Title

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fp — Fractional polynomial regression

Description

`fp <term>: est_cmd` fits models with the "best"-fitting fractional polynomial substituted for `<term>` wherever it appears in *est_cmd*. `fp <weight>: regress mpg <weight> foreign` would fit a regression model of `mpg` on a fractional polynomial in `weight` and (linear) `foreign`.

By specifying option `fp()`, you may set the exact powers to be used. Otherwise, a search through all possible fractional polynomials up to the degree set by `dimension()` with the powers set by `powers()` is performed.

`fp` without arguments redisplays the previous estimation results, just as typing *est_cmd* would. You can type either one. `fp` will include a fractional polynomial comparison table.

`fp generate` creates fractional polynomial power variables for a given set of powers. For instance, `fp <weight>: regress mpg <weight> foreign` might produce the fractional polynomial $\texttt{weight}^{(-2,-1)}$ and store $\texttt{weight}^{-2}$ in `weight_1` and $\texttt{weight}^{-1}$ in `weight_2`. Typing `fp generate weight^(-2 -1)` would allow you to create the same variables in another dataset.

See [R] **mfp** for multivariable fractional polynomial models.

Quick start

Fit models with fractional polynomials

Find optimal second-degree fractional polynomial of `x1` in regression of `y` on `x2` and `x3`

```
fp <x1>: regress y <x1> x2 x3
```

As above, but search only powers of -1, -0.5, 1, and 2.

```
fp <x1>, power(-1 -.5 1 2): regress y <x1> x2 x3
```

As above, but allow search to include third-degree fractional polynomials

```
fp <x1>, power(-1 -.5 1 2) dimension(3): regress y <x1> x2 x3
```

Fit model including $\texttt{x1}^{-2}$ and $\texttt{x1}^{2}$ without performing search

```
fp <x1>, fp(-2 2): regress y <x1> x2 x3
```

Rescale `x1` to nonextreme positive values when computing fractional polynomials

```
fp <x1>, scale: regress y <x1> x2 x3
```

As above, and center fractional polynomial of `x1` at its scaled mean

```
fp <x1>, center scale: regress y <x1> x2 x3
```

Set fractional polynomial to zero for nonpositive values of `x1`

```
fp <x1>, zero: regress y <x1> x2 x3
```

As above, and include an indicator variable in the model for nonpositive values of `x1`

```
fp <x1>, catzero: regress y <x1> x2 x3
```

Create variables corresponding to fractional polynomial powers

Generate `x1_1` and `x1_2` corresponding to $\texttt{x1}^{-2}$ and $\texttt{x1}^{2}$

```
fp generate x1^(-2 2)
```

As above, but generate fractional polynomial variables with automatic scaling and centering

```
fp generate x1^(-2 2), center scale
```

Note: In the above examples, `regress` could be replaced with any estimation command allowing the `fp` prefix.

Menu

fp

Statistics > Linear models and related > Fractional polynomials > Fractional polynomial regression

fp generate

Statistics > Linear models and related > Fractional polynomials > Create fractional polynomial variables

Syntax

Estimation

```
fp <term> [ , est_options ] :  est_cmd
```

est_cmd may be almost any estimation command that stores the `e(ll)` result. To confirm whether `fp` works with a specific *est_cmd*, see the documentation for that *est_cmd*.

Instances of `<term>` (with the angle brackets) that occur within *est_cmd* are replaced in *est_cmd* by a varlist containing the fractional powers of the variable *term*. These variables will be named *term*`_1`, *term*`_2`, ...

`fp` performs *est_cmd* with this substitution, fitting a fractional polynomial regression in *term*.

est_cmd in either this or the following syntax may not contain other prefix commands; see [U] **11.1.10 Prefix commands**.

Estimation (alternate syntax)

```
fp <term>(varname) [ , est_options ] :  est_cmd
```

Use this syntax to specify that fractional powers of *varname* are to be calculated. The fractional polynomial power variables will still be named *term*`_1`, *term*`_2`, ...

Replay estimation results

```
fp [ , replay_options ]
```

Create specified fractional polynomial power variables

```
fp generate [type] [newvar =] varname^(numlist) [if] [in] [ , gen_options ]
```

est_options	Description
Main	
Search	
`powers(# # ... #)`	powers to be searched; default is `powers(-2 -1 -.5 0 .5 1 2 3)`
`dimension(#)`	maximum degree of fractional polynomial; default is `dimension(2)`
Or specify	
`fp(# # ...#)`	use specified fractional polynomial
And then specify any of these options	
Options	
`classic`	perform automatic scaling and centering and omit comparison table
`replace`	replace existing fractional polynomial power variables named *term*_1, *term*_2, ...
`all`	generate *term*_1, *term*_2, ... in all observations; default is in observations `if esample()`
`scale(#_a #_b)`	use (*term*+*a*)/*b*; default is to use variable term as is
`scale`	specify *a* and *b* automatically
`center(#_c)`	report centered-on-*c* results; default is uncentered results
`center`	specify *c* to be the mean of (scaled) *term*
`zero`	set *term*_1, *term*_2, ... to zero if scaled *term* ≤ 0; default is to issue an error message
`catzero`	same as `zero` and include *term*_0 $= (term \leq 0)$ among fractional polynomial power variables
Reporting	
replay_options	specify how results are displayed

replay_options	Description
Reporting	
`nocompare`	do not display model-comparison test results
reporting_options	any options allowed by *est_cmd* for replaying estimation results

gen_options	Description
Main	
`replace`	replace existing fractional polynomial power variables named *term*_1, *term*_2, ...
`scale(#_a #_b)`	use (*term*+*a*)/*b*; default is to use variable term as is
`scale`	specify *a* and *b* automatically
`center(#_c)`	report centered-on-*c* results; default is uncentered results
`center`	specify *c* to be the mean of (scaled) *term*
`zero`	set *term*_1, *term*_2, ... to zero if scaled *term* ≤ 0; default is to issue an error message
`catzero`	same as `zero` and include *term*_0 $= (term \leq 0)$ among fractional polynomial power variables

Options for fp

Main

`powers(# # ... #)` specifies that a search be performed and details about the search provided. `powers()` works with the `dimension()` option; see below. The default is `powers(-2 -1 -.5 0 .5 1 2 3)`.

`dimension(#)` specifies the maximum degree of the fractional polynomial to be searched. The default is `dimension(2)`.

If the defaults for both `powers()` and `dimension()` are used, then the fractional polynomial could be any of the following 44 possibilities:

$$
\begin{gathered}
term^{(-2)} \\
term^{(-1)} \\
\vdots \\
term^{(3)} \\
term^{(-2)}, term^{(-2)} \\
term^{(-2)}, term^{(-1)} \\
\vdots \\
term^{(-2)}, term^{(3)} \\
term^{(-1)}, term^{(-2)} \\
\vdots \\
term^{(3)}, term^{(3)}
\end{gathered}
$$

`fp(# # ... #)` specifies that no search be performed and that the fractional polynomial specified be used. `fp()` is an alternative to `powers()` and `dimension()`.

Options

`classic` performs automatic scaling and centering and omits the comparison table. Specifying `classic` is equivalent to specifying `scale`, `center`, and `nocompare`.

`replace` replaces existing fractional polynomial power variables named *term*_1, *term*_2, ...

`all` specifies that *term*_1, *term*_2, ... be filled in for all observations in the dataset rather than just for those in `e(sample)`.

`scale(#_a #_b)` specifies that *term* be scaled in the way specified, namely, that (*term*+*a*)/*b* be calculated. All values of the scaled term are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large or too close to zero, because by default, cubic powers and squared reciprocal powers will be considered. When `scale(`*a b*`)` is specified, values in the variable *term* are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

You will probably not use `scale(`*a b*`)` for values of *a* and *b* that you create yourself, although you could. It is usually easier just to `generate` a scaled variable. For instance, if *term* is `age`, and `age` in your data is required to be greater than or equal to 20, you might generate an `age5` variable, for use as *term*:

```
. generate age5 = (age-19)/5
```

`scale(`*a b*`)` is useful when you previously fit a model using automatic scaling (option `scale`) in one dataset and now want to create the fractional polynomials in another. In the first dataset, `fp` with `scale` added `notes` to the dataset concerning the values of *a* and *b*. You can see them by typing

```
. notes
```

You can then use `fp generate, scale(`*a b*`)` in the second dataset.

The default is to use *term* as it is used in calculating fractional powers; thus *term*'s values are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large, because by default, cubic powers will be considered.

`scale` specifies that *term* be scaled to be greater than zero and not too large in calculating fractional powers. See *Scaling* for more details. When `scale` is specified, values in the variable *term* are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

`center(`*#_c*`)` reports results for the fractional polynomial in (scaled) *term*, centered on *c*. The default is to perform no centering.

$term^{(p_1,p_2,\ldots,p_m)}$-$c^{(p_1,p_2,\ldots,p_m)}$ is reported. This makes the constant coefficient (intercept) easier to interpret. See *Centering* for more details.

`center` performs `center(`*c*`)`, where *c* is the mean of (scaled) *term*.

`zero` and `catzero` specify how nonpositive values of *term* are to be handled. By default, nonpositive values of *term* are not allowed, because we will be calculating natural logarithms and fractional powers of *term*. Thus an error message is issued.

`zero` sets the fractional polynomial value to zero for nonpositive values of (scaled) *term*.

`catzero` sets the fractional polynomial value to zero for nonpositive values of (scaled) *term* and includes a dummy variable indicating where nonpositive values of (scaled) *term* appear in the model.

Reporting

`nocompare` suppresses display of the comparison tests.

reporting_options are any options allowed by *est_cmd* for replaying estimation results.

Options for fp generate

Main

`replace` replaces existing fractional polynomial power variables named *term*_1, *term*_2, ...

`scale(`*#_a #_b*`)` specifies that *term* be scaled in the way specified, namely, that (*term*+*a*)/*b* be calculated. All values of the scaled term are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large or too close to zero, because by default, cubic powers and squared reciprocal powers will be considered. When `scale(`*a b*`)` is specified, values in the variable *term* are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

You will probably not use `scale(`*a b*`)` for values of *a* and *b* that you create yourself, although you could. It is usually easier just to `generate` a scaled variable. For instance, if *term* is `age`, and `age` in your data is required to be greater than or equal to 20, you might generate an `age5` variable, for use as *term*:

```
. generate age5 = (age-19)/5
```

`scale(`*a b*`)` is useful when you previously fit a model using automatic scaling (option `scale`) in one dataset and now want to create the fractional polynomials in another. In the first dataset, `fp` with `scale` added `notes` to the dataset concerning the values of *a* and *b*. You can see them by typing

```
. notes
```

You can then use `fp generate, scale(`*a b*`)` in the second dataset.

The default is to use *term* as it is used in calculating fractional powers; thus *term*'s values are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large, because by default, cubic powers will be considered.

`scale` specifies that *term* be scaled to be greater than zero and not too large in calculating fractional powers. See *Scaling* for more details. When `scale` is specified, values in the variable *term* are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

`center(#_`*c*`)` reports results for the fractional polynomial in (scaled) *term*, centered on *c*. The default is to perform no centering.

$term^{(p_1,p_2,\ldots,p_m)}$-$c^{(p_1,p_2,\ldots,p_m)}$ is reported. This makes the constant coefficient (intercept) easier to interpret. See *Centering* for more details.

`center` performs `center(`*c*`)`, where *c* is the mean of (scaled) *term*.

`zero` and `catzero` specify how nonpositive values of *term* are to be handled. By default, nonpositive values of *term* are not allowed, because we will be calculating natural logarithms and fractional powers of *term*. Thus an error message is issued.

`zero` sets the fractional polynomial value to zero for nonpositive values of (scaled) *term*.

`catzero` sets the fractional polynomial value to zero for nonpositive values of (scaled) *term* and includes a dummy variable indicating where nonpositive values of (scaled) *term* appear in the model.

Remarks and examples

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Remarks are presented under the following headings:

Fractional polynomial regression

Regression models based on fractional polynomial functions of a continuous covariate are described by Royston and Altman (1994).

Fractional polynomials increase the flexibility afforded by the family of conventional polynomial models. Although polynomials are popular in data analysis, linear and quadratic functions are limited in their range of curve shapes, whereas cubic and higher-order curves often produce undesirable artifacts such as edge effects and waves.

Fractional polynomials differ from regular polynomials in that 1) they allow logarithms, 2) they allow noninteger powers, and 3) they allow powers to be repeated.

We will write a fractional polynomial in x as

$$x^{(p_1,p_2,\ldots,p_m)\prime}\boldsymbol{\beta}$$

We will write $x^{(p)}$ to mean a regular power except that $x^{(0)}$ is to be interpreted as meaning $\ln(x)$ rather than $x^{(0)} = 1$.

Then if there are no repeated powers in $(p_1, p_2, \ldots, p_m)$,

$$x^{(p_1,p_2,\ldots,p_m)\prime}\boldsymbol{\beta} = \beta_0 + \beta_1 x^{(p_1)} + \beta_2 x^{(p_2)} + \cdots + \beta_m x^{(p_m)}$$

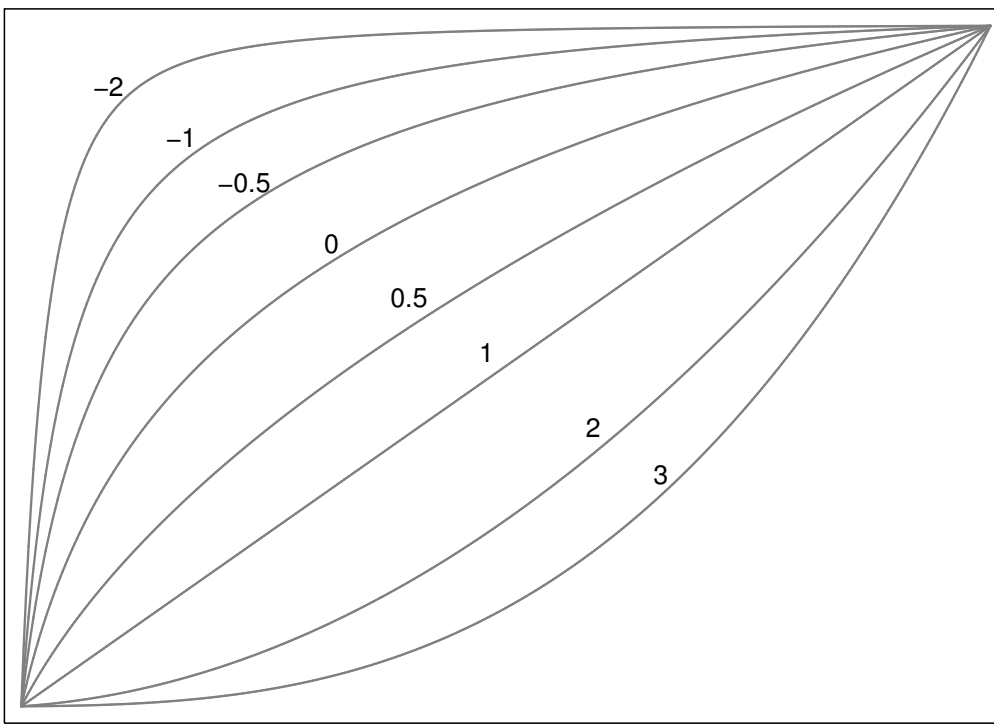
Powers are allowed to repeat in fractional polynomials. Each time a power repeats, it is multiplied by another $\ln(x)$. As an extreme case, consider the fractional polynomial with all-repeated powers, say, m of them,

$$x^{(p,p,\ldots,p)\prime}\boldsymbol{\beta} = \beta_0 + \beta_1 x^{(p)} + \beta_2 x^{(p)} \ln(x) + \cdots + \beta_m x^{(p)} \{\ln(x)\}^{m-1}$$

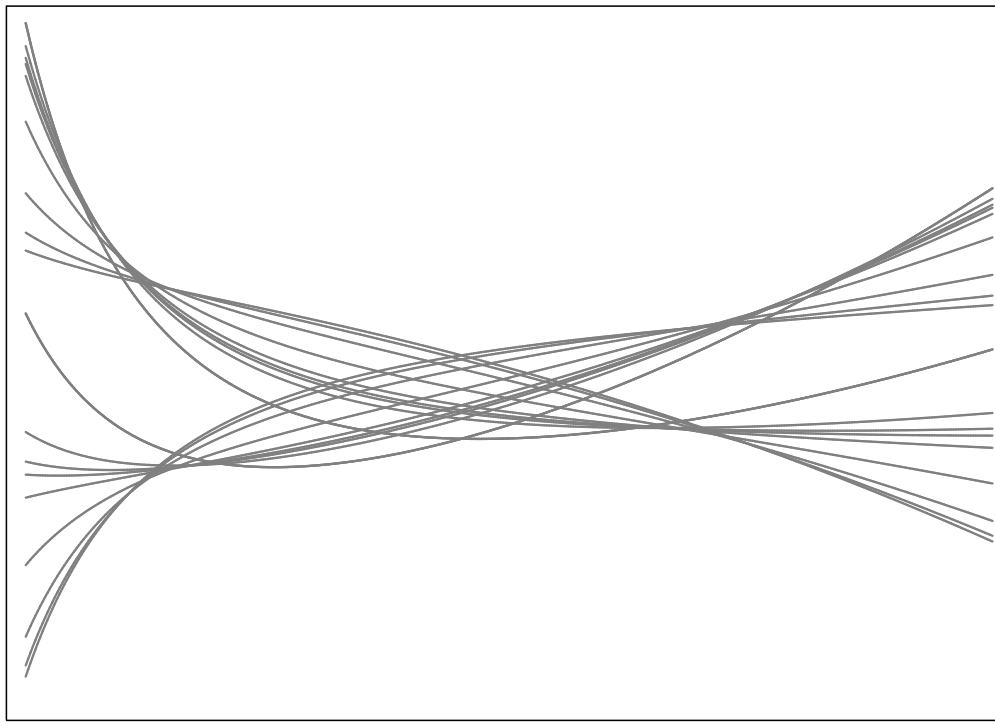
Thus the fractional polynomial $x^{(0,0,2)\prime}\boldsymbol{\beta}$ would be

$$\begin{aligned} x^{(0,0,2)\prime}\boldsymbol{\beta} &= \beta_0 + \beta_1 x^{(0)} + \beta_2 x^{(0)} \ln(x) + \beta_3 x^{(2)} \\ &= \beta_0 + \beta_1 \ln(x) + \beta_2 \{\ln(x)\}^2 + \beta_3 x^2 \end{aligned}$$

With this definition, we can obtain a much wider range of shapes than can be obtained with regular polynomials. The following graphs appeared in Royston and Sauerbrei (2008, sec. 4.5). The first graph shows the shapes of differing fractional polynomials.



The second graph shows some of the curve shapes available with different βs for the degree-2 fractional polynomial, $x^{(-2,2)}$.



In modeling a fractional polynomial, Royston and Sauerbrei (2008) recommend choosing powers from among $\{-2, -1, -0.5, 0, 0.5, 1, 2, 3\}$. By default, `fp` chooses powers from this set, but other powers can be explicitly specified in the `powers()` option.

`fp` <*term*>`:` *est_cmd* fits models with the terms of the best-fitting fractional polynomial substituted for <*term*> wherever it appears in *est_cmd*. We will demonstrate with `auto.dta`, which contains repair records and other information about a variety of vehicles in 1978.

We use `fp` to find the best fractional polynomial in automobile weight (lbs.) (`weight`) for the linear regression of miles per gallon (`mpg`) on `weight` and an indicator of whether the vehicle is foreign (`foreign`).

By default, `fp` will fit degree-2 fractional polynomial (FP2) models and choose the fractional powers from the set $\{-2, -1, -0.5, 0, 0.5, 1, 2, 3\}$. Because car weight is measured in pounds and will have a cubic transformation applied to it, we shrink it to a smaller scale before estimation by dividing by 1,000.

We modify the existing `weight` variable for conciseness and to facilitate the comparison of tables. When applying a data transformation in practice, rather than modifying the existing variables, you should create new variables that hold the transformed values.

```
. use http://www.stata-press.com/data/r15/auto
(1978 Automobile Data)
. replace weight = weight/1000
variable weight was int now float
(74 real changes made)
. fp <weight>: regress mpg <weight> foreign
(fitting 44 models)
(....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%)
Fractional polynomial comparisons:
```

weight	df	Deviance	Res. s.d.	Dev. dif.	P(*)	Powers
omitted	0	456.347	5.356	75.216	0.000	
linear	1	388.366	3.407	7.236	0.082	1
m = 1	2	381.806	3.259	0.675	0.733	-.5
m = 2	4	381.131	3.268	0.000	--	-2 -2

(*) P = sig. level of model with m = 2 based on F with 68 denominator dof.

Source	SS	df	MS		
				Number of obs	= 74
				F(3, 70)	= 52.95
Model	1696.05949	3	565.353163	Prob > F	= 0.0000
Residual	747.399969	70	10.6771424	R-squared	= 0.6941
				Adj R-squared	= 0.6810
Total	2443.45946	73	33.4720474	Root MSE	= 3.2676

mpg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
weight_1	15.88527	20.60329	0.77	0.443	-25.20669	56.97724
weight_2	127.9349	47.53106	2.69	0.009	33.13723	222.7326
foreign	-2.222515	1.053782	-2.11	0.039	-4.324218	-.1208131
_cons	3.705981	3.367949	1.10	0.275	-3.011182	10.42314

`fp` begins by showing the model-comparison table. This table shows the best models of each examined degree, obtained by searching through all possible power combinations. The fractional powers of the models are shown in the `Powers` column. A separate row is provided for the linear fractional polynomial because it is often the default used when including a predictor in the model. The null model does not include any fractional polynomial terms for `weight`. The `df` column shows the count of the additional parameters used in each model beyond the quantity of parameters used in the null model. The model deviance, which we define as twice the negative log likelihood, is given in the `Deviance` column. The difference of the model deviance from the deviance of the model with the lowest deviance is given in the `Dev. dif.` column.

The p-value for the partial F test comparing the models and the lowest deviance model is given in the `P(*)` column. An estimate of the residual standard error is given in the `Res. Sd.` column. Under linear regression, a partial F test is used in the model-comparison table. In other settings, a likelihood-ratio test is performed. Then a χ^2 statistic is reported.

Under robust variance estimation and some other cases (see [R] **lrtest**), the likelihood-ratio test cannot be performed. When the likelihood-ratio test cannot be performed on the model specified in *est_cmd*, `fp` still reports the model-comparison table, but the comparison tests are not performed.

`fp` reports the "best" model as the model with the lowest deviance; however, users may choose a more efficient model based on the comparison table. They may choose the lowest degree model that the partial F test (or likelihood-ratio test) fails to reject in favor of the lowest deviance model.

After the comparison table, the results of the estimation command for the lowest deviance model are shown. Here the best model has terms $\texttt{weight}^{(-2,-2)}$. However, based on the model-comparison

table, we can reject the model without weight and the linear model at the 0.1 significance level. We fail to reject the m = 1 model at any reasonable level. We will choose the FP1 model, which includes weight$^{(-.5)}$.

We use fp again to estimate the parameters for this model. We use the fp() option to specify what powers we want to use; this option specifies that we do not want to perform a search for the best powers. We also specify the replace option to overwrite the previously created fractional polynomial power variables.

```
. fp <weight>, fp(-.5) replace: regress mpg <weight> foreign
-> regress mpg weight_1 foreign
```

Source	SS	df	MS	Number of obs	=	74
				F(2, 71)	=	79.51
Model	1689.20865	2	844.604325	Prob > F	=	0.0000
Residual	754.25081	71	10.6232508	R-squared	=	0.6913
				Adj R-squared	=	0.6826
Total	2443.45946	73	33.4720474	Root MSE	=	3.2593

mpg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
weight_1	66.89665	6.021749	11.11	0.000	54.88963	78.90368
foreign	-2.095622	1.043513	-2.01	0.048	-4.176329	-.0149157
_cons	-17.58651	3.397992	-5.18	0.000	-24.36192	-10.81111

Alternatively, we can use fp generate to create the fractional polynomial variable corresponding to weight$^{(-.5)}$ and then use regress. We store weight$^{(-.5)}$ in the new variable wgt_nsqrt.

```
. fp generate wgt_nsqrt=weight^(-.5)
. regress mpg wgt_nsqrt foreign
```

Source	SS	df	MS	Number of obs	=	74
				F(2, 71)	=	79.51
Model	1689.20874	2	844.604371	Prob > F	=	0.0000
Residual	754.250718	71	10.6232495	R-squared	=	0.6913
				Adj R-squared	=	0.6826
Total	2443.45946	73	33.4720474	Root MSE	=	3.2593

mpg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
wgt_nsqrt_1	66.89665	6.021748	11.11	0.000	54.88963	78.90368
foreign	-2.095622	1.043513	-2.01	0.048	-4.176328	-.0149155
_cons	-17.58651	3.397991	-5.18	0.000	-24.36191	-10.81111

Scaling

Fractional polynomials are only defined for positive *term* variables. By default, fp will assume that the variable x is positive and attempt to compute fractional powers of x. If the positive value assumption is incorrect, an error will be reported and estimation will not be performed.

If the values of the variable are too large or too small, the reported results of fp may be difficult to interpret. By default, cubic powers and squared reciprocal powers will be considered in the search for the best fractional polynomial in *term*.

We can scale the variable x to 1) make it positive and 2) ensure its magnitude is not too large or too small.

Suppose you have data on hospital patients with age as a fractional polynomial variable of interest. age is required to be greater than or equal to 20, so you might generate an age5 variable by typing

```
. generate age5 = (age-19)/5
```

A unit change in `age5` is equivalent to a five-year change in age, and the minimum value of `age5` is $1/5$ instead of 20.

In the automobile example of *Fractional polynomial regression*, our *term* variable was automobile weight (lbs.). Cars weigh in the thousands of pounds, so cubing their weight figures results in large numbers. We prevented this from being a problem by shrinking the weight by 1,000; that is, we typed

```
. replace weight = weight/1000
```

Calendar year is another type of variable that can have a problematically large magnitude. We can shrink this by dividing by 10, making a unit change correspond to a decade.

```
. generate decade = calendar_year/10
```

You may also have a variable that measures deviation from zero. Perhaps `x` has already been demeaned and is symmetrically about zero. The fractional polynomial in `x` will be undefined for half of its domain. We can shift the location of `x`, making it positive by subtracting its minimum and adding a small number to it. Suppose `x` ranges from -4 to 4; we could use

```
. generate newx = x+5
```

Rescaling ourselves provides easily communicated results. We can tell exactly how the scaling was performed and how it should be performed in similar applications.

Alternatively, `fp` can scale the fractional polynomial variable so that its values are positive and the magnitude of the values are not too large. This can be done automatically or by directly specifying the scaling values.

Scaling can be automatically performed with `fp` by specifying the `scale` option. If *term* has nonpositive values, the minimum value of *term* is subtracted from each observation of *term*. In this case, the counting interval, the minimum distance between the sorted values of *term*, is also added to each observation of *term*.

After adjusting the location of *term* so that its minimum value is positive, creating *term**, automatic scaling will divide each observation of *term* by a power of ten. The exponent of this scaling factor is given by

$$p = \log_{10}\{\max(term^*) - \min(term^*)\}$$

$$p^* = \text{sign}(p)\text{floor}(|p|)$$

Rather than letting `fp` automatically choose the scaling of *term*, you may specify adjustment and scale factors a and b by using the `scale(`*a b*`)` option. Fractional powers are then calculated using the (*term*+*a*)/*b* values.

When `scale` or `scale(`*a b*`)` is specified, values in the variable *term* are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

In addition to `fp`, both `scale` and `scale(`*a b*`)` may be used with `fp generate`.

You will probably not use `scale(`*a b*`)` with `fp` for values of a and b that you create yourself, although you could. As we demonstrated earlier, it is usually easier just to `generate` a scaled variable.

`scale(`*a b*`)` is useful when you previously fit a model using `scale` in one dataset and now want to create the fractional polynomials in another. In the first dataset, `fp` with `scale` added `notes` to the dataset concerning the values of a and b. You can see them by typing

```
. notes
```

You can then use `fp generate, scale(`*a b*`)` in the second dataset.

When you apply the scaling rules of a previously fit model to new data with the `scale(`*a b*`)` option, it is possible that the scaled term may have nonpositive values. `fp` will be unable to calculate the fractional powers of the term in this case and will issue an error.

The options `zero` and `catzero` cause `fp` and `fp generate` to output zero values for each fractional polynomial variable when the input (scaled) fractional polynomial variable is nonpositive. Specifying `catzero` causes a dummy variable indicating nonpositive values of the (scaled) fractional polynomial variable to be included in the model. A detailed example of the use of `catzero` and `zero` is shown in example 3 below.

Using the scaling options, we can fit our previous model again using the `auto.dta`. We specify `scale(0 1000)` so that `fp` will shrink the magnitude of `weight` in estimating the regression. This is done for demonstration purposes because our scaling rule is simple. As mentioned before, in practice, you would probably only use `scale(`*a b*`)` when applying the scaling rules from a previous analysis. Allowing `fp` to scale does have the advantage of not altering the original variable, `weight`.

```
. use http://www.stata-press.com/data/r15/auto, clear
(1978 Automobile Data)
. fp <weight>, fp(-.5) scale(0 1000): regress mpg <weight> foreign
-> regress mpg weight_1 foreign
```

Source	SS	df	MS	Number of obs	=	74
				F(2, 71)	=	79.51
Model	1689.20861	2	844.604307	Prob > F	=	0.0000
Residual	754.250846	71	10.6232514	R-squared	=	0.6913
				Adj R-squared	=	0.6826
Total	2443.45946	73	33.4720474	Root MSE	=	3.2593

mpg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
weight_1	66.89665	6.021749	11.11	0.000	54.88963	78.90368
foreign	-2.095622	1.043513	-2.01	0.048	-4.176329	-.0149159
_cons	-17.58651	3.397992	-5.18	0.000	-24.36192	-10.81111

The scaling is clearly indicated in the variable notes for the generated variable `weight_1`.

```
. notes weight_1
weight_1:
  1.  fp term 1 of x^(-.5), where x is weight scaled.
  2.  Scaling was user specified: x = (weight+a)/b where a=0 and b=1000
  3.  Fractional polynomial variables created by fp <weight>, fp(-.5) scale(0
      1000): regress mpg <weight> foreign
  4.  To re-create the fractional polynomial variables, for instance, in
      another dataset, type fp gen double weight^(-.5), scale(0 1000)
```

Centering

The fractional polynomial of *term*, centered on c is

$$\left(term^{(p_1,\ldots,p_m)} - c^{(p_1,\ldots,p_m)}\right)'\boldsymbol{\beta}$$

The intercept of a centered fractional polynomial can be interpreted as the effect at zero for all the covariates. When we center the fractional polynomial terms using c, the intercept is now interpreted as the effect at $term = c$ and zero values for the other covariates.

Suppose we wanted to center the fractional polynomial of x with powers (0, 0, 2) at $x = c$.

$$\begin{aligned}
&\left(x^{(0,0,2)} - c^{(0,0,2)}\right)'\boldsymbol{\beta} \\
&= \beta_0 + \beta_1\left(x^{(0)} - c^{(0)}\right) + \beta_2\left\{x^{(0)}\ln(x) - c^{(0)}\ln(c)\right\} + \beta_3\left(x^{(2)} - c^{(2)}\right) \\
&= \beta_0 + \beta_1\{\ln(x) - \ln(c)\} + \beta_2\left[\{\ln(x)\}^2 - \{\ln(c)\}^2\right] + \beta_3\left(x^2 - c^2\right)
\end{aligned}$$

When `center` is specified, `fp` centers based on the sample mean of (scaled) *term*. A previously chosen value for centering, c, may also be specified in `center(`c`)`. This would be done when applying the results of a previous model fitting to a new dataset.

The `center` and `center(`c`)` options may be used in `fp` or `fp generate`.

Returning to the model of mileage per gallon based on automobile weight and foreign origin, we refit the model with the fractional polynomial of `weight` centered at its scaled mean.

```
. use http://www.stata-press.com/data/r15/auto, clear
(1978 Automobile Data)
. fp <weight>, fp(-.5) scale(0 1000) center: regress mpg <weight> foreign
-> regress mpg weight_1 foreign
```

Source	SS	df	MS	Number of obs	=	74
				F(2, 71)	=	79.51
Model	1689.20861	2	844.604307	Prob > F	=	0.0000
Residual	754.250846	71	10.6232514	R-squared	=	0.6913
				Adj R-squared	=	0.6826
Total	2443.45946	73	33.4720474	Root MSE	=	3.2593

mpg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
weight_1	66.89665	6.021749	11.11	0.000	54.88963	78.90368
foreign	-2.095622	1.043513	-2.01	0.048	-4.176329	-.0149159
_cons	20.91163	.4624143	45.22	0.000	19.9896	21.83366

Note that the coefficients for `weight_1` and `foreign` do not change. Only the intercept `_cons` changes. It can be interpreted as the estimated average miles per gallon of an American-made car of average weight.

Examples

▷ Example 1: Linear regression

Consider the serum immunoglobulin G (IgG) dataset from Isaacs et al. (1983), which consists of 298 independent observations in young children. The dependent variable `sqrtigg` is the square root of the IgG concentration, and the independent variable `age` is the age of each child. (Preliminary Box–Cox analysis shows that a square root transformation removes the skewness in IgG.)

The aim is to find a model that accurately predicts the mean of `sqrtigg` given `age`. We use `fp` to find the best FP2 model (the default option). We specify `center` for automatic centering. The age of each child is small in magnitude and positive, so we do not use the scaling options of `fp` or scale ourselves.

```
. use http://www.stata-press.com/data/r15/igg, clear
(Immunoglobulin in children)
. fp <age>, scale center: regress sqrtigg <age>
(fitting 44 models)
(....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%)
Fractional polynomial comparisons:
```

age	df	Deviance	Res. s.d.	Dev. dif.	P(*)	Powers
omitted	0	427.539	0.497	108.090	0.000	
linear	1	337.561	0.428	18.113	0.000	1
m = 1	2	327.436	0.421	7.987	0.020	0
m = 2	4	319.448	0.416	0.000	--	-2 2

(*) P = sig. level of model with m = 2 based on F with 293 denominator dof.

Source	SS	df	MS		
				Number of obs	= 298
				F(2, 295)	= 64.49
Model	22.2846976	2	11.1423488	Prob > F	= 0.0000
Residual	50.9676492	295	.172771692	R-squared	= 0.3042
				Adj R-squared	= 0.2995
Total	73.2523469	297	.246640898	Root MSE	= .41566

sqrtigg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age_1	-.1562156	.027416	-5.70	0.000	-.2101713	-.10226
age_2	.0148405	.0027767	5.34	0.000	.0093757	.0203052
_cons	2.283145	.0305739	74.68	0.000	2.222974	2.343315

The new variables created by `fp` contain the best-fitting fractional polynomial powers of `age`, as centered by `fp`. For example, `age_1` is centered by subtracting the mean of `age` raised to the power -2.

The variables created by `fp` and `fp generate` are centered or scaled as specified by the user, which is reflected in the estimated regression coefficients and intercept. Centering does have its advantages (see *Centering* earlier in this entry). By default, `fp` will not perform scaling or centering. For a more detailed discussion, see Royston and Sauerbrei (2008, sec. 4.11).

The fitted curve has an asymmetric S shape. The best model has powers $(-2, 2)$ and deviance 319.448. We reject lesser degree models: the null, linear, and natural log power models at the 0.05 level. As many as 44 models have been fit in the search for the best powers. Now let's look at models of degree ≤ 4. The highest allowed degree is specified in `dimension()`. We overwrite the previously generated fractional polynomial power variables by including `replace`.

```
. fp <age>, dimension(4) center replace: regress sqrtigg <age>
(fitting 494 models)
(....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%)
Fractional polynomial comparisons:
```

age	df	Deviance	Res. s.d.	Dev. dif.	P(*)	Powers
omitted	0	427.539	0.497	109.795	0.000	
linear	1	337.561	0.428	19.818	0.007	1
m = 1	2	327.436	0.421	9.692	0.149	0
m = 2	4	319.448	0.416	1.705	0.798	-2 2
m = 3	6	319.275	0.416	1.532	0.476	-2 1 1
m = 4	8	317.744	0.416	0.000	--	0 3 3 3

```
(*) P = sig. level of model with m = 4 based on F with 289
    denominator dof.
```

Source	SS	df	MS		
				Number of obs	= 298
				F(4, 293)	= 32.63
Model	22.5754541	4	5.64386353	Prob > F	= 0.0000
Residual	50.6768927	293	.172958678	R-squared	= 0.3082
				Adj R-squared	= 0.2987
Total	73.2523469	297	.246640898	Root MSE	= .41588

sqrtigg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age_1	.8761824	.1898721	4.61	0.000	.5024962	1.249869
age_2	-.1922029	.0684934	-2.81	0.005	-.3270044	-.0574015
age_3	.2043794	.074947	2.73	0.007	.0568767	.3518821
age_4	-.0560067	.0212969	-2.63	0.009	-.097921	-.0140924
_cons	2.238735	.0482705	46.38	0.000	2.143734	2.333736

It appears that the FP4 model is not significantly different from the other fractional polynomial models (at the 0.05 level).

Let's compare the curve shape from the $m = 2$ model with that from a conventional quartic polynomial whose fit turns out to be significantly better than a cubic (not shown). We use the ability of `fp` both to generate the required powers of `age`, namely, $(1, 2, 3, 4)$ for the quartic and $(-2, 2)$ for the second-degree fractional polynomial, and to fit the model. The `fp()` option is used to specify the powers. We use `predict` to obtain the fitted values of each regression. We fit both models with `fp` and graph the resulting curves with `twoway scatter`.

```
. fp <age>, center fp(1 2 3 4) replace: regress sqrtigg <age>
-> regress sqrtigg age_1 age_2 age_3 age_4
```

Source	SS	df	MS	Number of obs	=	298
				F(4, 293)	=	32.65
Model	22.5835458	4	5.64588646	Prob > F	=	0.0000
Residual	50.668801	293	.172931061	R-squared	=	0.3083
				Adj R-squared	=	0.2989
Total	73.2523469	297	.246640898	Root MSE	=	.41585

sqrtigg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age_1	2.047831	.4595962	4.46	0.000	1.143302	2.952359
age_2	-1.058902	.2822803	-3.75	0.000	-1.614456	-.5033479
age_3	.2284917	.0667591	3.42	0.001	.0971037	.3598798
age_4	-.0168534	.0053321	-3.16	0.002	-.0273475	-.0063594
_cons	2.240012	.0480157	46.65	0.000	2.145512	2.334511

```
. predict fit1
(option xb assumed; fitted values)
. label variable fit1 "Quartic"
. fp <age>, center fp(-2 2) replace: regress sqrtigg <age>
-> regress sqrtigg age_1 age_2
```

Source	SS	df	MS	Number of obs	=	298
				F(2, 295)	=	64.49
Model	22.2846976	2	11.1423488	Prob > F	=	0.0000
Residual	50.9676492	295	.172771692	R-squared	=	0.3042
				Adj R-squared	=	0.2995
Total	73.2523469	297	.246640898	Root MSE	=	.41566

sqrtigg	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age_1	-.1562156	.027416	-5.70	0.000	-.2101713	-.10226
age_2	.0148405	.0027767	5.34	0.000	.0093757	.0203052
_cons	2.283145	.0305739	74.68	0.000	2.222974	2.343315

```
. predict fit2
(option xb assumed; fitted values)
. label variable fit2 "FP 2"
. scatter sqrtigg fit1 fit2 age, c(. l l) m(o i i) msize(small)
> lpattern(. -_.) ytitle("Square root of IgG") xtitle("Age, years")
```

The quartic curve has an unsatisfactory wavy appearance that is implausible for the known behavior of IgG, the serum level of which increases throughout early life. The fractional polynomial curve (FP2) increases monotonically and is therefore biologically the more plausible curve. The two models have approximately the same deviance.

◁

▷ Example 2: Cox regression

Data from Smith et al. (1992) contain times to complete healing of leg ulcers in a randomized, controlled clinical trial of two treatments in 192 elderly patients. Several covariates were available, of which an important one is `mthson`, the number of months since the recorded onset of the ulcer. This time is recorded in whole months, not fractions of a month; therefore, some zero values are recorded.

Because the response variable is time to an event of interest and some (in fact, about one-half) of the times are censored, using Cox regression to analyze the data is appropriate. We consider fractional polynomials in `mthson`, adjusting for four other covariates: `age`; `ulcarea`, the area of tissue initially affected by the ulcer; `deepppg`, a binary variable indicating the presence or absence of deep vein involvement; and `treat`, a binary variable indicating treatment type.

We fit fractional polynomials of degrees 1 and 2 with `fp`. We specify `scale` to perform automatic scaling on `mthson`. This makes it positive and ensures that its magnitude is not too large. (See *Scaling* for more details.) The display option `nohr` is specified before the colon so that the coefficients and not the hazard ratios are displayed.

The `center` option is specified to obtain automatic centering. `age` and `ulcarea` are also demeaned by using `summarize` and then subtracting the returned result `r(mean)`.

In Cox regression, there is no constant term, so we cannot see the effects of centering in the table of regression estimates. The effects would be present if we were to graph the baseline hazard or survival function because these functions are defined with all predictors set equal to 0.

In these graphs, we will see the estimated baseline hazard or survival function under no deep vein involvement or treatment and under mean age, ulcer area, and number of months since the recorded onset of the ulcer.

```
. use http://www.stata-press.com/data/r15/legulcer1, clear
(Leg ulcer clinical trial)
. stset ttevent, fail(cens)

     failure event:  censored != 0 & censored < .
obs. time interval:  (0, ttevent]
 exit on or before:  failure

------------------------------------------------------------------------------
        192  total observations
          0  exclusions
------------------------------------------------------------------------------
        192  observations remaining, representing
         92  failures in single-record/single-failure data
     13,825  total analysis time at risk and under observation
                                             at risk from t =         0
                                  earliest observed entry t =         0
                                       last observed exit t =       206

. quietly sum age
. replace age = age - r(mean)
variable age was byte now float
(192 real changes made)
. quietly sum ulcarea
. replace ulcarea = ulcarea - r(mean)
variable ulcarea was int now float
(192 real changes made)
. fp <mthson>, center scale nohr: stcox <mthson> age ulcarea deepppg treat
(fitting 44 models)
(....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%)
Fractional polynomial comparisons:
```

mthson	df	Deviance	Dev. dif.	P(*)	Powers
omitted	0	754.345	17.636	0.001	
linear	1	751.680	14.971	0.002	1
m = 1	2	738.969	2.260	0.323	-.5
m = 2	4	736.709	0.000	--	.5 .5

```
(*) P = sig. level of model with m = 2 based on chi^2 of dev. dif.
Cox regression -- Breslow method for ties
No. of subjects =          192                 Number of obs    =         192
No. of failures =           92
Time at risk    =        13825
                                               LR chi2(6)       =      108.59
Log likelihood  =   -368.35446                 Prob > chi2      =      0.0000
```

_t	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
mthson_1	-2.81425	.6996385	-4.02	0.000	-4.185516	-1.442984
mthson_2	1.541451	.4703143	3.28	0.001	.6196521	2.46325
age	-.0261111	.0087983	-2.97	0.003	-.0433556	-.0088667
ulcarea	-.0017491	.000359	-4.87	0.000	-.0024527	-.0010455
deepppg	-.5850499	.2163173	-2.70	0.007	-1.009024	-.1610758
treat	-.1624663	.2171048	-0.75	0.454	-.5879838	.2630513

The best-fitting fractional polynomial of degree 2 has powers $(0.5, 0.5)$ and deviance 736.709. However, this model does not fit significantly better than the fractional polynomial of degree 1 (at the 0.05 level), which has power -0.5 and deviance 738.969. We prefer the model with $m = 1$.

```
. fp <mthson>, replace center scale nohr fp(-.5): stcox <mthson> age ulcarea
> deepppg treat
-> stcox mthson_1 age ulcarea deepppg treat
Cox regression -- Breslow method for ties
No. of subjects =          192                 Number of obs    =         192
No. of failures =           92
Time at risk    =        13825
                                               LR chi2(5)       =      106.33
Log likelihood  =   -369.48426                 Prob > chi2      =      0.0000
```

_t	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
mthson_1	.1985592	.0493922	4.02	0.000	.1017523	.2953662
age	-.02691	.0087875	-3.06	0.002	-.0441331	-.0096868
ulcarea	-.0017416	.0003482	-5.00	0.000	-.0024241	-.0010591
deepppg	-.5740759	.2185134	-2.63	0.009	-1.002354	-.1457975
treat	-.1798575	.2175726	-0.83	0.408	-.6062921	.246577

The hazard for healing is much higher for patients whose ulcer is of recent onset than for those who have had an ulcer for many months.

A more appropriate analysis of this dataset, if one wanted to model all the predictors, possibly with fractional polynomial functions, would be to use `mfp`; see [R] **mfp**.

◁

▷ Example 3: Logistic regression

The `zero` option permits fitting a fractional polynomial model to the positive values of a covariate, taking nonpositive values as zero. An application is the assessment of the effect of cigarette smoking as a risk factor. Whitehall 1 is an epidemiological study, which was examined in Royston and Sauerbrei (2008), of 18,403 male British Civil Servants employed in London. We examine the data collected in Whitehall 1 and use logistic regression to model the odds of death based on a fractional polynomial in the number of cigarettes smoked.

Nonsmokers may be qualitatively different from smokers, so the effect of smoking (regarded as a continuous variable) may not be continuous between zero cigarettes and one cigarette. To allow for this possibility, we model the risk as a constant for the nonsmokers and as a fractional polynomial function of the number of cigarettes for the smokers, adjusted for age.

The dependent variable `all10` is an indicator of whether the individual passed away in the 10 years under study. `cigs` is the number of cigarettes consumed per day. After loading the data, we demean `age` and create a dummy variable, `nonsmoker`. We then use `fp` to fit the model.

```
. use http://www.stata-press.com/data/r15/smoking, clear
(Smoking and mortality data)
. quietly sum age
. replace age = age - r(mean)
variable age was byte now float
(17,260 real changes made)
. generate byte nonsmoker = cond(cigs==0, 1, 0) if cigs < .
. fp <cigs>, zero: logit all10 <cigs> nonsmoker age
(fitting 44 models)
(....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%)
Fractional polynomial comparisons:
```

cigs	df	Deviance	Dev. dif.	P(*)	Powers
omitted	0	9990.804	46.096	0.000	
linear	1	9958.801	14.093	0.003	1
m = 1	2	9946.603	1.895	0.388	0
m = 2	4	9944.708	0.000	--	-1 -1

```
(*) P = sig. level of model with m = 2 based on chi^2 of dev. dif.
Logistic regression                             Number of obs     =     17,260
                                                LR chi2(4)        =    1029.03
                                                Prob > chi2       =     0.0000
Log likelihood = -4972.3539                     Pseudo R2         =     0.0938
```

all10	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
cigs_1	-1.285867	.3358483	-3.83	0.000	-1.944117	-.6276162
cigs_2	-1.982424	.572109	-3.47	0.001	-3.103736	-.8611106
nonsmoker	-1.223749	.1119583	-10.93	0.000	-1.443183	-1.004315
age	.1194541	.0045818	26.07	0.000	.1104739	.1284343
_cons	-1.591489	.1052078	-15.13	0.000	-1.797693	-1.385286

Omission of the `zero` option would cause `fp` to halt with an error message because nonpositive covariate values (for example, values of `cigs`) are invalid unless the `scale` option is specified.

A closely related approach involves the `catzero` option. Here we no longer need to have `nonsmoker` in the model, because `fp` creates its own dummy variable `cigs_0` to indicate whether the individual does not smoke on that day.

```
. fp <cigs>, catzero replace: logit all10 <cigs> age
(fitting 44 models)
(....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%)
Fractional polynomial comparisons:
```

cigs	df	Deviance	Dev. dif.	P(*)	Powers
omitted	0	10175.75	231.047	0.000	
linear	2	9958.80	14.093	0.003	1
m = 1	3	9946.60	1.895	0.388	0
m = 2	5	9944.71	0.000	--	-1 -1

```
(*) P = sig. level of model with m = 2 based on chi^2 of dev. dif.
Logistic regression                               Number of obs     =     17,260
                                                  LR chi2(4)        =    1029.03
                                                  Prob > chi2       =     0.0000
Log likelihood = -4972.3539                       Pseudo R2         =     0.0938
```

all10	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
cigs_0	-1.223749	.1119583	-10.93	0.000	-1.443183	-1.004315
cigs_1	-1.285867	.3358483	-3.83	0.000	-1.944117	-.6276162
cigs_2	-1.982424	.572109	-3.47	0.001	-3.103736	-.8611106
age	.1194541	.0045818	26.07	0.000	.1104739	.1284343
_cons	-1.591489	.1052078	-15.13	0.000	-1.797693	-1.385286

Under both approaches, the comparison table suggests that we can accept the FP1 model instead of the FP2 model. We estimate the parameters of the accepted model—that is, the one that uses the natural logarithm of `cigs`—with `fp`.

```
. fp <cigs>, catzero replace fp(0): logit all10 <cigs> age
-> logit all10 cigs_0 cigs_1 age
Logistic regression                               Number of obs     =     17,260
                                                  LR chi2(3)        =    1027.13
                                                  Prob > chi2       =     0.0000
Log likelihood = -4973.3016                       Pseudo R2         =     0.0936
```

all10	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
cigs_0	.1883732	.1553093	1.21	0.225	-.1160274	.4927738
cigs_1	.3469842	.0543552	6.38	0.000	.2404499	.4535185
age	.1194976	.0045818	26.08	0.000	.1105174	.1284778
_cons	-3.003767	.1514909	-19.83	0.000	-3.300683	-2.70685

The high p-value for `cigs_0` in the output indicates that we cannot reject that there is no extra effect at zero for nonsmokers.

◁

Stored results

In addition to the results that *est_cmd* stores, `fp` stores the following in `e()`:

Scalars

`e(fp_dimension)`	degree of fractional polynomial
`e(fp_center_mean)`	value used for centering or .
`e(fp_scale_a)`	value used for scaling or .
`e(fp_scale_b)`	value used for scaling or .
`e(fp_compare_df2)`	denominator degree of freedom in F test

Macros

`e(fp_cmd)`	`fp, search():` or `fp, powers():`
`e(fp_cmdline)`	full `fp` command as typed
`e(fp_variable)`	fractional polynomial variable
`e(fp_terms)`	generated `fp` variables
`e(fp_gen_cmdline)`	`fp generate` command to re-create `e(fp_terms)` variables
`e(fp_catzero)`	`catzero`, if specified
`e(fp_zero)`	`zero`, if specified
`e(fp_compare_type)`	`F` or `chi2`

Matrices

`e(fp_fp)`	powers used in fractional polynomial
`e(fp_compare)`	results of model comparisons
`e(fp_compare_stat)`	F test statistics
`e(fp_compare_df1)`	numerator degree of F test
`e(fp_compare_fp)`	powers of comparison models
`e(fp_compare_length)`	encoded string for display of row titles
`e(fp_powers)`	powers that are searched

`fp generate` stores the following in `r()`:

Scalars

`r(fp_center_mean)`	value used for centering or .
`r(fp_scale_a)`	value used for scaling or .
`r(fp_scale_b)`	value used for scaling or .

Macros

`r(fp_cmdline)`	full `fp generate` command as typed
`r(fp_variable)`	fractional polynomial variable
`r(fp_terms)`	generated `fp` variables
`r(fp_catzero)`	`catzero`, if specified
`r(fp_zero)`	`zero`, if specified

Matrices

`r(fp_fp)`	powers used in fractional polynomial

Methods and formulas

The general definition of a fractional polynomial, accommodating possible repeated powers, may be written for functions $H_1(x), \ldots, H_m(x)$ of $x > 0$ as

$$\beta_0 + \sum_{j=1}^{m} \beta_j H_j(x)$$

where $H_1(x) = x^{(p_1)}$ and for $j = 2, \ldots, m$,

$$H_j(x) = \begin{cases} x^{(p_j)} & \text{if } p_j \neq p_{j-1} \\ H_{j-1}(x)\ln(x) & \text{if } p_j = p_{j-1} \end{cases}$$

For example, a fractional polynomial of degree 3 with powers $(1, 3, 3)$ has $H_1(x) = x$, $H_2(x) = x^3$, and $H_3(x) = x^3 \ln(x)$ and equals $\beta_0 + \beta_1 x + \beta_2 x^3 + \beta_3 x^3 \ln(x)$.

We can express a fractional polynomial in vector notation by using $\mathbf{H}(x) = [H_1(x), \ldots, H_d(x)]'$. We define $x^{(p_1,p_2,\ldots,p_m)} = [\mathbf{H}(x)', 1]'$. Under this notation, we can write

$$x^{(1,3,3)\prime}\boldsymbol{\beta} = \beta_0 + \beta_1 x + \beta_2 x^3 + \beta_3 x^3 \ln(x)$$

The fractional polynomial may be centered so that the intercept can be more easily interpreted. When centering the fractional polynomial of x at c, we subtract $c^{(p_1,p_2,\ldots,p_m)}$ from $x^{(p_1,p_2,\ldots,p_m)}$, where $c^{(p_1,p_2,\ldots,p_d)} = [\mathbf{H}(x)', 0]'$. The centered fractional polynomial is

$$\left(x^{(p_1,\ldots,p_d)} - c^{(p_1,\ldots,p_d)}\right)'\boldsymbol{\beta}$$

The definition may be extended to allow $x \leq 0$ values. For these values, the fractional polynomial is equal to the intercept β_0 or equal to a zero-offset term α_0 plus the intercept β_0.

A fractional polynomial model of degree m is taken to have $2m + 1$ degrees of freedom (df): one for β_0 and one for each β_j and its associated power. Because the powers in a fractional polynomial are chosen from a finite set rather than from the entire real line, the df defined in this way are approximate.

The deviance D of a model is defined as -2 times its maximized log likelihood. For normal-errors models, we use the formula

$$D = n\left(1 - \bar{l} + \ln\frac{2\pi\text{RSS}}{n}\right)$$

where n is the sample size, $\bar{l}$ is the mean of the lognormalized weights ($\bar{l} = 0$ if the weights are all equal), and RSS is the residual sum of squares as fit by `regress`.

`fp` reports a table comparing fractional polynomial models of degree $k < m$ with the degree m fractional polynomial model, which will have the lowest deviance.

The p-values reported by `fp` are calculated differently for normal and nonnormal regressions. Let D_k and D_m be the deviances of the models with degrees k and m, respectively. For normal-errors models, a variance ratio F is calculated as

$$F = \frac{n_2}{n_1}\left\{\exp\left(\frac{D_k - D_m}{n}\right) - 1\right\}$$

where n_1 is the numerator df, the quantity of the additional parameters that the degree m model has over the degree k model. n_2 is the denominator df and equals the residual degrees of freedom of the degree m model, minus the number of powers estimated, m. The p-value is obtained by referring F to an F distribution on (n_1, n_2) df.

For nonnormal models, the p-value is obtained by referring $D_k - D_m$ to a χ^2 distribution on $2m - 2k$ df. These p-values for comparing models are approximate and are typically somewhat conservative (Royston and Altman 1994).

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Also see

[R] **fp postestimation** — Postestimation tools for fp

[R] **mfp** — Multivariable fractional polynomial models

[U] **20 Estimation and postestimation commands**