  $\rho$  defined below. This confidence interval is also available for unequal groups. It is not applicable and, therefore, not computed for the estimates of  $\rho$  provided by the mean and median options. Again, because the intraclass coefficient is nonnegative, if the lower bound is negative for either confidence interval, it is truncated to zero. As might be expected, the coverage probability of a truncated interval is higher than its nominal value.

The asymptotic standard error of  $\rho_A$ , assuming that the  $y_{ij}$  are normally distributed, is also computed when appropriate, namely, for unweighted data and when  $\rho_A$  is computed (neither the mean option nor the median option is specified):

$$V(\rho_A) = \frac{2(1-\rho)^2}{g^2} \left( A + B + C \right)$$

with

$$\begin{split} A &= \frac{\{1+\rho(g-1)\}^2}{N-k} \\ B &= \frac{(1-\rho)\{1+\rho(2g-1)\}}{k-1} \\ C &= \frac{\rho^2\{\sum n_i^2 - 2N^{-1}\sum n_i^3 + N^{-2}(\sum n_i^2)^2\}}{(k-1)^2} \end{split}$$

and  $\rho_A$  is substituted for  $\rho$  (Donner 1986).

The estimated reliability of the group-averaged score, known as the Spearman–Brown prediction formula in the psychometric literature (Winer, Brown, and Michels 1991, 1014), is

$$\rho_t = \frac{t\rho}{1+(t-1)\rho}$$

for group size t. loneway computes  $\rho_t$  for t = g.

The estimated standard deviation of the group effect is  $\sigma_{\alpha} = \sqrt{(MS_{\alpha} - MS_{\epsilon})/g}$ . This deviation comes from the assumption that an observation is derived by adding a group effect to a within-group effect.

The estimated standard deviation within group is the square root of the mean square due to error, or  $\sqrt{MS_{\epsilon}}$ .

## Acknowledgment

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### References

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### Also see

- [R] anova Analysis of variance and covariance
- [R] icc Intraclass correlation coefficients
- [R] oneway One-way analysis of variance

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