Title

ivqregress — Instrumental-variables quantile regression

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Description

`ivqregress` fits a linear instrumental-variables quantile regression (IVQR) model that accounts for endogenous covariates using two estimators: the inverse quantile regression (IQR) estimator proposed in Chernozhukov and Hansen (2006) and the smoothed estimating equations (SEE) estimator outlined in Kaplan and Sun (2017).

Quick start

Use the IQR estimator to fit the median IVQR model of `y1` on exogenous `x1` and endogenous `y2` with instruments `z1` and `z2`

```
ivqregress iqr y1 x1 (y2 = z1 z2)
```

Same as above, but estimate the 0.75 quantile

```
ivqregress iqr y1 x1 (y2 = z1 z2), quantile(0.75)
```

Same as above, but estimate the 0.1, 0.2, ..., 0.9 quantiles

```
ivqregress iqr y1 x1 (y2 = z1 z2), quantile(10(10)90)
```

Use the SEE estimator to estimate the 0.6 quantile regression of `y1` on exogenous `x1` and endogenous `y2` and `y3` with instruments `z1` and `z2`

```
ivqregress smooth y1 x1 (y2 y3 = z1 z2), quantile(0.6)
```

Same as above, but estimate the 0.1, 0.2, ..., 0.9 quantiles

```
ivqregress smooth y1 x1 (y2 y3 = z1 z2), quantile(10(10)90)
```

IQR options to control optimization

Use 50 grid points in the IQR estimator to fit the 0.5 and 0.75 IVQR model

```
ivqregress iqr y1 x1 (y2 = z1 z2), ngrid(50) quantile(50 75)
```

Same as above, but construct grid points between 1 and 5 for all the quantiles

```
ivqregress iqr y1 x1 (y2 = z1 z2), ngrid(50) quantile(50 75) bound(1 5)
```

Same as above, but construct grid points using different bounds for different quantiles

```
ivqregress iqr y1 x1 (y2 = z1 z2), ngrid(50) quantile(50 75)  ///
     bound(1 5, at(50)) bound(2 6, at(75))
```

SEE options to control optimization

Use 2 as the initial bandwidth in the SEE estimator to fit the 0.5 and 0.75 IVQR model

```
ivqregress smooth y x1 (d1 d2 = z1 z2), quantile(50 75)  ///
      initbwidth(2)
```

Same as above, but use different initial bandwidths for different quantiles

```
ivqregress smooth y x1 (d1 d2 = z1 z2), quantile(50 75) ///
    initbwidth(2, at(50)) initbwidth(1, at(75))
```

Menu

Statistics > Endogenous covariates > Quantile regression with endogenous covariates

Syntax

Inverse quantile regression (IQR) estimator

`ivqregress iqr` *depvar* [*varlist*$_1$] `(`*varname* `=` *varlist*$_{\mathrm{iv}}$`)` [*if*] [*in*] [`,` *options* *IQR_options*]

Smoothed estimating equations (SEE) estimator

`ivqregress smooth` *depvar* [*varlist*$_1$] `(`*varlist*$_2$ `=` *varlist*$_{\mathrm{iv}}$`)` [*if*] [*in*] [`,` *options* *SEE_options*]

varlist$_1$ is the list of exogenous variables.

varname is an endogenous variable.

varlist$_2$ is the list of endogenous variables.

varlist$_{\mathrm{iv}}$ is the list of exogenous variables used with *varlist*$_1$ as instruments for *varlist*$_2$ and *varname*.

options	Description
Model	
`quantile(`*numlist*`)`	estimate quantiles specified in *numlist*; default is `quantile(0.5)`
SE/Robust	
`vce(`[*vcetype*] [`,` *vceopts*]`)`	technique used to estimate standard errors; *vcetype* may be `robust` (the default) or `bootstrap`
Reporting	
`level(`*#*`)`	set confidence level; default is `level(95)`
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
[`no`] `log`	suppress or display the iteration log
`verbose`	display a verbose iteration log
`coeflegend`	display legend instead of statistics

IQR_options	Description
Options	
bound($\#_{min}$ $\#_{max}$ [, at($\#_q$)])	specify the lower and upper bounds for the grid in the $\#_q$th quantile estimation; may be repeated
ngrid($\#_g$)	use $\#_g$ grid points; default is ngrid(30)

SEE_options	Description
Options	
initbwidth($\#_b$ [, at($\#_q$)])	specify initial bandwidth $\#_b$ to smooth the estimating equations for the $\#_q$th quantile estimation; default is the theoretical optimal bandwidth; may be repeated
iterate(#)	perform maximum of # iterations when solving the estimating equation; default is iterate(100)
nosearchbwidth	do not search for feasible bandwidth if the initial bandwidth is not feasible; default is to search for feasible bandwidth
tolerance(#)	specify the tolerance for the coefficient vector; default is tolerance(1e-9)
ztolerance(#)	specify the tolerance to determine whether the proposed solution for a zero-finding problem is sufficiently close to 0; default is ztolerance(1e-9)

vceopts	Description
kernel(*kernel*)	use a nonparametric kernel density estimator; default is epanechnikov
bwidth(# | *bwrule*)	specify the bandwidth to be used by the kernel density estimator; default is silverman, which is Silverman's rule of thumb

kernel	Description
epanechnikov	Epanechnikov kernel function; the default
epan2	alternative Epanechnikov kernel function
biweight	biweight kernel function
cosine	cosine trace kernel function
gaussian	Gaussian kernel function
parzen	Parzen kernel function
rectangle	rectangle kernel function
triangle	triangle kernel function

bwrule	Description
silverman	Silverman's rule of thumb; the default
hsheather	Hall–Sheather's bandwidth
bofinger	Bofinger's bandwidth

varlist$_1$, *varname*, *varlist*$_2$, and *varlist*$_{\mathrm{iv}}$ may contain factor variables; see **[U] 11.4.3 Factor variables**.
`bootstrap`, `by`, `collect`, `rolling`, and `statsby` are allowed; see **[U] 11.1.10 Prefix commands**.
`coeflegend` does not appear in the dialog box.
See **[U] 20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

`quantile(`*numlist*`)` specifies the quantiles to be estimated and should contain numbers between 0 and 1, exclusive. Numbers larger than 1 are interpreted as percentages. The default is `quantile(0.5)`, which corresponds to the median.

Options

The following options apply only to the IQR estimator.

`bound(`$\#_{\mathrm{min}}$ $\#_{\mathrm{max}}$ [`, at(`$\#_q$`)`]`)` specifies the lower bound ($\#_{\mathrm{min}}$) and the upper bound ($\#_{\mathrm{max}}$) for the grid in the $\#_q$th quantile estimation. By default, the bounds are determined by the two-stage quantile regression, extending the two-stage median regression in Amemiya (1982). This option is repeatable as long as different quantiles $\#_q$ are given in each specification.

The specified bound is required to be wider than the $\#_{\mathrm{level}}$ confidence interval (CI) that is robust to the weak instruments, which is also known as dual CI. The value of $\#_{\mathrm{level}}$ can be specified in the `level()` option; the default is 95% CI.

The grid points are $\#_g$ equally spaced points between $\#_{\mathrm{min}}$ and $\#_{\mathrm{max}}$, where $\#_g$ is specified by the `ngrid()` option.

`ngrid(`$\#_g$`)` specifies the number of grid points in the IQR estimator. The default is `ngrid(30)`; that is, 30 grid points are used.

The following options apply only to the SEE estimator.

`initbwidth(`$\#_b$ [`, at(`$\#_q$`)`]`)` specifies initial bandwidth $\#_b$ to smooth the estimating equations for the $\#_q$th quantile estimation. The default is the theoretical optimal bandwidth that minimizes the mean squared errors of the estimating equations; see Kaplan and Sun (2017). This option is repeatable as long as different quantiles $\#_q$ are given in each specification.

`iterate(`*#*`)` specifies the maximum number of iterations to perform when solving the estimating equation; the default is `iterate(100)`.

`nosearchbwidth` specifies to not search for a feasible bandwidth if the initial bandwidth is not estimable; the default is to search for a feasible bandwidth.

`tolerance(`*#*`)` specifies the tolerance used to determine whether successive estimates of the solution have converged. The default is `tolerance(1e-9)`.

`ztolerance(`*#*`)` specifies the tolerance used to determine whether the proposed solution to a zero-finding problem is sufficiently close to 0; the default is `ztolerance(1e-9)`.

SE/Robust

`vce(`[*vcetype*] [`,` *vceopts*]`)` specifies the type of VCE to compute and the density estimation method to use in computing the VCE.

vcetype specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`) and that use bootstrap methods (`bootstrap`); see [R] ***vce_option***.

vceopts available with *vcetype* `robust` are the following:

`kernel(`*kernel*`)` specifies the kernel method to be used by the nonparametric density estimator. The available kernel functions are `epanechnikov`, `epan2`, `biweight`, `cosine`, `gaussian`, `parzen`, `rectangle`, and `triangle`. The default is `epanechnikov`. See [R] **kdensity** for the kernel function forms.

`bwidth(`*#* | *bwrule*`)` specifies the bandwidth to be used by the nonparametric density estimator. If specified as a number, it is used as the bandwidth for the nonparametric density estimator. Otherwise, *bwrule* specifies the method used to compute the bandwidth. Available methods are `silverman` for Silverman's rule of thumb, `hsheather` for the Hall–Sheather bandwidth, and `bofinger` for the Bofinger bandwidth.

See [R] **kdensity** for Silverman's rule of thumb. See Koenker (2005, sec. 4.10) for a description of the Hall–Sheather and Bofinger bandwidth formulas.

Reporting

`level(`*#*`)`; see [R] **Estimation options**.

display_options: `noci`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(`*#*`)`, `fvwrapon(`*style*`)`, `cformat(`*%fmt*`)`, `pformat(`*%fmt*`)`, `sformat(`*%fmt*`)`, and `nolstretch`; see [R] **Estimation options**.

Optimization

`log` and `nolog` specify whether to display the log showing the progress of the estimation. By default, for the IQR estimator, one dot is shown for each grid point; for the SEE estimator, one line is shown for each bandwidth. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] ***set iter***.

`verbose` displays a verbose log showing the iterations of each computation step. For the IQR estimator, each line is shown for each grid point. For the SEE estimator, iteration logs are shown when solving the estimating equations.

The following option is available with `ivqregress` but is not shown in the dialog box:

`coeflegend`; see [R] **Estimation options**.

Remarks and examples

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Remarks are presented under the following headings:

Overview
When quantile regression matters
Examples

Overview

`ivqregress` fits a linear IVQR model when some covariates are endogenous. The general IVQR model was first proposed by Chernozhukov and Hansen (2005). `ivqregress` is based on the linear IVQR model described in Chernozhukov and Hansen (2006, 2008). For an introduction to the IVQR model, see Chernozhukov, Hansen, and Wüthrich (2018). `ivqregress` implements two estimators: the IQR estimator proposed in Chernozhukov and Hansen (2006) and the SEE estimator outlined in Kaplan and Sun (2017).

In empirical applications, we are usually interested in the effects of some covariate on the outcome variable. The traditional linear regression model is an excellent way to model how the covariate affects the outcome's conditional mean. However, sometimes we would like to study features of the outcome distribution other than the mean to have a complete picture of the effects of covariates. For example, a policymaker may want to learn how participation in a 401(k) would affect the lower-level, median, and upper-level conditional quantiles of net wealth.

Quantile regression in Koenker and Bassett (1978) can help us grasp a better picture than regular linear regression by estimating the effects on different quantiles of the outcome's conditional distribution. For a general discussion, see [R] **qreg**. For an illustration of when quantile regression matters, see *When quantile regression matters* below.

In practice, some covariates of interest are often endogenous for reasons such as self-selection, omission of some relevant variable, and measurement error. For example, participation in a voluntary savings plan for retirement, such as participation in a 401(k) program, may be endogenous because the people who do and do not participate may have different saving preferences, which will affect net wealth growth.

Endogenous covariates make quantile regression estimates inconsistent, as is the case for the linear regression model. Analogous to the instrumental-variable least-squares estimator, there are IVQR model estimators to consistently estimate the effects at different quantiles. For a discussion of instrumental-variables estimation, see [R] **ivregress**.

`ivqregress` fits a quantile regression model that accounts for endogenous covariates using two estimators: the IQR estimator proposed in Chernozhukov and Hansen (2006) and the SEE estimator outlined in Kaplan and Sun (2017). Intuitively, `ivqregress` can be thought of as the `ivregress` version of `qreg`.

Here we outline the Stata commands to fit, visualize, infer, and diagnose the IVQR model. In particular, these Stata commands can be grouped into the following categories.

Estimation: `ivqregress iqr` fits the IVQR model by the IQR estimator proposed in Chernozhukov and Hansen (2006, 2008).

`ivqregress smooth` fits the IVQR model by the SEE estimator proposed in Kaplan and Sun (2017).

Visualization: `estat coefplot` allows us to visualize how one covariate's effects vary at different quantiles of the outcome.

Inference: `estat endogeffects` tests if

1. the endogenous variable does not affect the outcome variable,
2. the effects of the endogenous variable do not vary across estimated quantiles,
3. the effects of the endogenous variable are greater than zero across estimated quantiles, and
4. the variable is exogenous instead of endogenous.

`estat dualci` provides CIs that are robust to weak instruments for the effects of the endogenous variable. It is allowed only after `ivqregress iqr`.

Diagnosis: `estat waldplot` helps diagnose the convergence of the IQR estimator (`ivqregress iqr`). In particular, `estat waldplot` allows us to visualize the optimization process during the computation in `ivqregress iqr` and shows if the searching domain contains the true value of the parameter with a predefined probability level.

In addition, some other classical postestimation tools are also available; see [R] **ivqregress postestimation**.

When quantile regression matters

Here is an example illustrating the advantages of quantile regressions. Suppose we have a simple model $E(y|x) = \beta_0 + x\beta_1$, where y is the outcome variable and x is a covariate. For simplicity, we assume x can only take values in $\{0, 1, 2, 3, 4, 5, 6\}$. By definition, β_1 fully characterizes the effects of increasing one unit of x on the conditional mean of outcome y; that is, $\beta_1 = E(y|x = a+1) - E(y|x = a)$. Now we consider two scenarios of the data-generating process.

1. The probability density function of the outcome conditional on $x = a+1$, $f(y|x = a+1)$, is only location shifted relative to $f(y|x = a)$. In this case, β_1 summarizes the effect of x not only on the conditional mean but also on each conditional quantile of y. This case is illustrated in the left panel of figure 1.
2. The probability density function of the outcome conditional on $x = a+1$, $f(y|x = a+1)$, is both location shifted and rescaled relative to $f(y|x = a)$. In this case, β_1 summarizes the effect of x only on the conditional mean but not on conditional quantiles of y. This case is illustrated in the right panel of figure 1.

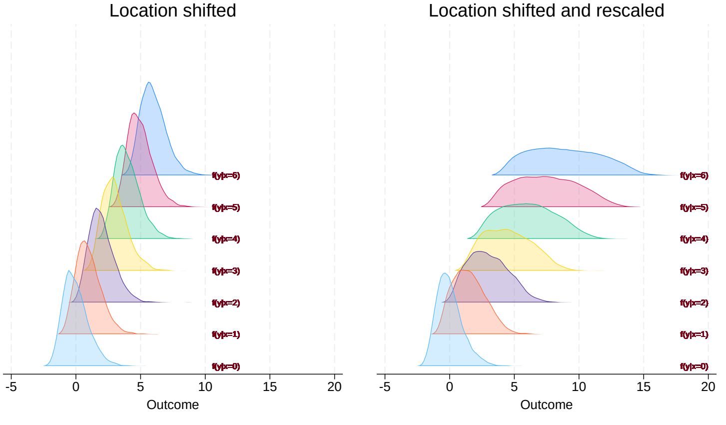


Figure 1.

In the left panel, we see that each conditional density is parallel relative to each other, and only the location has been shifted. In this case, β_1 captures the shift in both conditional mean and any conditional quantiles of the outcome. As a result, running a linear regression provides as much information on β_1 as quantile regression.

In contrast, in the right panel, conditional density for each level of x has different locations and different shapes. Thus, β_1 can only summarize the shifts in conditional mean, which are generally different from the shifts in conditional quantiles. Quantile regression becomes necessary to learn about the effects of x on the conditional quantiles of the outcome.

Examples

▷ Example 1: IVQR with the IQR estimator

Suppose that we want to estimate the effect of 401(k) participation (`p401k`) on different conditional quantiles of net financial assets (`assets`). We use data reported by Chernozhukov and Hansen (2004). These data are from a sample of households in the 1990 Survey of Income and Program Participation (SIPP). For the head of household, we have data on income (`income`), age (`age`), number of people in the family (`familysize`), years of education (`educ`), marital status (`married`), whether participated in an IRA (`ira`), whether received a pension benefit (`pension`), and whether owned a home (`ownhome`).

We suspect 401(k) participation is endogenous because it may depend on unobserved factors such as saving preference that also impact financial assets. Using 401(k) eligibility (`e401k`) as an instrument for 401(k) participation, we use `ivqregress` to estimate how `p401k` affects the entire range of `assets`' conditional distribution. One concern about using `e401k` as an instrument is that choosing to work for a company that offers a 401(k) program is not randomly assigned. Poterba, Venti, and Wise (1995) suggest that after conditioning on income, we can take working for a company that offers a 401(k) plan as exogenous.

The IVQR model we want to fit is

$$\texttt{assets}_i = \texttt{p401k}_i\alpha(U) + \textbf{covariates}_i'\boldsymbol{\beta}(U)$$

where the distribution of U conditional on the instrument `e401k` and the covariates is assumed to be uniform between 0 and 1. The covariates `income`, `age`, `familysize`, and `educ` are included in the model as continuous variables. The covariates `i.married`, `i.ira`, `i.pension`, and `i.ownhome` are included as categorical (factor) variables. As discussed above, `e401k` is the instrument for `p401k`. The coefficients $\alpha(U)$ and $\boldsymbol{\beta}(U)$ are random because they depend on the unobserved random variable U. In practice, U can be considered a ranking variable for the asset. When U is set to a fixed level τ, we fit an IVQR model at a specific quantile index τ. For example, when $\tau = 0.5$, we estimate how 401(k) participation affects the median of net financial assets conditional on other covariates.

The objective of the analysis is to estimate the quantile treatment effects of 401k participation on net financial assets. By definition, the τth conditional quantile of the asset when everyone participates in a 401(k) plan is

$$\texttt{assets}_{401(\text{k})} = \alpha(\tau) + \textbf{covariates}_i'\boldsymbol{\beta}(\tau)$$

In contrast, the τth conditional quantile of the asset when everyone does not participate in a 401(k) plan is

$$\texttt{assets}_{\text{no }401(\text{k})} = \textbf{covariates}_i'\boldsymbol{\beta}(\tau)$$

Thus, the coefficient $\alpha(\tau)$ can fully summarize the quantile treatment effect of `p401k` on `assets`. That is

$$\alpha(\tau) = \texttt{assets}_{401(\text{k})} - \texttt{assets}_{\text{no }401(\text{k})}$$

In this example, we use the IQR estimator (`ivqregress iqr`) to estimate the effect of 401(k) participation on the conditional median of the net financial assets. The dependent variable is `assets`. The endogenous variable `i.p401k` and the instrument `i.e401k` are specified in parentheses; the other covariates follow as a regular variable list. `ivqregress` fits the IV median regression model by default. The estimation result is stored as `est_iqr` for later use.

```
. use https://www.stata-press.com/data/r18/assets2
(Excerpt from Chernozhukov and Hansen (2004))

. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ

Initial grid:
Quantile = 0.50: .........10.........20.........30 done

Adaptive grid:
Quantile = 0.50: .........10.........20.........30 done

IV median regression                              Number of obs =    9,913
Estimator: Inverse quantile regression            Wald chi2(9)  = 1289.75
                                                  Prob > chi2   =  0.0000

------------------------------------------------------------------------------
             |               Robust
      assets | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       p401k |
        Yes  |   5313.397   573.2818     9.27   0.000     4189.786    6437.009
      income |   .1577512   .0124889    12.63   0.000     .1332735    .1822289
         age |   99.96526   8.561923    11.68   0.000      83.1842    116.7463
  familysize |  -197.8251   54.36773    -3.64   0.000    -304.3838   -91.26627
             |
     married |
    Married  |  -1359.124   227.3366    -5.98   0.000    -1804.696   -913.5528
             |
         ira |
        Yes  |   22629.61   1022.706    22.13   0.000     20625.15    24634.08
             |
     pension |
Receives ..  |  -693.8347   210.6176    -3.29   0.001    -1106.638   -281.0317
             |
     ownhome |
        Yes  |  -30.29657   154.7265    -0.20   0.845     -333.555    272.9618
        educ |  -96.43983   32.09465    -3.00   0.003    -159.3442   -33.53547
       _cons |  -4998.673   570.1315    -8.77   0.000     -6116.11   -3881.236
------------------------------------------------------------------------------
Endogenous: 1.p401k
Exogenous:  income age familysize 1.married 1.ira 1.pension 1.ownhome educ
            1.e401k

. estimates store est_iqr
```

The coefficient for p401k is 5,313. It means participation in a 401(k) would increase the median net financial assets by $5,313, conditional on other covariates, relative to a scenario where no one participates. We store the estimation result as est_iqr for later use.

After ivqregress iqr, we can use estat dualci to obtain the dual CI, which is robust to weak instruments, for the coefficient on the endogenous variable.

```
. estat dualci
Dual confidence interval                                Number of obs = 9,913

                           Robust                                Dual
      assets   Coefficient  std. err.      z    P>|z|     [95% conf. interval]

       p401k
         Yes     5313.397   573.2818     9.27   0.000     3683.916    7304.986
```

The dual CI is usually wider than the regular CI, but it provides a more robust inference if the instruments are weak. In this example, we see that the dual 95% CI is [3684, 7305], which is wider than the regular 95% CI [4190, 6437].

Example 2: IVQR with the smooth estimator

In this example, we use `ivqregress` to fit the IVQR model as in example 1 but using the SEE estimator (`ivqregress smooth`). The model specification is the same as in example 1. The estimation result is stored as `est_smooth` for later use.

```
. ivqregress smooth assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ
Fitting smoothed IV quantile regression:
Quantile = .5:
Step 1: Bandwidth = 1302.9736    GMM criterion Q(b) = 2.617e-08
Step 2: Bandwidth = 6079.6881    GMM criterion Q(b) = 2.391e-12
Step 3: Bandwidth = 1438.3068    GMM criterion Q(b) = 8.068e-13
IV median regression                                 Number of obs =    9,913
Estimator: Smoothed estimating equations             Wald chi2(9)  =  1243.05
                                                     Prob > chi2   =   0.0000

                           Robust
      assets   Coefficient  std. err.      z    P>|z|     [95% conf. interval]

       p401k
         Yes     5364.468   573.3728     9.36   0.000     4240.678    6488.258
      income     .1679934    .013419    12.52   0.000     .1416925    .1942942
         age     113.6318   9.352867    12.15   0.000     95.30052    131.9631
  familysize    -228.7766   57.61072    -3.97   0.000    -341.6916   -115.8617

     married
    Married      -1362.56   238.5988    -5.71   0.000    -1830.205   -894.9153

         ira
         Yes     22402.04   1043.504    21.47   0.000     20356.81    24447.27

     pension
Receives ..      -713.996    220.476    -3.24   0.001    -1146.121   -281.8709

     ownhome
         Yes    -12.71396   161.3703    -0.08   0.937     -328.994    303.5661
        educ    -102.2889   34.18527    -2.99   0.003    -169.2908   -35.28701
       _cons    -5672.645   619.7049    -9.15   0.000    -6887.244   -4458.045

Endogenous: 1.p401k
Exogenous:  income age familysize 1.married 1.ira 1.pension 1.ownhome educ
            1.e401k
. estimates store est_smooth
```

The interpretation of the coefficient estimates is the same as in example 1. For example, the coefficient for p401k is 5,364. So participation in a 401(k) would increase the median of net financial assets by $5,364, conditional on other covariates, relative to a scenario where no one participates.

Now we can compare the coefficient on p401k between the SEE estimator and the IQR estimator.

```
. estimates table est_iqr est_smooth, keep(i.p401k) se
```

Variable	est_iqr	est_smooth
p401k		
Yes	5313.3974	5364.468
	573.28183	573.37279

Legend: b/se

We see that the point estimates from these two estimators are similar but not the same. It is normal to see different results from the IQR and SEE estimators because these two estimators approximate the original exact estimating equation differently. On one hand, the IQR estimator tries to find the solution by an exhaustive grid search. The estimation result critically depends on the range and finesse of grid points. On the other hand, the SEE estimator uses a kernel method to smooth the original estimating equation. Its result depends on how well the SEE approximates the original, mainly controlled by the bandwidth.

Both the IQR and SEE estimators have their advantages and weaknesses. The IQR estimator is numerically stable, and it allows computing the dual CI, which is robust to weak instruments (use estat dualci). However, the IQR becomes computationally intensive when there is more than one endogenous variable. Thus, ivqregress iqr allows only one endogenous variable. In contrast, the SEE estimator can handle multiple endogenous variables within a reasonable computation time. However, it does not allow estat dualci for inference that is robust to weak instruments. Suppose there is only one endogenous variable in the model. We recommend using both estimators, comparing the results, and using the IQR estimator as a benchmark because it can provide valid inference even if the instrument is weak. If there is more than one endogenous variable, only ivqregress smooth is available.

◁

▷ Example 3: IVQR at different quantiles

In the first two examples, we estimated the 401(k) participation (p401k) treatment effect on the conditional median of net financial assets (assets). From a policy designer's point of view, we may be more interested in estimating the treatment effect of p401k on other conditional quantiles of assets. For example, we can ask questions like 1) how 401(k) participation affects the lower quantile of assets and 2) whether 401(k) participation is unambiguously beneficial for the asset's lower and upper conditional quantiles. In addition, we might also want to know whether the 401(k) participation is endogenous in our model. In this example, we will show how to use ivqregress to fit the IVQR model at different quantiles and how to use the postestimation tools to answer the above questions.

First, we use the IQR estimator to fit the model at different quantiles. In particular, we specify the `quantile(10(10)90)` option to fit the IVQR model at the 10th, 20th, ..., 90th quantiles.

```
. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, quantile(10(10)90)
Initial grid:
Quantile = 0.10: .........10.........20.........30 done
Quantile = 0.20: .........10.........20.........30 done
Quantile = 0.30: .........10.........20.........30 done
Quantile = 0.40: .........10.........20.........30 done
Quantile = 0.50: .........10.........20.........30 done
Quantile = 0.60: .........10.........20.........30 done
Quantile = 0.70: .........10.........20.........30 done
Quantile = 0.80: .........10.........20.........30 done
Quantile = 0.90: .........10.........20.........30 done
Adaptive grid:
Quantile = 0.10: .........10.........20.........30 done
Quantile = 0.20: .........10.........20.........30 done
Quantile = 0.30: .........10.........20.........30 done
Quantile = 0.40: .........10.........20.........30 done
Quantile = 0.50: .........10.........20.........30 done
Quantile = 0.60: .........10.........20.........30 done
Quantile = 0.70: .........10.........20.........30 done
Quantile = 0.80: .........10.........20.........30 done
Quantile = 0.90: .........10.........20.........30 done
IV quantile regression                              Number of obs =    9,913
Estimator: Inverse quantile regression              Wald chi2(81) = 5121.46
                                                    Prob > chi2   =   0.0000
```

assets	Coefficient	Robust std. err.	z	P>|z|	[95% conf. interval]	
q10						
p401k						
Yes	3240.08	475.6184	6.81	0.000	2307.885	4172.275
income	.0303072	.0123138	2.46	0.014	.0061725	.0544419
age	131.5908	15.13725	8.69	0.000	101.9223	161.2592
familysize	-329.2838	123.4665	-2.67	0.008	-571.2737	-87.29385
married						
Married	-1504.648	380.0373	-3.96	0.000	-2249.508	-759.7886
ira						
Yes	7864.15	344.2198	22.85	0.000	7189.492	8538.809
pension						
Receives ..	63.88643	326.6017	0.20	0.845	-576.2412	704.0141
ownhome						
Yes	969.6861	300.4319	3.23	0.001	380.8503	1558.522
educ	-301.1635	52.02897	-5.79	0.000	-403.1384	-199.1885
_cons	-7455.806	1192.112	-6.25	0.000	-9792.302	-5119.311

(output omitted)

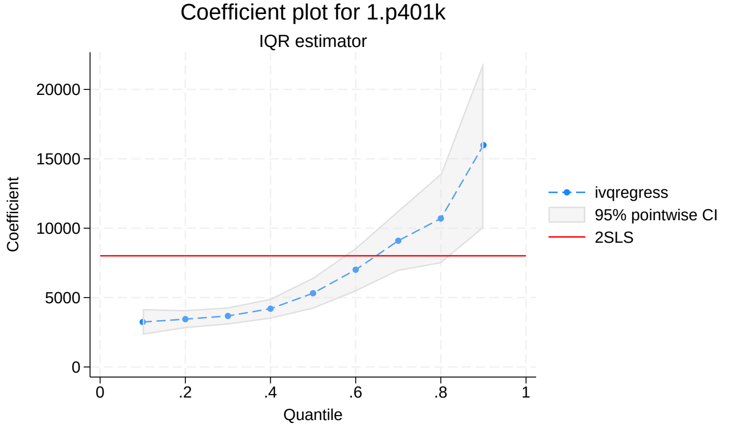
q90						
p401k						
Yes	15983.42	3046.028	5.25	0.000	10013.32	21953.53
income	.8247356	.0570029	14.47	0.000	.713012	.9364593
age	485.8734	48.99224	9.92	0.000	389.8504	581.8965
familysize	-646.4962	185.913	-3.48	0.001	-1010.879	-282.1134
married						
Married	-3265.007	753.4701	-4.33	0.000	-4741.782	-1788.233
ira						
Yes	68543.44	4952.261	13.84	0.000	58837.18	78249.69
pension						
Receives ..	-4656.177	869.4887	-5.36	0.000	-6360.343	-2952.01
ownhome						
Yes	400.1957	680.2776	0.59	0.556	-933.124	1733.515
educ	48.4205	106.2844	0.46	0.649	-159.8931	256.7341
_cons	-20594.85	2260.983	-9.11	0.000	-25026.3	-16163.41

```
Endogenous: 1.p401k
Exogenous:  income age familysize 1.married 1.ira 1.pension 1.ownhome educ
            1.e401k
```

The results show the estimates for the effect of 401(k) participation on each conditional quantile of the asset. The interpretation of the coefficient is similar to example 1, except we are looking at different conditional quantiles. For example, for quantile `q90`, the estimate for the coefficient on `p401k` is 15,983. Thus, 401(k) participation would increase the 90% conditional quantile of net financial assets by $15,983.

In addition to looking at the numerical estimates from the coefficient table, we can use `estat coefplot` to visualize the trend of `p401k`'s treatment effect from the lower to the upper quantile. By default, `estat coefplot` shows the first endogenous variable, which is `1.p401k` in our example. We specify the `name()` option for later reference of this graph and add a subtitle indicating which estimator we used.

```
. estat coefplot, name(cp_iqr) subtitle(IQR estimator)
```



The dots in the plot show the point estimates of `p401k`'s treatment effect on different conditional quantiles of `assets`, and the gray bound shows the 95% pointwise CI. We see that there is an upward trend of `p401k`'s treatment effect. At lower-level quantiles such as the 10th, 20th, ..., 40th quantiles, the treatment effect is relatively flat. However, we see the treatment effect increases significantly in the upper-level quantiles. The red line shows the two-stage least-squares estimates, which can be used as a benchmark.

`estat coefplot` is a good way to visualize the treatment effect's trend. If we want to test some hypotheses regarding the trend and the model statistically, we can use `estat endogeffects`. For example, we are interested in testing the following hypotheses:

No effect: 401(k) participation does not affect net financial assets for all the estimated quantiles.

Constant effect: 401(k) participation's treatment effect is constant for all the estimated quantiles.

Dominance: 401(k) participation is unambiguously positive for all the estimated quantiles; that is, the coefficient values are strictly positive.

Exogeneity: 401(k) participation is exogenous.

We will use `estat endogeffects` to show the Kolmogorov–Smirnov statistic and the 95% critical value for each hypothesis. We can reject the null hypothesis if the test statistic is greater than the critical value; otherwise, we cannot reject the null hypothesis. We specify the `rseed()` option to make the results reproducible because the critical values are generated from a bootstrap sample.

```
. estat endogeffects, rseed(12345671)
Tests for endogenous effects          Replications = 100
```

Null hypothesis	KS statistic	95% critical value
No effect	11.271	2.554
Constant effect	5.395	2.446
Dominance	0.000	2.467
Exogeneity	4.145	2.478

```
Note: If the KS statistic < critical value, there is
      insufficient evidence to reject the null
      hypothesis. (KS = Kolmogorov–Smirnov)
```

In particular, we see that the test statistics are greater than the critical values in testing the hypotheses of no effect, constant effect, and exogeneity. Thus, with a 95% confidence level, we can reject these three hypotheses. In other words, we find that 401(k) participation has some effect, treatment is not constant across different quantiles, and 401(k) participation is endogenous. In contrast, we cannot reject the dominance hypothesis. Thus, we find that 401(k) participation is unambiguously beneficial for all the estimated quantiles of assets.

The test results are consistent with the result of the coefficient plot produced by `estat coefplot`, where we saw that the treatment effects are positive (dominance and no effect hypotheses) and upward trended (constant effect hypothesis).

For comparison, we can also use the SEE estimator to fit the model.

```
. ivqregress smooth assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, quantile(10(10)90)
Fitting smoothed IV quantile regression:
Quantile = .1:
Step 1: Bandwidth = 1327.0069    GMM criterion Q(b) = 9.224e-11
Step 2: Bandwidth = 1311.3131    GMM criterion Q(b) = 1.995e-10
Quantile = .2:
Step 1: Bandwidth = 1272.5204    GMM criterion Q(b) = 2.089e-10
Step 2: Bandwidth = 1237.7195    GMM criterion Q(b) = 3.075e-19
Quantile = .3:
Step 1: Bandwidth = 1504.4065    GMM criterion Q(b) = 5.407e-13
Step 2: Bandwidth = 1486.4224    GMM criterion Q(b) = 1.136e-10
Quantile = .4:
Step 1: Bandwidth = 1362.7753    GMM criterion Q(b) = 5.511e-17
Step 2: Bandwidth = 1362.6479    GMM criterion Q(b) = 8.561e-16
Quantile = .5:
Step 1: Bandwidth = 1302.9736    GMM criterion Q(b) = 2.617e-08
Step 2: Bandwidth = 6079.6881    GMM criterion Q(b) = 2.391e-12
Step 3: Bandwidth = 1438.3068    GMM criterion Q(b) = 8.068e-13
Quantile = .6:
Step 1: Bandwidth = 1533.5129    GMM criterion Q(b) = 2.679e-18
Step 2: Bandwidth = 1520.1182    GMM criterion Q(b) = 1.141e-19
Quantile = .7:
Step 1: Bandwidth = 2044.8617    GMM criterion Q(b) = 1.391e-10
Step 2: Bandwidth = 1977.2482    GMM criterion Q(b) = 1.827e-11
Quantile = .8:
Step 1: Bandwidth = 2503.7256    GMM criterion Q(b) = 3.623e-10
Step 2: Bandwidth = 2458.6714    GMM criterion Q(b) = 2.317e-10
Quantile = .9:
Step 1: Bandwidth = 3560.2178    GMM criterion Q(b) = 4.301e-12
Step 2: Bandwidth = 3529.3557    GMM criterion Q(b) = 2.932e-10
```

```
IV quantile regression                              Number of obs =    9,913
Estimator: Smoothed estimating equations            Wald chi2(81) = 4932.84
                                                    Prob > chi2   =  0.0000

------------------------------------------------------------------------------
             |               Robust
      assets | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
q10          |
        p401k |
         Yes |   3191.667   486.2193     6.56   0.000     2238.695    4144.639
      income |   .0318585   .0123707     2.58   0.010     .0076124    .0561046
         age |   128.9268   15.42632     8.36   0.000     98.69178    159.1618
  familysize |  -329.8374   125.4774    -2.63   0.009    -575.7687   -83.90615
             |
     married |
     Married |  -1480.013   386.4611    -3.83   0.000    -2237.463   -722.5635
             |
         ira |
         Yes |   7914.049   342.9506    23.08   0.000     7241.878     8586.22
             |
     pension |
 Receives .. |  -5.356704   334.9869    -0.02   0.987     -661.919    651.2056
             |
     ownhome |
         Yes |   1043.279    308.722     3.38   0.001     438.1945    1648.363
        educ |  -289.8807   53.06713    -5.46   0.000    -393.8904   -185.8711
       _cons |  -7631.313   1214.725    -6.28   0.000    -10012.13   -5250.496
-------------+----------------------------------------------------------------
```

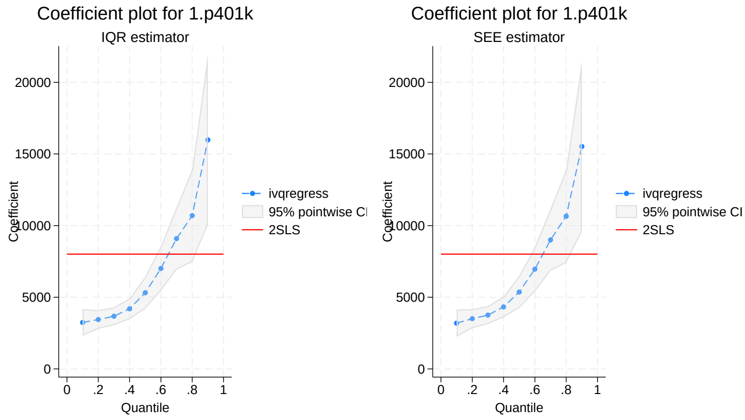
(output omitted)

```
-------------+----------------------------------------------------------------
q90          |
        p401k |
         Yes |   15525.23   3035.965     5.11   0.000     9574.848    21475.61
      income |   .8311508   .0574108    14.48   0.000     .7186277    .9436738
         age |   486.9876   51.61654     9.43   0.000      385.821    588.1541
  familysize |  -586.2617   193.5936    -3.03   0.002    -965.6983   -206.8252
             |
     married |
     Married |  -3877.165   781.2296    -4.96   0.000    -5408.347   -2345.983
             |
         ira |
         Yes |   67888.86   4902.106    13.85   0.000     58280.91    77496.81
             |
     pension |
 Receives .. |  -4829.506   898.9147    -5.37   0.000    -6591.346   -3067.665
             |
     ownhome |
         Yes |   715.6272   722.8727     0.99   0.322    -701.1773    2132.432
        educ |    14.5293   110.8781     0.13   0.896    -202.7878    231.8464
       _cons |  -19953.21   2326.698    -8.58   0.000    -24513.45   -15392.96
------------------------------------------------------------------------------
Endogenous: 1.p401k
Exogenous:  income age familysize 1.married 1.ira 1.pension 1.ownhome educ
            1.e401k
```

After `ivqregress smooth`, we can also use `estat coefplot` to visualize the treatment effect and `estat endogeffects` to test some hypotheses of particular interest in the context of the IVQR model.

First, we use `estat coefplot` to plot the coefficients and then use `graph combine` so that we can visually compare this plot with the coefficients plot for the IQR estimates.

```
. estat coefplot, name(cp_smooth) subtitle(SEE estimator)
. graph combine cp_iqr cp_smooth, xcommon ycommon altshrink
```



The left and right panels of the figure show the coefficient plots for the IQR and SEE estimates, respectively. We see that both estimators produce similar trends for the coefficients on `1.p401k` at different quantiles.

Next, we can use `estat endogeffects` to see if we draw the same conclusion regarding the four hypotheses of interest as we did with the IQR estimator.

```
. estat endogeffects, rseed(12345671)
Tests for endogenous effects                Replications = 100
```

Null hypothesis	KS statistic	95% critical value
No effect	11.507	2.593
Constant effect	5.351	2.391
Dominance	0.000	2.556
Exogeneity	4.195	2.526

```
Note: If the KS statistic < critical value, there is
      insufficient evidence to reject the null
      hypothesis. (KS = Kolmogorov-Smirnov)
```

The results align with those produced after `ivqregress iqr`. That is, the treatment effects are positive (dominance and no effect hypotheses), upward trended (constant effect hypothesis), and endogenous (exogeneity hypothesis).

◁

▷ Example 4: Robustness checks and diagnostics for the IQR estimator

In this example, we will take a closer look at the IQR estimator and show how to use `estat waldplot` to inspect the convergence visually. Nevertheless, let's first briefly discuss the intuition and algorithm behind the IQR estimator.

The IVQR model satisfies the following conditional probability:

$$\Pr(y \leq d\alpha(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau)|\mathbf{x}, \mathbf{z}) = \tau$$

where y is the outcome variable, d is an endogenous variable, $\mathbf{x}$ is a vector of exogenous covariates, and $\mathbf{z}$ is a vector of instruments. The coefficients $\alpha(\tau)$ and $\boldsymbol{\beta}(\tau)$ are indexed with the quantile level τ to indicate that they are for the model of the τ conditional quantile of the outcome y. We cannot fit the above model using the regular quantile regression because the conditional set is on $\mathbf{x}$ and $\mathbf{z}$ but the covariates contain $\mathbf{x}$ and d. Now suppose we know the value of $\alpha(\tau)$. We can then rewrite this conditional probability as

$$\Pr(y - d\alpha(\tau) \leq \mathbf{x}'\boldsymbol{\beta}(\tau) + \mathbf{z}'0|\mathbf{x}, \mathbf{z}) = \tau$$

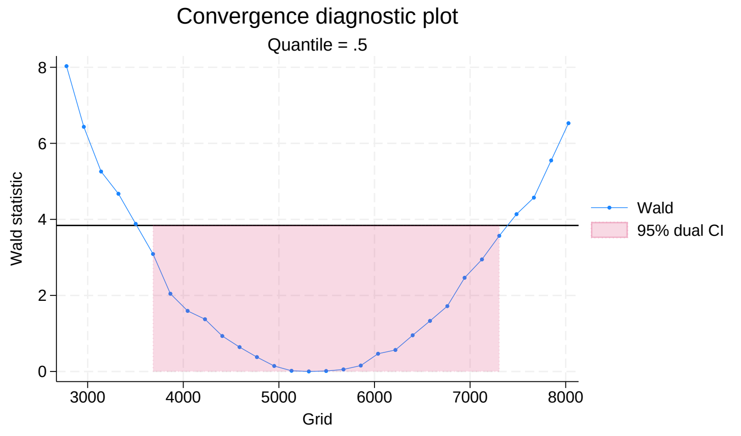
By the definition of quantile regression, we can fit this model by running a quantile regression of the transformed outcome variable, $y - d\alpha(\tau)$, on the covariates $\mathbf{x}$ and instruments $\mathbf{z}$. Notice that if $\alpha(\tau)$ is the true value, the coefficient on the instruments, which we denote as $\gamma(\tau)$, should be 0. In other words, to solve the original moment conditional for the IVQR model, we need to find a $\alpha(\tau)$ such that the auxiliary quantile regression of $y - d\alpha(\tau)$ on $\mathbf{x}$ and $\mathbf{z}$ produces 0s for the coefficients on the instrument $\mathbf{z}$. In practice, we want $\gamma(\tau)$ as close to 0 as possible, where the closeness to 0 can be measured by the Wald statistic on $\gamma(\tau)$.

Based on the above intuition, here is an outline of the IQR estimator's algorithm.

1. Define a grid of $A = \{\alpha_1, \ldots, \alpha_J\}$ (see *IQR default grid algorithm* in *Methods and formulas*).
2. For each α_j in A, run an auxiliary quantile regression of $y - d\alpha_j$ on covariates $\mathbf{x}$ and instruments $\mathbf{z}$.
3. IQR finds $\alpha_k \in A$ as a solution such that the coefficient on $\mathbf{z}$ is as close to 0 as possible in the corresponding auxiliary quantile regression, where the Wald statistic measures the closeness to 0.
4. The grid points boundary must be wider than the dual CI, which is robust to weak instruments; otherwise, `ivqregress iqr` will error out. Dual CI means it covers the true value of $\alpha(\tau)$ with 95% probability (see Chernozhukov and Hansen [2008] and `estat dualci`).

We can use estat waldplot to visualize the above procedure. Using the estimation result in example 1, we first restore the result est_iqr and then use estat waldplot to plot the Wald statistics corresponding to each grid point.

```
. estimates restore est_iqr
(results est_iqr are active now)
. estat waldplot
```



The horizontal axis shows the grid points for α, and the vertical axis shows the values of the Wald statistics. The dots in the plot show the Wald statistics corresponding to each grid point. The red line is the 95% critical value of the Wald test. Thus, only the Wald statistics below the red line will not reject the hypothesis that γ_j equals 0. Respectively, the 95% dual CI corresponds to the α's for which the Wald statistics are below the critical value. See example 1 for the use of estat dualci to show the numerical values of the dual CI.

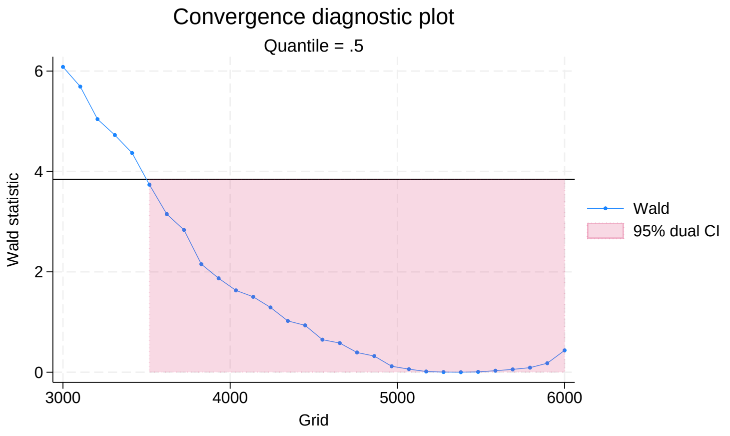
By default, ivqregress iqr uses the dual CI to generate the lower and upper bounds for the grid points to make sure that the grid covers the true value of parameter α with a large probability. Sometimes, we may want to customize the bounds. For example, suppose we want to search grid points between 3,000 and 6,000. We can use the bound() option for this purpose.

```
. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, bound(3000 6000)
Initial grid:
Quantile = 0.50: .........10.........20.........30
convergence not achieved
    The grid interval should be wider than the 95% dual confidence interval.
    Try to set a wider bound using option bound(). Use estat waldplot for
    diagnosis.
r(430);
```

We see that ivqregress iqr stops with a "convergence not achieved" error message. The reason is that the specified bound is too narrow to cover the true value of the parameter with a 95% probability.

We can now use `estat waldplot` to further visualize the issue.

```
. estat waldplot
```



The graph shows that the upper bound of 6,000 is too small because we need the Wald statistics to intersect with the 95% critical value at the lower and upper bounds.

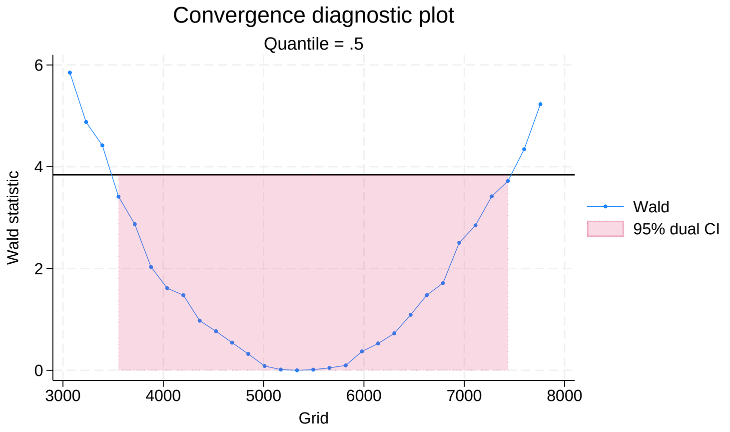
We can increase the upper bound and see if the IQR estimator converges. For example, below we increase the upper bound to 8,000.

```
. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, bound(3000 8000)
Initial grid
quantile = 0.50: .........10.........20.........30
Adaptive grid
quantile = 0.50: .........10.........20.........30
IV median regression                                Number of obs =    9,913
Estimator: Inverse quantile regression              Wald chi2(9)  = 1290.41
                                                    Prob > chi2   =   0.0000

------------------------------------------------------------------------------
             |               Robust
      assets | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       p401k |
         Yes |   5332.937   574.5175     9.28   0.000     4206.903    6458.971
      income |    .157381    .012478    12.61   0.000     .1329246    .1818374
         age |   99.78981   8.553978    11.67   0.000     83.02432    116.5553
  familysize |  -199.6165    54.3519    -3.67   0.000    -306.1442   -93.08872
             |
     married |
    Married  |  -1351.309   227.0824    -5.95   0.000    -1796.382   -906.2357
             |
         ira |
         Yes |   22631.85   1022.023    22.14   0.000     20628.72    24634.98
             |
     pension |
Receives ..  |  -694.1447    210.533    -3.30   0.001    -1106.782   -281.5077
             |
     ownhome |
         Yes |  -30.67158   154.6947    -0.20   0.843    -333.8676    272.5244
        educ |  -96.30363    32.0715    -3.00   0.003    -159.1626   -33.44465
       _cons |  -4983.758   569.4043    -8.75   0.000     -6099.77   -3867.746
------------------------------------------------------------------------------
Endogenous: 1.p401k
 Exogenous: income age familysize 1.married 1.ira 1.pension 1.ownhome educ
            1.e401k
```

Now that the IQR estimator converges, we can redraw the Wald plot to confirm that the proposed grid points interval is indeed wider than the dual CI.

```
. estat waldplot
```



◁

Stored results

`ivqregress iqr` and `ivqregress smooth` store the following in `e()`:

Scalars

`e(N)`	number of observations
`e(q#)`	the quantiles requested
`e(n_q)`	number of quantiles requested
`e(bwidth_q#)`	bandwidth used in standard errors for `q_#`th quantile
`e(sm_init_bwidth_q#)`	initial bandwidth used in smoothing the indicator function in q#th quantile estimation (`smooth` only)
`e(sm_bwidth_q#)`	bandwidth used in smoothing the indicator function in q#th quantile estimation (`smooth` only)
`e(convcode)`	`0` if converged; otherwise, return code for why nonconvergence
`e(p)`	*p*-value for model test
`e(df_m)`	model degrees of freedom
`e(chi2)`	χ^2
`e(rank)`	rank of `e(V)`

Macros

`e(cmd)`	`ivqregress`
`e(cmdline)`	command as typed
`e(depvar)`	name of dependent variable
`e(inst)`	names of instrumental variables
`e(bwrule)`	method to compute the bandwidth in standard errors
`e(kernel)`	kernel function
`e(title)`	title in estimation output
`e(vce)`	*vcetype* specified in `vce()`
`e(vcetype)`	title used to label Std. err.
`e(estimator)`	`iqr` or `smooth`
`e(exogr)`	exogenous regressors
`e(endog)`	endogenous regressors
`e(properties)`	`b V`
`e(estat_cmd)`	program used to implement `estat`
`e(predict)`	program used to implement `predict`

`e(marginsok)`	predictions allowed by `margins`
`e(marginsnotok)`	predictions disallowed by `margins`
Matrices	
`e(b)`	coefficient vector
`e(V)`	variance–covariance matrix of the estimators
Functions	
`e(sample)`	marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices	
`r(table)`	matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

Methods and formulas are presented under the following headings:

The model
The IQR estimator
The IQR algorithm
The IQR default grid algorithm
The SEE estimator
The bandwidth selection algorithm
The robust standard errors

The model

The general IVQR model was first proposed by Chernozhukov and Hansen (2005). `ivqregress` fits a linear IVQR model described in Chernozhukov and Hansen (2006, 2008). For notational simplicity, we drop the observational subscript i to refer to a random variable and add the subscript i to refer to a realization of a random variable.

We can write the linear IVQR model in the form of a "random coefficients" model as

$$y = \mathbf{d}'\boldsymbol{\alpha}(u) + \mathbf{x}'\boldsymbol{\beta}(u)$$

where

1. y is a scalar outcome variable, $\mathbf{d}$ is a vector of endogenous variables, $\mathbf{x}$ is a vector of exogenous variables, and u is the unobserved error term;
2. $\mathbf{d}$ depends on the exogenous covariates $\mathbf{x}$, and the instrumental variables $\mathbf{z}$ and unobserved error term are correlated with u;
3. u is a scalar random variable that characterizes the heterogeneity of the outcome and captures all the unobservables in the outcome from item 1 above. Conditional on $\mathbf{z}$ and $\mathbf{x}$, u is uniformly distributed between 0 and 1;
4. $\boldsymbol{\alpha}(\cdot)$ and $\boldsymbol{\beta}(\cdot)$ are random coefficient vectors that depend on u;
5. the function $\tau \to \mathbf{d}'\boldsymbol{\alpha}(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau)$ is strictly increasing in τ; and
6. the observable variables are $\{y_i, \mathbf{x}_i, \mathbf{d}_i, \mathbf{z}_i\}_{i=1}^N$ with a sample of size N.

Under some regularity conditions (see Chernozhukov and Hansen [2005]), the IVQR model satisfies the conditional probability

$$\Pr\{y \leq \mathbf{d}'\boldsymbol{\alpha}(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau)|\mathbf{x},\mathbf{z}\} = \tau \tag{1}$$

By the definition of probability and the law of iterated expectation, (1) implies the following unconditional moment condition:

$$E\left([\tau - I\{y \leq \mathbf{d}'\boldsymbol{\alpha}(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau)\}]\,\Psi\right) = 0 \tag{2}$$

where $I(\cdot)$ is the indicator function, $\Psi = (\widehat{\mathbf{d}}', \mathbf{x}')'$, and $\widehat{\mathbf{d}}$ is some function of $\mathbf{x}$ and $\mathbf{z}$ and can be treated as instruments for $\mathbf{d}$. In practice, $\widehat{\mathbf{d}}$ is the linear prediction of $\mathbf{d}$ using $\mathbf{x}$ and $\mathbf{z}$.

Equation (2) can be used as the estimating equation for the IVQR model. However, the objective function based on (2) is nonconvex and nonsmooth because of the indicator function. Thus, it is computationally challenging to fit the IVQR model by directly using (2).

`ivqregress` implements two estimators that approximately solve the original moment condition in (2). In particular, `ivqregress iqr` implements the IQR estimator proposed in Chernozhukov and Hansen (2006), and `ivqregress smooth` implements the SEE estimator outlined in Kaplan and Sun (2017). Here are the main ideas behind these two estimators.

The IQR estimator reduces the original p-dimensional (where p is the dimension of $\mathbf{x}$ and $\mathbf{d}$) nonconvex problem into a low-dimensional nonconvex problem. Then, it solves the problem by doing an exhaustive grid search over a high-quality grid. The grid is high quality in the sense that it covers the true value of the parameter for $\alpha(\tau)$ with a high probability (Chernozhukov and Hansen 2008). As a byproduct, the IQR estimator can also provide the CI that is robust to the weak instruments, which is also known as dual CI (see `estat dualci`). However, the IQR estimator becomes computationally intensive if there is more than one endogenous variable. As a result, `ivqregress iqr` allows only one endogenous variable.

The SEE estimator smooths the original moment condition in (2) using a kernel method to approximate the indicator function. Thus, the optimization problem reduces to solving a system of smooth nonlinear equations. One advantage of the SEE estimator compared with the IQR estimator is that it can handle more than one endogenous variable. However, it cannot provide the dual CI, which is robust to weak instruments like the IQR estimator.

While the IQR and SEE estimators are consistent for the IVQR model, their results are generally different. The reason is that the two estimators approximate the original moment condition in different ways. On one hand, the IQR estimator tries to find the solution by an exhaustive grid search. The estimation result critically depends on the range and finesse of grid points. On the other hand, the SEE estimator uses a kernel method to smooth the original estimating equation. Its result depends on how well the SEE estimator approximates the original, mainly controlled by the bandwidth.

In practice, suppose there is only one endogenous variable in the model. We recommend using both estimators, comparing the results, and using the IQR estimator as a benchmark because it can provide valid inference even if the instruments are weak.

The IQR estimator

Before diving into the details, we discuss the intuition of the IQR estimator. The IVQR model satisfies the conditional probability

$$\Pr\{y \leq d\alpha(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau)|\mathbf{x},\mathbf{z}\} = \tau$$

We cannot fit the above model using regular quantile regression because the conditional set is on $\mathbf{x}$ and $\mathbf{z}$, but the covariates contain $\mathbf{x}$ and d. Now suppose we know the value of $\alpha(\tau)$. We can then rewrite this conditional probability as

$$\Pr\{y - d\alpha(\tau) \leq \mathbf{x}'\boldsymbol{\beta}(\tau) + \mathbf{z}' * 0|\mathbf{x}, \mathbf{z}\} = \tau$$

By the definition of quantile regression, we can fit this model by running a quantile regression of the transformed outcome variable, $y - d\alpha(\tau)$, on the covariates $\mathbf{x}$ and instruments $\mathbf{z}$. Notice that if $\alpha(\tau)$ is the actual value, the coefficient on the instruments, which we denote as $\gamma(\tau)$, should be 0. In other words, to solve the original moment conditional for the IVQR model, we need to find a $\alpha(\tau)$ such that the auxiliary quantile regression of $y - d\alpha(\tau)$ on $\mathbf{x}$ and $\mathbf{z}$ produces zeros for the coefficients on the instruments $\mathbf{z}$. In practice, we want $\gamma(\tau)$ as close to 0 as possible, where the closeness to 0 can be measured by the Wald statistic on $\gamma(\tau)$.

Based on the above intuition, here is an outline of the IQR estimator's algorithm.

The IQR algorithm

1. Compute $\widehat{\mathbf{d}}$, which is the linear projection of $\mathbf{d}$ on $\mathbf{x}$ and $\mathbf{z}$. $\widehat{\mathbf{d}}$ can be treated as instruments for $\mathbf{d}$.
2. Define a grid of $A = \{\alpha_1, \ldots, \alpha_J\}$. For the algorithm of the default grid generation, see *The IQR default grid algorithm*.
3. For each α_j in A, run an auxiliary quantile regression of $y - d\alpha_j$ on covariates $\mathbf{x}$ and instruments $\widehat{\mathbf{d}}$.
4. IQR finds $\alpha_k \in A$ as a solution such that the coefficient on $\widehat{\mathbf{d}}$ is as close to 0 as possible in the corresponding auxiliary quantile regression, where the Wald statistic measures the closeness to 0.
5. The grid points boundary must be wider than the dual CI, which is robust to weak instruments; otherwise, `ivqregress iqr` will error out. Dual CI means it covers the true value of $\alpha(\tau)$ with $\#_{\text{level}}$ probability (see Chernozhukov and Hansen [2008] and `estat dualci`). The `level()` option specifies the confidence level $\#_{\text{level}}$; the default is `level(95)`.

The IQR default grid algorithm

The default grid algorithm can be divided into two stages: 1) the initial grid generation based on the two-stage quantile regression, which extends the two-stage median regression in Amemiya (1982); and 2) the adaptive grid that depends on the dual CI, which is robust to weak instruments (Chernozhukov and Hansen 2008).

1. Initial grid based on two-stage quantile regression
   a. Run a quantile regression of y on $\mathbf{x}$ and $\widehat{\mathbf{d}}$. Denote $\widetilde{\alpha}$ as the point estimate for the coefficient on $\widehat{\mathbf{d}}$ and $\widetilde{s}$ as its standard errors. $\widetilde{s}$ is computed by assuming the error term is normally distributed.
   b. Compute the lower and upper bounds of the grid. The lower bound is $\text{lb} = \widetilde{\boldsymbol{\alpha}} - 4\widetilde{s}$, and the upper bound is $\text{ub} = \widetilde{\boldsymbol{\alpha}} + 4\widetilde{s}$.
   c. By default, the grid points are $\#_g$ equally spaced points between lb and ub, where the `ngrid()` option specifies the number of grid points $\#_g$.

2. Adaptive grid based on the dual CI
   a. Given the initial grid, go through the steps 3–5 in *The IQR algorithm*.
   b. Obtain the dual CI based on the initial grid.
   c. Use the dual CI as the bound for the adaptive grid points and generate $\#_g$ equally spaced points.

The SEE estimator

The basic idea of the SEE estimator is to replace the indicator function in (2) with a smooth function. To be precise, we replace the moment condition in (2) with

$$E\left(\left[\tau - \widetilde{I}\{y - \mathbf{d}'\boldsymbol{\alpha}(\tau) - \mathbf{x}'\boldsymbol{\beta}(\tau) \leq 0\}\right]\Psi\right) = 0 \tag{3}$$

where $\widetilde{I}(v/h) = \max[0, \min\{1, (1 - v/h)/2\}]$ and h is the bandwidth. By default, the bandwidth is computed using the theoretical optimal bandwidth that minimizes the mean squared errors of the estimating equations. See proposition 2 in Kaplan and Sun (2017) for the optimal bandwidth.

Because $\widetilde{I}(\cdot)$ is a smooth function, the SEE estimator reduces to solve a system of smooth nonlinear equations. Let $F(\theta)$ denote the left-hand side of (3), where $\theta = \{\boldsymbol{\alpha}(\tau), \boldsymbol{\beta}(\tau)\}$. Let $\theta(i)$ denote the proposed solution at iteration i, and let $\theta(i-1)$ denote the proposed solution at the previous iteration. The convergence is declared if `mreldif(`$\theta(i), \theta(i-1)$`)` $<$ *itol* or $F(\theta)'F(\theta) <$ *ztol*, where *itol* and *ztol* can be specified by using the `tolerance()` and `ztolerance()` options, respectively. The maximum number of iterations can be specified by using the `iterate()` option.

By default, the SEE estimator searches for the bandwidth as follows.

The bandwidth selection algorithm

Denote $\widehat{\theta}_0$ as the initial values for the parameters $\alpha(\tau)$ and $\beta(\tau)$. Denote $\widehat{\mathbf{h}_{\text{opt}}}(\widehat{\theta}_0)$ as the optimal bandwidths based on the initial values $\widehat{\theta}_0$. $\widehat{\mathbf{h}_{\text{opt}}}(\widehat{\theta}_0)$ is a vector with elements (h_1, h_2, h_3), where h_1 is a nonparametrically estimated bandwidth, h_2 assumes Gaussian distribution, and h_3 uses the Silverman rule of thumb. Regardless of the assumption used, each element in $\widehat{\mathbf{h}_{\text{opt}}}(\widehat{\theta}_0)$ requires initial estimates of the error term $\epsilon = y - \mathbf{d}'\alpha(\tau) - \mathbf{x}'\beta(\tau)$. Thus, the optimal bandwidth is a function of the initial estimates for $\alpha(\tau)$ and $\beta(\tau)$. For details, see section 5.4 in Kaplan (2022).

1. Let $\widehat{\theta}_0$ be the estimates of a quantile regression of y on $\mathbf{d}$ and $\mathbf{x}$.
2. Based on $\widehat{\theta}_0$, compute the optimal bandwidths $\widehat{\mathbf{h}_{\text{opt}}}(\widehat{\theta}_0)$.
3. Define the initial bandwidth set as $\mathbf{h}_0 = \{\widehat{\mathbf{h}_{\text{opt}}}(\widehat{\theta}_0), h_{\text{init}}\}$, where h_{init} is the bandwidth in the `initbwidth()` option if specified. $\mathbf{h}_0 = \widehat{\mathbf{h}_{\text{opt}}}(\widehat{\theta}_0)$ if `initbwidth()` is not specified.
4. Find the smallest element in $\mathbf{h}_0$ such that it solves (3). The estimates for $\alpha(\tau)$ are within the dual CI with $\#_{\text{level}}$ probability (see Chernozhukov and Hansen [2008]).
   a. If a valid bandwidth is found, go to step 5.
   b. Otherwise, do a bisection search of the bandwidth with the upper bound as $100 \times \min(\mathbf{h}_0)$ and the lower bound as $\min(\mathbf{h}_0)/100$.

   If a valid bandwidth is found, denote it as h^*.

5. Update $\widehat{\theta}_0$ as the solution for the SEE estimator based on bandwidth h^*. Repeat steps 2–4 based on the updated $\widehat{\theta}_0$.

By default, steps 1–5 are used to select the bandwidth. If the `nosearchbwidth` and `initbwidth()` options are both specified, steps 2, 4b, and 5 are omitted. Thus, in this case, `ivqregress smooth` will try to solve (3) with the specified initial bandwidth without searching for the optimal or feasible bandwidths. If only the `nosearchbwidth` option is specified, step 4b is omitted.

The robust standard errors

The robust asymptotic variance–covariance estimator for the IQR and SEE estimators can be estimated as follows (see Chernozhukov and Hansen [2006] and de Castro et al. [2019]). Let $\theta = \{\boldsymbol{\alpha}(\tau), \boldsymbol{\beta}(\tau)\}$ be the true values of parameters and $\widehat{\theta} = \{\widehat{\boldsymbol{\alpha}(\tau)}, \widehat{\boldsymbol{\beta}(\tau)}\}$ be the IQR or the SEE estimator. For any finite collection of quantile indices $\tau_j, j \in T$

$$[\sqrt{n}\{\widehat{\boldsymbol{\theta}}(\tau) - \boldsymbol{\theta}(\tau)\}]_{j\in T} \rightarrow N(0, [\mathbf{J}(\tau_k)^{-1}\mathbf{S}(\tau_j,\tau_k)\{\mathbf{J}(\tau_j)^{-1}\}']_{k,j\in T})$$

where

$$\mathbf{J}(\tau) = E\left\{f_{\epsilon(\tau)}(0|\mathbf{x},\mathbf{d},\mathbf{z})\Psi[\mathbf{d}',\mathbf{x}']\right\}$$
$$\mathbf{S}(\tau_j,\tau_k) = \{\min(\tau_j,\tau_k) - \tau_j\tau_k\}E(\Psi\Psi')$$

and $f_{\epsilon(\tau)}(0|\mathbf{x},\mathbf{d},\mathbf{z})$ is the conditional density of $\epsilon(\tau)$ evaluated at 0, with $\epsilon_i(\tau) = y_i - \mathbf{d}_i'\boldsymbol{\alpha}(\tau) - \mathbf{x}_i'\boldsymbol{\beta}(\tau)$.

The components in the variance can be obtained by their sample counterparts. In particular, $\mathbf{S}(\cdot)$ can be estimated as

$$\widehat{\mathbf{S}}(\tau_k,\tau_j) = \{\min(\tau_k,\tau_j) - \tau_k\tau_j\}\frac{1}{N}\sum_{i=1}^{N}\Psi_i\Psi_i'$$

$\mathbf{J}(\cdot)$ can be estimated as

$$\widehat{\mathbf{J}}(\tau) = \frac{1}{Nh_N}\sum_{i}^{N} K\left\{\frac{-\widehat{\epsilon_i(\tau)}}{h_n}\right\}\Psi_i[\mathbf{d}_i',\mathbf{x}_i']$$

where $\widehat{\epsilon_i(\tau)} = y_i - \mathbf{d}_i'\widehat{\boldsymbol{\alpha}(\tau)} - \mathbf{x}_i'\widehat{\boldsymbol{\beta}(\tau)}$, $K(\cdot)$ is a kernel function, and h_n is the bandwidth.

`vce(`*vcetype*`, kernel())` specifies the kernel function form $K(\cdot)$. See [R] **kdensity** for the function forms of the eight kernels.

`vce(`*vcetype*`, bwidth())` specifies which bandwidth to use: `silverman` specifies to use h_{s}, `hsheather` specifies to use h_{k} with h_1 replaced by h_{hs}, and `bofinger` specifies to use h_{k} with h_1 replaced by h_{bo}.

Silverman's rule of thumb bandwidth is

$$h_{\mathrm{s}} = 0.9\min\left\{\widehat{\sigma(\epsilon)}, \frac{M}{1.349}\right\}N^{-\frac{1}{5}}$$

where $\widehat{\sigma(\epsilon)}$ is the standard deviation of $\widehat{\epsilon}$ and M is the interquartile range of $\widehat{\epsilon}$.

The bandwidth in Koenker (2005, 81) is

$$h_{\rm k} = \min\left\{\widehat{\sigma(\epsilon)}, \frac{M}{1.349}\right\}\{\Phi^{-1}(\tau + h_1) - \Phi^{-1}(\tau - h_1)\}$$

where $\Phi^{-1}(\cdot)$ is the inverse cumulative standard normal distribution and h_1 can be one of the bandwidths in Hall and Sheather (1988) ($h_{\rm hs}$) or Bofinger (1975) ($h_{\rm bo}$). In particular,

$$h_{\rm hs} = N^{-1/3}\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)^{2/3}\left[\frac{3}{2} \times \frac{\phi\left\{\Phi^{-1}(\tau)\right\}^2}{2\Phi^{-1}(\tau)^2 + 1}\right]^{1/3}$$

$$h_{\rm bo} = N^{-1/5}\left[\frac{9}{2} \times \frac{\phi\left\{\Phi^{-1}(\tau)\right\}^4}{\left\{2\Phi^{-1}(\tau)^2 + 1\right\}^2}\right]^{1/5}$$

where $\phi(\cdot)$ is the standard normal probability density function.

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Also see

[R] **ivqregress postestimation** — Postestimation tools for ivqregress

[R] **ivregress** — Single-equation instrumental-variables regression

[R] **qreg** — Quantile regression

[U] **20 Estimation and postestimation commands**