contrast — Contrasts and linear hypothesis tests after estimation							
Description	Quick start	Menu	Syntax	Options			
Remarks and examples	Stored results	Methods and formulas	References	Also see			

# Description

contrast tests linear hypotheses and forms contrasts involving factor variables and their interactions from the most recently fit model. The tests include ANOVA-style tests of main effects, simple effects, interactions, and nested effects. contrast can use named contrasts to decompose these effects into comparisons against reference categories, comparisons of adjacent levels, comparisons against the grand mean, orthogonal polynomials, and such. Custom contrasts may also be specified.

contrast can be used with svy estimation results; see [SVY] svy postestimation.

Contrasts can also be computed for margins of linear and nonlinear responses; see [R] margins, contrast.

# Quick start

Contrasts for one-way models

Test the main effect of categorical variable a after regress y i.a or anova y a

contrast a

Reference category contrasts of cell means of y with the smallest value of a as the base category contrast r.a

Same as above, but specify a = 3 as the base category for comparisons contrast rb3.a

Report tests instead of confidence intervals for each contrast

```
contrast r.a, pveffects
```

Report tests and confidence intervals for each contrast

```
contrast r.a, effects
```

Contrasts of the cell mean of y for each level of a with the grand mean of y

contrast g.a

Same as above, but compute grand mean as a weighted average of cell means with weights based on the number of observations for each level of a

contrast gw.a

User-defined contrast comparing the cell mean of y for a = 1 with the average of the cell means for a = 3 and a = 4

contrast {a -1 0 .5 .5}

Contrasts for two-way models
Test of the interaction effect after regress y a##b or anova y a##b contrast a#b
Test of the main and interaction effects contrast a b a#b
Same as above contrast a##b
Individual reference category contrasts for the interaction of a and b contrast r.a#r.b
Joint tests of the simple effects of a within each level of b contrast a@b
Individual reference category contrasts for the simple effects of a within each level of b contrast r.a@b
Orthogonal polynomial contrasts for a within each level of b contrast p.a@b
Reference category contrasts of the marginal means of y for levels of a contrast r.a
Same as above, but with marginal means for a computed as a weighted average of cell means, using the marginal frequencies of b rather than equal weights for each level contrast r.a, asobserved
Contrasts of the marginal mean of y for each level of a with the previous level—reverse-adjacent con- trasts
contrast ar.a
Contrasts for models with continuous covariates
Test of the interaction effect after regress y a##c.x or anova y a##c.x contrast a#c.x
Reference category effects of a on the slope of x contrast r.a#c.x

Reference category effects of a on the intercept

contrast r.a

Contrasts for nonlinear models
Orthogonal polynomial contrasts of log odds across levels of a after logit y i.a contrast p.a
Test the main and interaction effects after logit y a##b contrast a##b
Simple reference category effects for a within each level of b contrast r.a@b
Contrasts for multiple-equation models
Test the main and interaction effects in the equation for y2 after mvreg y1 y2 y3 = a##b contrast a##b, equation(y2)
Reference category contrasts of estimated marginal means of y3 for levels of a contrastr.a, equation(y3)
Test for a difference in the overall estimated marginal means of y1, y2, and y3 contrast _eqns
Contrasts of estimated marginal means of y2 and y3 with y1 contrast reqns
Test whether interaction effects differ across equations contrast a#b#_eqns

## Menu

Statistics > Postestimation

## **Syntax**

contrast termlist [, options]

where *termlist* is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without contrast operators, and you may use any factor-variable syntax:

See the *operators (op.)* table below for the list of contrast operators.

options	Description
Main	
<u>over</u> all	add a joint hypothesis test for all specified contrasts
<u>asobs</u> erved	treat all factor variables as observed
lincom	treat user-defined contrasts as linear combinations
Equations	
equation(eqspec)	perform contrasts in <i>termlist</i> for equation <i>eqspec</i>
atequations	perform contrasts in termlist within each equation
Advanced	
<pre>emptycells(empspec)</pre>	treatment of empty cells for balanced factors
noestimcheck	suppress estimability checks
Reporting	
<u>l</u> evel(#)	confidence level; default is level(95)
<pre>mcompare(method)</pre>	adjust for multiple comparisons; default is mcompare(noadjust)
noeffects	suppress table of individual contrasts
<u>ci</u> effects	show effects table with confidence intervals
pveffects	show effects table with <i>p</i> -values
effects	show effects table with confidence intervals and <i>p</i> -values
nowald	suppress table of Wald tests
<u>noatlev</u> els	report only the overall Wald test for terms that use the within @ or nested   operator
nosvyadjust	compute unadjusted Wald tests for survey results
sort	sort the individual contrast values in each term
post	post contrasts and their VCEs as estimation results
display_options	control column formats, row spacing, line width, and factor-variable labeling
eform_option	report exponentiated contrasts
df(#)	use $t$ distribution with # degrees of freedom for computing $p$ -values and confidence intervals

collect is allowed; see [U] **11.1.10 Prefix commands**. df (#) does not appear in the dialog box.

Term	Description
Main effects	
Α	joint test of the main effects of A
r.A	individual contrasts that decompose A using $r$ .
Interaction effects	
A#B	joint test of the two-way interaction effects of A and B
A#B#C	joint test of the three-way interaction effects of A, B, and C
<i>r</i> .A# <i>g</i> .B	individual contrasts for each interaction of A and B defined by $r$ . and $g$ .
Partial interaction effect	S
<i>r</i> .A#B	joint tests of interactions of A and B within each contrast defined by $r$ . A
A#r.B	joint tests of interactions of A and B within each contrast defined by $r.B$
Simple effects	
A@B	joint tests of the effects of A within each level of B
A@B#C	joint tests of the effects of A within each combination of the levels of B and C
r.A@B	individual contrasts of A that decompose A@B using $r$ .
r.A@B#C	individual contrasts of A that decompose A@B#C using $r$ .
Other conditional effects	s
A#B@C	joint tests of the interaction effects of A and B within each level of C
A#B@C#D	joint tests of the interaction effects of A and B within each combination of the levels of C and D
r.A#g.B@C	individual contrasts for each interaction of A and B that decompose A#B@C using $r$ . and $g$ .
Nested effects	
A B	joint tests of the effects of A nested in each level of B
A B#C	joint tests of the effects of A nested in each combination of the levels of B and C
A#B C	joint tests of the interaction effects of A and B nested in each level of C
A#B C#D	joint tests of the interaction effects of A and B nested in each
	combination of the levels of C and D
<i>r</i> .A B	individual contrasts of A that decompose $A \mid B$ using $r$ .
r.A B#C	individual contrasts of A that decompose A   $B$ #C using $r$ .
r.A#g.B C	individual contrasts for each interaction of A and B defined by $r$ . and $g$ .
	nested in each level of C
Slope effects	
A#c.x	joint test of the effects of A on the slopes of x
A#c.x#c.y	joint test of the effects of A on the slopes of the product (interaction) of x and y
A#B#c.x	joint test of the interaction effects of A and B on the slopes of x
A#B#c.x#c.y	joint test of the interaction effects of A and B on the slopes of the product (interaction) of x and y
r.A#c.x	individual contrasts of A's effects on the slopes of $x$ using $r$ .
Denominators	
/ term2	use $term2$ as the denominator in the $F$ tests of the preceding terms
/	use the residual as the denominator in the $F$ tests of the preceding terms (the default if no other /s are specified)

A, B, C, and D represent any factor variable in the current estimation results.

x and y represent any continuous variable in the current estimation results.

r. and g. represent any contrast operator. See the table below.

c. specifies that a variable be treated as continuous; see [U] 11.4.3 Factor variables.

Operators are allowed on any factor variable that does not appear to the right of @ or |. Operators decompose the effects of the associated factor variable into one-degree-of-freedom effects (contrasts).

Higher-level interactions are allowed anywhere an interaction operator (#) appears in the table.

Time-series operators are allowed if they were used in the estimation.

\_eqns designates the equations in manova, mlogit, mprobit, and mvreg and can be specified anywhere a factor variable appears.

/ is allowed only after anova, cnsreg, manova, mvreg, or regress.

operators (op.)	Description
r.	differences from the reference (base) level; the default
a.	differences from the next level (adjacent contrasts)
ar.	differences from the previous level (reverse adjacent contrasts)
As-balanced operators	
g.	differences from the balanced grand mean
h.	differences from the balanced mean of subsequent levels (Helmert contrasts)
j.	differences from the balanced mean of previous levels (reverse Helmert contrasts)
р.	orthogonal polynomial in the level values
q.	orthogonal polynomial in the level sequence
As-observed operators	
gw.	differences from the observation-weighted grand mean
hw.	differences from the observation-weighted mean of subsequent levels
jw.	differences from the observation-weighted mean of previous levels
pw.	observation-weighted orthogonal polynomial in the level values
qw.	observation-weighted orthogonal polynomial in the level sequence

One or more individual contrasts may be selected by using the op#. or op(numlist). syntax. For example, a3. A selects the adjacent contrast for level 3 of A, and p(1/2). B selects the linear and quadratic effects of B. Also see Orthogonal polynomial contrasts and Beyond linear models.

Custom contrasts	Description
{A numlist}	user-defined contrast on the levels of factor A
{A#B numlist}	user-defined contrast on the levels of interaction between A and B

Custom contrasts may be part of a term, such as {A numlist}#B, {A numlist}@B, {A numlist}|B, {A#B numlist}, and {A numlist}#{B numlist}. The same is true of higher-order custom contrasts, such as {A#B numlist}@C, {A#B numlist}#r.C, and {A#B numlist}#c.x.

Higher-order interactions with at most eight factor variables are allowed with custom contrasts.

method	Description
noadjust	do not adjust for multiple comparisons; the default
bonferroni [adjustall]	Bonferroni's method; adjust across all terms
<u>sid</u> ak [adjustall]	Šidák's method; adjust across all terms
<u>sch</u> effe	Scheffé's method

# Options

ſ	Main	J

overall specifies that a joint hypothesis test over all terms be performed.

- asobserved specifies that factor covariates be evaluated using the cell frequencies observed in the estimation sample. The default is to treat all factor covariates as though there were an equal number of observations in each level.
- lincom specifies that user-defined contrasts be treated as linear combinations. The default is to require that all user-defined contrasts sum to zero. (Summing to zero is part of the definition of a contrast.)

Equations

equation(*eqspec*) specifies the equation from which contrasts are to be computed. The default is to compute contrasts from the first equation.

atequations specifies that the contrasts be computed within each equation.

Advanced

- emptycells(*empspec*) specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced.
  - emptycells(strict) is the default; it specifies that contrasts involving empty cells be treated as not estimable.
  - emptycells(reweight) specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the contrast estimable but changes its interpretation.
- noestimcheck specifies that contrast not check for estimability. By default, the requested contrasts are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If noestimcheck is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations.

Reporting

- level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.
- mcompare(*method*) specifies the method for computing *p*-values and confidence intervals that account for multiple comparisons within a factor-variable term.

Most methods adjust the comparisonwise error rate,  $\alpha_c$ , to achieve a prespecified experimentwise error rate,  $\alpha_e$ .

mcompare(noadjust) is the default; it specifies no adjustment.

$$\alpha_c=\alpha_e$$

mcompare(bonferroni) adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality

 $\alpha_e \leq m \alpha_c$ 

where m is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = \alpha_e/m$$

mcompare(sidak) adjusts the comparisonwise error rate based on the upper limit of the probability
inequality

$$\alpha_e \le 1 - (1 - \alpha_c)^m$$

where m is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = 1 - (1 - \alpha_e)^{1/m}$$

This adjustment is exact when the m comparisons are independent.

- mcompare(scheffe) controls the experimentwise error rate using the F or  $\chi^2$  distribution with degrees of freedom equal to the rank of the term.
- mcompare(method adjustall) specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in marginslist. This option is compatible only with the bonferroni and sidak methods.
- noeffects suppresses the table of individual contrasts with confidence intervals. This table is produced by default when the mcompare() option is specified or when a term in *termlist* implies all individual contrasts.
- cieffects specifies that a table containing a confidence interval for each individual contrast be reported.
- pveffects specifies that a table containing a p-value for each individual contrast be reported.
- effects specifies that a single table containing a confidence interval and *p*-value for each individual contrast be reported.
- nowald suppresses the table of Wald tests.
- noatlevels indicates that only the overall Wald test be reported for each term containing within or nested (@ or |) operators.
- nosvyadjust is for use with svy estimation commands. It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is to say the test is carried out as  $W/k \sim F(k, d)$  rather than as  $(d k + 1)W/(kd) \sim F(k, d k + 1)$ , where k is the dimension of the test and d is the total number of sampled PSUs minus the total number of strata.
- sort specifies that the table of individual contrasts be sorted by the contrast values within each term.

- post causes contrast to behave like a Stata estimation (e-class) command. contrast posts the vector of estimated contrasts along with the estimated variance-covariance matrix to e(), so you can treat the estimated contrasts just as you would results from any other estimation command. For example, you could use test to perform simultaneous tests of hypotheses on the contrasts, or you could use lincom to create linear combinations.
- - vsquish specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.
  - nofvlabel displays factor-variable level values rather than attached value labels. This option overrides the fvlabel setting; see [R] set showbaselevels.
  - fvwrap(#) specifies how many lines to allow when long value labels must be wrapped. Labels
    requiring more than # lines are truncated. This option overrides the fvwrap setting; see [R] set
    showbaselevels.
  - fvwrapon(*style*) specifies whether value labels that wrap will break at word boundaries or break based on available space.

fvwrapon(word), the default, specifies that value labels break at word boundaries.

fvwrapon(width) specifies that value labels break based on available space.

This option overrides the fvwrapon setting; see [R] set showbaselevels.

- cformat (% fint) specifies how to format contrasts, standard errors, and confidence limits in the table of estimated contrasts.
- pformat (% fmt) specifies how to format p-values in the table of estimated contrasts.
- sformat (% fmt) specifies how to format test statistics in the table of estimated contrasts.
- nolstretch specifies that the width of the table of estimated contrasts not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically widen the table of estimated contrasts up to the width of the Results window. Specifying lstretch or nolstretch overrides the setting given by set lstretch. If set lstretch has not been set, the default is lstretch. nolstretch is not shown in the dialog box.
- *eform\_option* specifies that the contrasts table be displayed in exponentiated form.  $e^{\text{contrast}}$  is displayed rather than contrast. Standard errors and confidence intervals are also transformed. See [R] *eform\_option* for the list of available options.

The following option is available with contrast but is not shown in the dialog box:

df(#) specifies that the t distribution with # degrees of freedom be used for computing p-values and confidence intervals. The default is to use e(df\_r) degrees of freedom or the standard normal distribution if e(df\_r) is missing.

## **Remarks and examples**

Remarks are presented under the following headings:

Introduction One-way models Estimated cell means Testing equality of cell means Reference category contrasts Reverse adjacent contrasts Orthogonal polynomial contrasts Two-way models Estimated interaction cell means Simple effects Interaction effects Main effects Partial interaction effects Three-way and higher-order models Contrast operators Differences from a reference level (r.) Differences from the next level (a.) Differences from the previous level (ar.) Differences from the grand mean (g.) Differences from the mean of subsequent levels (h.) Differences from the mean of previous levels (j.) Orthogonal polynomials (p. and q.) User-defined contrasts Empty cells Empty cells, ANOVA style Nested effects Multiple comparisons Unbalanced data Using observed cell frequencies Weighted contrast operators Testing factor effects on slopes Chow tests Beyond linear models Multiple equations Video example

#### Introduction

contrast performs ANOVA-style tests of main effects, interactions, simple effects, and nested effects. It can easily decompose these tests into constituent contrasts using either named contrasts (codings) or user-specified contrasts. Comparing levels of factor variables—whether as main effects, interactions, or simple effects—is as easy as adding a contrast operator to the variable. The operators can compare each level with the previous level, each level with a reference level, each level with the mean of previous levels, and more.

contrast tests and estimates contrasts. A contrast of the parameters  $\mu_1, \mu_2, \ldots, \mu_p$  is a linear combination  $\sum_i c_i \mu_i$  whose  $c_i$  sum to zero. A difference of population means such as  $\mu_1 - \mu_2$  is a contrast, as are most other comparisons of population or model quantities (Coster 2005). Some contrasts may be estimated with lincom, but contrast is much more powerful. contrast can handle multiple contrasts simultaneously, and the command's contrast operators make it easy to specify complicated linear combinations.

Both the contrast operation and the creation of the margins for comparison can be performed as though the data were balanced (typical for experimental designs) or using the observed frequencies in the estimation sample (typical for observational studies). contrast can perform these analyses on the results of almost all of Stata's estimators, not just the linear-models estimators.

Most of contrast's computations can be considered comparisons of estimated cell means from a model fit. Tests of interactions are tests of whether the cell means for the interaction are all equal. Tests of main effects are tests of whether the marginal cell means for the factor are all equal. More focused comparisons of cell means (for example, is level 2 equal to level 1) are specified using contrast operators. More formally, all of contrast's computations are comparisons of conditional expectations; cell means are one type of conditional expectation.

All contrasts can also easily be graphed; see [R] marginsplot.

For a discussion of contrasts and testing for linear models, see Searle and Gruber (2017) and Searle (1997). For discussions specifically related to experimental design, see Winer, Brown, and Michels (1991) and Milliken and Johnson (2009). Rosenthal, Rosnow, and Rubin (2000) focus on contrasts with applications in behavioral sciences. Mitchell (2021, 2015) and Baldwin (2019) focus on contrasts in Stata.

contrast is a flexible tool for understanding the effects of categorical covariates. If your model contains categorical covariates, and especially if it contains interactions, you will want to use contrast.

#### One-way models

Suppose we have collected data on cholesterol levels for individuals from five age groups. To study the effect of age group on cholesterol, we can begin by fitting a one-way model using regress:

```
. use https://www.stata-press.com/data/r19/cholesterol
(Artificial cholesterol data)
. label list ages
ages:
           1 10-19
           2 20-29
           3 30-39
           4 40-59
           5 60-79
 regress chol i.agegrp
      Source
                      SS
                                    df
                                              MS
                                                      Number of obs
                                                                        =
                                                                                 75
                                                      F(4, 70)
                                                                              35.02
                                                                        =
       Model
                 14943.3997
                                     4
                                        3735.84993
                                                      Prob > F
                                                                        =
                                                                             0.0000
    Residual
                 7468.21971
                                    70
                                        106.688853
                                                      R-squared
                                                                        =
                                                                             0.6668
                                                       Adj R-squared
                                                                        =
                                                                             0.6477
       Total
                 22411.6194
                                    74
                                        302.859722
                                                      Root MSE
                                                                        =
                                                                             10.329
        chol
                Coefficient
                              Std. err.
                                                   P>|t|
                                                              [95% conf. interval]
                                              t.
      agegrp
      20-29
                  8.203575
                              3.771628
                                            2.18
                                                   0.033
                                                              .6812991
                                                                           15.72585
      30-39
                  21.54105
                              3.771628
                                            5.71
                                                   0.000
                                                              14.01878
                                                                           29.06333
                                                                           37.67295
      40 - 59
                  30.15067
                              3.771628
                                            7.99
                                                   0.000
                                                               22.6284
      60-79
                  38.76221
                              3.771628
                                           10.28
                                                   0.000
                                                              31.23993
                                                                           46.28448
                              2.666944
                                           67.69
                                                   0.000
                                                              175.2007
       _cons
                  180.5198
                                                                           185.8388
```

### Estimated cell means

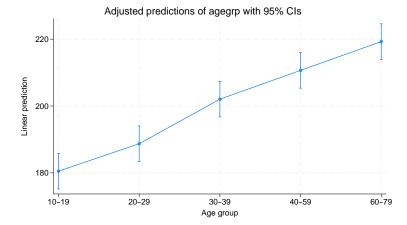
margins will show us the estimated cell means for each age group based on our fitted model:

. margins agegrp								
Adjusted predi Model VCE: OLS	Number c	of obs = 75						
Expression: Li	inear predict	ion, predict	()					
	I	Delta-method						
	Margin	std. err.	t	P> t	[95% conf.	interval]		
agegrp								
10-19	180.5198	2.666944	67.69	0.000	175.2007	185.8388		
20-29	188.7233	2.666944	70.76	0.000	183.4043	194.0424		
30-39	202.0608	2.666944	75.76	0.000	196.7418	207.3799		
40-59	210.6704	2.666944	78.99	0.000	205.3514	215.9895		
60-79	219.282	2.666944	82.22	0.000	213.9629	224.601		

We can graph those means with marginsplot:

```
. marginsplot
```

Variables that uniquely identify margins: agegrp



#### Testing equality of cell means

Are all the means equal? That is to say is there an effect of age group on cholesterol level? We can answer that by asking contrast to test whether the means of the age groups are identical.

```
. contrast agegrp
Contrasts of marginal linear predictions
Margins: asbalanced

df F P>F

agegrp 4 35.02 0.0000

Denominator 70
```

The means are clearly different. We could have obtained this same test directly had we fit our model using anova rather than regress.

Number of obs = Root MSE =		75 R-squa 10.329 Adj R-		red = squared =	
Source	Partial SS	df	MS	F	Prob>F
Model	14943.4	4	3735.8499	35.02	0.0000
agegrp	14943.4	4	3735.8499	35.02	0.0000
Residual	7468.2197	70	106.68885		
Total	22411.619	74	302.85972		

Achieving a more direct test result is why we recommend using anova instead of regress for models where our focus is on the categorical covariates. The models fit by anova and regress are identical; they merely parameterize the effects differently. The results of contrast will be identical regardless of which command is used to fit the model. If, however, we were fitting models whose responses are nonlinear functions of the covariates, such as logistic regression, then there would be no analogue to anova, and we would appreciate contrast's ability to quickly test main effects and interactions.

#### **Reference category contrasts**

. anova chol agegrp

Now that we know that the overall effect of age group is statistically significant, we can explore the effects of each age group. One way to do that is to use the reference category operator, r.:

```
. contrast r.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

			df	F	P>F	
age	grp					
(20-29 vs 10-19	<del>)</del> )		1	4.73	0.0330	
(30-39 vs 10-19	<del>)</del> )		1	32.62	0.0000	
(40-59 vs 10-19	<del>)</del> )		1	63.91	0.0000	
(60-79 vs 10-19	<del>)</del> )		1	105.62	0.0000	
Join	nt		4	35.02	0.0000	
Denomina	tor		70			
	0	Contrast	Std.	err.	[95% conf.	interval]
agegrp						
(20-29 vs 10-19)	8	3.203575	3.77	1628	.6812991	15.72585
(30-39 vs 10-19)		21.54105	3.77	1628	14.01878	29.06333
(40-59 vs 10-19)	3	30.15067	3.77	1628	22.6284	37.67295
(60-79 vs 10-19)	3	38.76221	3.77	1628	31.23993	46.28448
	1					

The cell mean of each age group is compared against the base age group (ages 10-19). The first table shows that each difference is significant. The second table gives an estimate and confidence interval for each contrast. These are the comparisons that linear regression gives with a factor covariate and no interactions. The contrasts are identical to the coefficients from our linear regression.

#### **Reverse adjacent contrasts**

. contrast ar.agegrp

We have far more flexibility with contrast. Age group is ordinal, so it is interesting to compare each age group with the preceding age group (rather than against one reference group). We specify that analysis by using the reverse adjacent operator, ar.:

Contrasts of margina Margins: asbalanced	l linear pr	edictio	ıs		
		df	F	P>F	
agegr			4 70	0.0000	
(20-29 vs 10-19)		1	4.73		
(30-39 vs 20-29)		1	12.51		
(40-59 vs 30-39)		1	5.21	0.0255	
(60-79 vs 40-59)		1	5.21	0.0255	
Joint	;	4	35.02	0.0000	
Denominato	or	70			
	Contrast	Std.	err.	[95% conf.	interval]
agegrp (20-29 vs 10-19)	8.203575			.6812991	15.72585
(30-39 vs 20-29)	13.33748			5.815204	
(40-59 vs 30-39)	8.60962			1.087345	16.1319
(60-79 vs 40-59)	8.611533	3.771	528	1.089257	16.13381

The 20–29 age group's cholesterol level is 8.2 points higher than the 10–19 age group's cholesterol level; the 30–39 age group's level is 13.3 points higher than the 20–29 age group's level; and so on. Each age group is statistically different from the preceding age group at the 5% level.

#### Orthogonal polynomial contrasts

The relationship between age group and cholesterol level looked almost linear in our graph. We can examine that relationship further by using the orthogonal polynomial operator, p.:

```
. contrast p.agegrp, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(linear)	1	139.11	0.0000
(quadratic)	1	0.15	0.6962
- (cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	4	35.02	0.0000
Denominator	70		

Only the linear effect is statistically significant.

We can even perform the joint test that all effects beyond linear are zero. We do that by selecting all polynomial contrasts above linear—that is, polynomial contrasts 2, 3, and 4.

P>F

```
. contrast p(2 3 4).agegrp, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced

df F

agegrp
```

agegrp			
(quadratic)	1	0.15	0.6962
(cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	3	0.32	0.8129
Denominator	70		

The joint test has three degrees of freedom and is clearly insignificant. A linear effect of age group seems adequate for this model.

### Two-way models

Suppose we are investigating the effects of different dosages of a blood pressure medication and believe that the effects may be different for men and women. We can fit the following ANOVA model for bpchange, the change in diastolic blood pressure. Change is defined as the after measurement minus the before measurement, so that negative values of bpchange correspond to decreases in blood pressure.

. anova bpchange dose##gender

-	Number of obs = Root MSE =	30 1.4677	1 1 1	red = squared =	0.9647 0.9573
Source	Partial SS	df	MS	F	Prob>F
Model	1411.9087	5	282.38174	131.09	0.0000
dose	963.48179	2	481.7409	223.64	0.0000
gender	355.11882	1	355.11882	164.85	0.0000
dose#gender	93.308093	2	46.654046	21.66	0.0000
Residual	51.699253	24	2.1541355		
Total	1463.608	29	50.46924		

#### Estimated interaction cell means

Everything is significant, including the interaction. So increasing dosage is effective and differs by gender. Let's explore the effects. First, let's look at the estimated cell mean of blood pressure change for each combination of gender and dosage.

. margins dose	e#gender					
Adjusted predictions					Number o	of obs = $30$
Expression: Li						
	I	)elta-method				
	Margin	std. err.	t	P> t	[95% conf	. interval]
dose#gender						
250#Male	-7.35384	.6563742	-11.20	0.000	-8.708529	-5.99915
250#Female	3.706567	.6563742	5.65	0.000	2.351877	5.061257
500#Male	-13.73386	.6563742	-20.92	0.000	-15.08855	-12.37917
500#Female	-6.584167	.6563742	-10.03	0.000	-7.938857	-5.229477
750#Male	-16.82108	.6563742	-25.63	0.000	-18.17576	-15.46639
750#Female	-14.38795	.6563742	-21.92	0.000	-15.74264	-13.03326

Our data are balanced, so these results will not be affected by the many different ways that margins can compute cell means. Moreover, because our model consists of only dose and gender, these are also the point estimates for each combination.

We can graph the results:

```
. marginsplot
Variables that uniquely identify margins: dose gender
                            Adjusted predictions of dose#gender with 95% CIs
                 5
                 0
           Linear prediction
                -5
                                                                                            Male

    Female

               -10
               -15
               -20
                   250
                                                  500
                                                                                  750
                                       Dosage in milligrams per day
```

The lines are not parallel, which we expected because the interaction term is significant. Males experience a greater decline in blood pressure at every dosage level, but the effect of increasing dosage is greater for females. In fact, it is not clear if we can tell the difference between male and female response at the maximum dosage.

#### Simple effects

We can contrast the male and female responses within dosage to see the simple effects of gender. Because there are only two levels in gender, the choice of contrast operator is largely irrelevant. Aside from orthogonal polynomials, all operators produce the same estimates, although the effects can change signs.

```
. contrast r.gender@dose
Contrasts of marginal linear predictions
Margins: asbalanced
                                  df
                                                F
                                                          P>F
          gender@dose
(Female vs Male) 250
                                   1
                                           141.97
                                                      0.0000
(Female vs Male) 500
                                   1
                                            59.33
                                                      0.0000
(Female vs Male) 750
                                   1
                                             6.87
                                                      0.0150
                Joint
                                   З
                                            69.39
                                                      0.0000
                                  24
          Denominator
                                       Std. err.
                                                       [95% conf. interval]
                           Contrast
          gender@dose
                                       .9282533
                                                                   12.97623
(Female vs Male) 250
                            11.06041
                                                      9.144586
(Female vs Male) 500
                           7.149691
                                        .9282533
                                                       5.23387
                                                                   9.065512
(Female vs Male) 750
                           2.433124
                                       .9282533
                                                      .5173031
                                                                   4.348944
```

The effect for females is about 11 points higher than for males at a dosage of 250, and that shrinks to 2.4 points higher at the maximum dosage of 750.

We can form the simple effects the other way by contrasting the effect of dose at each level of gender:

. contrast ar.dose@gender Contrasts of marginal linear predictions Margins: asbalanced

df	F	P>F	
		<u> </u>	
1	47.24	0.0000	
1	122.90	0.0000	
1	11.06	0.0028	
1	70.68	0.0000	
4	122.65	0.0000	
24			
Contrast	Std. err.	[95% conf	. interval]
-6.380018	.9282533	-8.295839	-4.464198
-10.29073	.9282533	-12.20655	-8.374914
-3.087217	.9282533	-5.003038	-1.171396
-7.803784	.9282533	-9.719605	-5.887963
	1 1 1 4 24 Contrast -6.380018 -10.29073 -3.087217	1       47.24         1       122.90         1       11.06         1       70.68         4       122.65         24         Contrast Std. err.         -6.380018       .9282533         -10.29073       .9282533         -3.087217       .9282533	1       47.24       0.0000         1       122.90       0.0000         1       11.06       0.0028         1       70.68       0.0000         4       122.65       0.0000         24      6.380018       -9282533         -6.380018       .9282533       -8.295839         -10.29073       .9282533       -12.20655         -3.087217       .9282533       -5.003038

Here we use the ar . reverse adjacent contrast operator so that first we are comparing a dosage of 500 with a dosage of 250, and then we are comparing 750 with 500. We see that increasing the dosage has a larger effect on females—10.3 points when going from 250 to 500 compared with 6.4 points for males, and 7.8 points when going from 500 to 750 versus 3.1 points for males.

### Interaction effects

By specifying contrast operators on both factors, we can decompose the interaction effect into separate interaction contrasts.

```
    contrast ar.dose#r.gender
    Contrasts of marginal linear predictions
    Margins: asbalanced
```

		df	F	P>F
dose#gender				
(500 vs 250) (Femal	le vs Male)	1	8.87	0.0065
(750 vs 500) (Female vs Male)		1	12.91	0.0015
	Joint	2	21.66	0.0000
	Denominator	24		
	Contrast	Std. err.	[95% conf.	intorwall
	CONTRAST	sta. ell.	[95% CON1.	Intervalj
dose#gender (500 vs 250)				
(Female vs Male) (750 vs 500)	-3.910716	1.312748	-6.620095	-1.201336
(Female vs Male)	-4.716567	1.312748	-7.425947	-2.007187

Look for departures from zero to indicate an interaction effect between dose and gender. Both contrasts are significantly different from zero. Of course, we already knew the overall interaction was significant from our ANOVA results. The effect of increasing dose from 250 to 500 is 3.9 points greater in females than in males, and the effect of increasing dose from 500 to 750 is 4.7 points greater in females than in males. The confidence intervals for both estimates easily exclude zero, meaning that there is an interaction effect.

The joint test of these two interaction effects reproduces the test of interaction effects in the anova output. We can see that the F statistic of 21.66 matches the statistic from our original ANOVA results.

#### Main effects

We can perform tests of the main effects by listing each variable individually in contrast.

```
. contrast dose gender
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
dose	2	223.64	0.0000
gender	1	164.85	0.0000
Denominator	24		

The F tests are equivalent to the tests of main effects in the anova output. This is true only for linear models. contrast provides an easy way to obtain main effects and other ANOVA-style tests for models whose responses are not linear in the parameters—logistic, probit, glm, etc.

If we include contrast operators on the variables, we can also decompose the main effects into individual contrasts:

. contrast ar.c Contrasts of ma		0	edictions		
Margins: asbala	-	-	041001010		
		df	F	P>F	
dose					
(500 vs 250)		1	161.27	0.0000	
(750 vs 500)		1	68.83	0.0000	
Joint		2	223.64	0.0000	
gender		1	164.85	0.0000	
Denominator		24			
		Contrast	Std. err.	[95% conf.	interval
da (500 vs 250 (750 vs 500		-8.335376 -5.4455	. 6563742 . 6563742		
gend (Female vs Male		6.881074	.5359273	5.774974	7.98717

By specifying the ar. operator on dose, we decompose the main effect for dose into two one-degreeof-freedom contrasts, comparing the marginal mean of blood pressure change for each dosage level with that of the previous level. Because gender has only two levels, we cannot decompose this main effect any further. However, specifying a contrast operator on gender allowed us to calculate the difference in the marginal means for women and men.

### Partial interaction effects

At this point, we have looked at the total interaction effects and at the main effects of each variable. The partial interaction effects are a midpoint between these two types of effects where we collect the individual interaction effects along the levels of one of the variables and perform a joint test of those interactions. If we think of the interaction effects as forming a table, with the levels of one factor variable forming the rows and the levels of the other forming the columns, partial interaction effects are joint tests of the interactions in a row or a column. To perform these tests, we specify a contrast operator on only one of the variables in our interaction. For this particular model, these are not very interesting because our variables have only two and three levels. Therefore, the tests of the partial interaction effects reproduce the tests that we obtained for the total interaction effects. We specify a contrast operator only on dose to decompose the overall test for interaction effects into joint tests for each ar.dose contrast:

. contrast ar.dose#gender						
Contrasts of marginal linear predictions						
Margins: asbalanced						
	df	F	P>F			
dose#gender						
(500 vs 250) (joint)	1	8.87	0.0065			
(750 vs 500) (joint)	1	12.91	0.0015			
Joint	2	21.66	0.0000			
Denominator	24					

The first row is a joint test of all the interaction effects involving the (500 vs 250) comparison of dosages. The second row is a joint test of all the interaction effects involving the (750 vs 500) comparison. If we look back at our output in *Interaction effects*, we can see that there was only one of each of these interaction effects. Therefore, each test labeled (joint) has only one degree-of-freedom.

We could have instead included a contrast operator on gender to compute the partial interaction effects along the other dimension:

. contrast dose#r.gender							
Contrasts of marginal linear predictions							
Margins: asbalanced							
		df	F	P>F			
dose#gender		2	21.66	0.0000			
Denominator		24					

Here we obtain a joint test of all the interaction effects involving the (Female vs Male) comparison for gender. Because gender has only two levels, the (Female vs Male) contrast is the only reference category contrast possible. Therefore, we obtain a single joint test of all the interaction effects.

Clearly, the partial interaction effects are not interesting for this particular model. However, if our factors had more levels, the partial interaction effects would produce tests that are not available in the total interaction effects. For example, if our model included factors for four dosage levels and three races, then typing

. contrast ar.dose#race

would produce three joint tests, one for each of the reverse adjacent contrasts for dosage. Each of these tests would be a two-degree-of-freedom test because race has three levels.

## Three-way and higher-order models

All the contrasts and tests that we reviewed above for two-way models can be used with models that have more terms. For instance, we could fit a three-way full factorial model by using the anova command:

. use https://www.stata-press.com/data/r19/cont3way

. anova y race##sex##group

We could then test the simple effects of race within each level of the interaction between sex and group:

. contrast race@sex#group

To see the reference category contrasts that decompose these simple effects, type

. contrast r.race@sex#group

We could test the three-way interaction effects by typing

. contrast race#sex#group

or the interaction effects for the interaction of race and sex by typing

. contrast race#sex

To see the individual reference category contrasts that decompose this interaction effect, type

. contrast r.race#r.sex

We could even obtain joint tests for the interaction of race and sex within each level of group by typing

. contrast race#sex@group

For tests of the main effects of each factor, we can type

. contrast race sex group

We can calculate the individual reference category contrasts that decompose these main effects:

. contrast r.race r.sex r.group

For the partial interaction effects, we could type

. contrast r.race#group

to obtain a joint test of the two-way interaction effects of race and group for each of the individual r.race contrasts.

We could type

. contrast r.race#sex#group

to obtain a joint test of all the three-way interaction terms for each of the individual r.race contrasts.

#### Contrast operators

contrast recognizes a set of contrast operators that are used to specify commonly used contrasts. When these operators are used, contrast will report a test for each individual contrast in addition to the joint test for the term. We have already seen a few of these, like r. and ar., in the previous examples. Here we will take a closer look at each of the unweighted operators.

Here we use the cholesterol dataset and the one-way ANOVA model from the example in One-way models:

```
. use https://www.stata-press.com/data/r19/cholesterol
(Artificial cholesterol data)
. anova chol agegrp
 (output omitted)
```

The margins command reports the estimated cell means,  $\hat{\mu}_1, \ldots, \hat{\mu}_5$ , for each of the five age groups.

```
. margins agegrp
Adjusted predictions
Expression: Linear prediction, predict()
```

219.282

Number of obs = 75

213,9629

224.601

-	1	× 1				
	l Margin	Delta-method std. err.	t	P> t	[95% conf.	interval]
agegrp						
10-19	180.5198	2.666944	67.69	0.000	175.2007	185.8388
20-29	188.7233	2.666944	70.76	0.000	183.4043	194.0424
30-39	202.0608	2.666944	75.76	0.000	196.7418	207.3799
40-59	210.6704	2.666944	78.99	0.000	205.3514	215.9895

2.666944

Contrast operators provide an easy way to make certain types of comparisons of these cell means. We use the ordinal factor agegrp to demonstrate these operators because some types of contrasts are meaningful only when the levels of the factor have a natural ordering. We demonstrate these contrast operators using a one-way model; however, they are equally applicable to main effects, simple effects, and interactions for more complicated models.

82.22

0.000

#### Differences from a reference level (r.)

60 - 79

The r. operator specifies that each level of the attached factor variable be compared with a reference level. These are referred to as reference-level or reference-category contrasts (or effects), and r. is the reference-level operator.

In the following, we use the r. operator to test the effect of each category of age group when that category is compared with a reference category.

. contrast r.agegrp								
Contrasts of marginal linear predictions								
Margins: asbalanced	1							
			df		F	P>F		
age (20-29 vs 10-19	-		1	Λ	73	0.0330		
(30-39 vs 10-19			1		62	0.0000		
(40-59 vs 10-19			1	63.				
(60-79 vs 10-19			1	105.				
Join			4	35.	02	0.0000		
Denominat	tor		70					
	(	Contrast	Std.	err.		[95% conf.	interval]	
agegrp (20-29 vs 10-19)		3.203575		1628		.6812991	15.72585	
(30-39 vs 10-19)	-	21.54105		1628		14.01878	29.06333	
(40-59 vs 10-19) (60-79 vs 10-19)		30.15067 38.76221		1628 1628		22.6284 31.23993	37.67295 46.28448	

In the first table, the row labeled (20-29 vs 10-19) is a test of  $\mu_2 = \mu_1$ , a test that the mean cholesterol levels for the 10–19 age group and the 20–29 age group are equal. The tests in the next three rows are defined similarly. The row labeled Joint provides the joint test for these four hypotheses, which is just the test of the main effects of age group.

The second table provides the contrasts of each category with the reference category along with confidence intervals. The contrast in the row labeled (20-29 vs 10-19) is the difference in the cell means of the second age group and the first age group,  $\hat{\mu}_2 - \hat{\mu}_1$ .

The first level of a factor is the default reference level, but we can specify a different reference level by using the b. operator; see [U] **11.4.3.2 Base levels**. Here we use the last age group, (60-79), instead of the first as the reference category. We also include the nowald option so that only the table of contrasts and their confidence intervals is produced.

```
. contrast rb5.agegrp, nowald
Contrasts of marginal linear predictions
Margins: asbalanced
```

	Contrast	Std. err.	[95% conf.	interval]
agegrp (10-19 vs 60-79) (20-29 vs 60-79) (30-39 vs 60-79) (40-59 vs 60-79)	-38.76221 -30.55863 -17.22115 -8.611533	3.771628 3.771628 3.771628 3.771628	-46.28448 -38.08091 -24.74343 -16.13381	-31.23993 -23.03636 -9.698877 -1.089257

Now, the first row is labeled (10-19 vs 60-79) and is the difference in the cell means of the first and fifth age groups.

#### Differences from the next level (a.)

The a. operator specifies that each level of the attached factor variable be compared with the next level. These are referred to as adjacent contrasts (or effects), and a. is the adjacent operator. This operator is meaningful only with factor variables that have a natural ordering in the levels.

We can use the a. operator to perform tests that each level of age group differs from the next adjacent level.

```
. contrast a.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

			df	1	F P>F	
age	grp					
(10-19 vs 20-29	<del>)</del> )		1	4.7	3 0.0330	
(20-29 vs 30-39	<del>)</del> )		1	12.5	1 0.0007	
(30-39 vs 40-59	9)		1	5.2	1 0.0255	
(40-59 vs 60-79	<del>)</del> )		1	5.2	1 0.0255	
Join	ıt		4	35.0	2 0.0000	
Denominat	tor		70			
	(	Contrast	Std.	err.	[95% conf.	interval]
agegrp (10-19 vs 20-29) (20-29 vs 30-39) (30-39 vs 40-59) (40-59 vs 60-79)	-:	3.203575 13.33748 -8.60962 3.611533	3.77 3.77 3.77 3.77	1628 1628	-15.72585 -20.85976 -16.1319 -16.13381	6812991 -5.815204 -1.087345 -1.089257

In the first table, the row labeled (10-19 vs 20-29) tests the effect of belonging to the 10-19 age group instead of the 20-29 age group. Likewise, the rows labeled (20-29 vs 30-39), (30-39 vs 40-59), and (40-59 vs 60-79) are tests for the effects of being in the younger of the two age groups instead of the older one.

In the second table, the contrast in the row labeled (10-19 vs 20-29) is the difference in the cell means of the first and second age groups,  $\hat{\mu}_1 - \hat{\mu}_2$ . The contrasts in the other rows are defined similarly.

#### Differences from the previous level (ar.)

(20-29 vs 10-19)

(30-39 vs 20-29)

(40-59 vs 30-39)

(60-79 vs 40-59)

The ar. operator specifies that each level of the attached factor variable be compared with the previous level. These are referred to as reverse adjacent contrasts (or effects), and ar. is the reverse adjacent operator. As with the a. operator, this operator is meaningful only with factor variables that have a natural ordering in the levels.

In the following, we use the ar. operator to report tests for the individual reverse adjacent effects of agegrp.

```
    contrast ar.agegrp
    Contrasts of marginal linear predictions
    Margins: asbalanced
```

			df	F	P>F	
age (20-29 vs 10-15 (30-39 vs 20-25 (40-59 vs 30-35 (60-79 vs 40-55 Join	9) 9) 9) 9)		1 1 1 1 4	4.73 12.51 5.21 5.21 35.02	0.0007 0.0255 0.0255	
Denominat	tor		70			
	(	Contrast	Std.	err.	[95% conf.	interval]
agegrp						

13.33748 3.771628

8.203575

8.60962

8.611533

Here the Wald tests in the first table for the individual reverse adjacent effects are equivalent to the tests for the adjacent effects in the previous example. However, if we compare values of the contrasts in the bottom tables, we see the difference between the r. and the ar. operators. This time, the contrast in the first row is labeled (20-29 vs 10-19) and is the difference in the cell means of the second and first age groups,  $\hat{\mu}_2 - \hat{\mu}_1$ . This is the estimated effect of belonging to the 20-29 age group instead of the 10-19 age group. The remaining rows make similar comparisons with the previous level.

.6812991

5.815204

1.087345

1.089257

15.72585

20.85976

16.1319

16.13381

3.771628

3.771628

3.771628

#### Differences from the grand mean (g.)

The g. operator specifies that each level of a factor variable be compared with the grand mean of all levels. For this operator, the grand mean is computed using a simple average of the cell means.

Here are the grand mean effects of agegrp:

```
. contrast g.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F	
agegrp				
(10-19 vs mean)	1	68.42	0.0000	
(20-29 vs mean)	1	23.36	0.0000	
(30-39 vs mean)	1	0.58	0.4506	
(40-59 vs mean)	1	19.08	0.0000	
(60-79 vs mean)	1	63.65	0.0000	
Joint	4	35.02	0.0000	
Denominator	70	)		
	Contrast	Std. err.	[95% conf.	interval]
agegrp				
(10-19 vs mean)	-19.7315	2.385387	-24.48901	-14.974
(20-29 vs mean)	-11.52793	2.385387	-16.28543	-6.770423
(30-39 vs mean)	1.809552	2.385387	-2.947953	6.567057
(40-59 vs mean)	10.41917	2.385387	5.661668	15.17668
(60-79 vs mean)	19.0307	2.385387	14.2732	23.78821

There are five age groups in our estimation sample. Thus, the row labeled (10-19 vs mean) tests  $\mu_1 = (\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)/5$ . The row labeled (20-29 vs mean) tests  $\mu_2 = (\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)/5$ . The remaining rows perform similar tests for the third, fourth, and fifth age groups. In our example, the means for all age groups except the 30-39 age group are statistically different from the grand mean.

#### Differences from the mean of subsequent levels (h.)

The h. operator specifies that each level of the attached factor variable be compared with the mean of subsequent levels. These are referred to as Helmert contrasts (or effects), and h. is the Helmert operator. For this operator, the mean is computed using a simple average of the cell means. This operator is meaningful only with factor variables that have a natural ordering in the levels.

Here are the Helmert contrasts for agegrp:

```
. contrast h.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

			df	I	F P>F	
age	grp					
(10-19 vs >10-1	9)		1	68.42	2 0.0000	
(20-29 vs >20-2	9)		1	50.79	0.0000	
(30-39 vs >30-3	9)		1	15.63	0.0002	
(40-59 vs 60-7	9)		1	5.23	0.0255	
Joi	nt		4	35.02	0.0000	
Denomina	tor		70			
	(	Contrast	Std.	err.	[95% conf.	interval]
agegrp						
(10-19 vs >10-19)	-2	24.66438	2.98	1734	-30.61126	-18.7175
(20-29 vs >20-29)	-2	21.94774	3.079	9522	-28.08965	-15.80583
(30-39 vs >30-39)	-:	12.91539	3.26	6326	-19.42987	-6.400905
(40-59 vs 60-79)	-8	3.611533	3.77	1628	-16.13381	-1.089257

The row labeled (10-19 vs >10-19) tests  $\mu_1 = (\mu_2 + \mu_3 + \mu_4 + \mu_5)/4$ , that is, that the cell mean for the youngest age group is equal to the average of the cell means for the older age groups. The row labeled (20-29 vs >20-29) tests  $\mu_2 = (\mu_3 + \mu_4 + \mu_5)/3$ . The tests in the other rows are defined similarly.

#### Differences from the mean of previous levels (j.)

The j. operator specifies that each level of the attached factor variable be compared with the mean of the previous levels. These are referred to as reverse Helmert contrasts (or effects), and j. is the reverse Helmert operator. For this operator, the mean is computed using a simple average of the cell means. This operator is meaningful only with factor variables that have a natural ordering in the levels.

Here are the reverse Helmert contrasts of agegrp:

. contrast j.agegrp					
Contrasts of marginal	linear pre	dictio	ns		
Margins: asbalanced					
		df	F	P>F	
agegr	b				
(20-29 vs 10-19)		1	4.73	0.0330	
(30-39 vs <30-39)		1	28.51	0.0000	
(40-59 vs <40-59)		1	43.18		
(60-79 vs <60-79)		1	63.65		
Joint		4	35.02	0.0000	
Denominato	r	70			
	Contrast	Std.	err.	[95% conf.	interval]
agegrp					
(20-29 vs 10-19)	8.203575	3.77	1628	.6812991	15.72585
(30-39 vs <30-39)	17.43927	3.26	6326	10.92479	23.95375
(40-59 vs <40-59)	20.2358	3.07	9522	14.09389	26.37771
(60-79 vs <60-79)	23.78838	2.98	1734	17.8415	29.73526

The row labeled (20-29 vs 10-19) tests  $\mu_2 = \mu_1$ , that is, that the cell means for the 20-29 and the 10-19 age groups are equal. The row labeled (30-39 vs <30-29) tests  $\mu_3 = (\mu_1 + \mu_2)/2$ , that is, that the cell mean for the 30-39 age group is equal to the average of the cell means for the 10-19 and 20-29 age groups. The tests in the remaining rows are defined similarly.

#### Orthogonal polynomials (p. and q.)

The p. and q. operators specify that orthogonal polynomials be applied to the attached factor variable. Orthogonal polynomial contrasts allow us to partition the effects of a factor variable into linear, quadratic, cubic, and higher-order polynomial components. The p. operator applies orthogonal polynomials using the values of the factor variable. The q. operator applies orthogonal polynomials using the level indices. If the level values of the factor variable are equally spaced, as with our agegrp variable, then the p. and q. operators yield the same result. These operators are meaningful only with factor variables that have a natural ordering in the levels.

Because agegrp has five levels, contrast can test the linear, quadratic, cubic, and quartic effects of agegrp.

. contrast p.agegrp, noeffects								
	Contrasts of marginal linear predictions							
Margins: asba	lanced							
	df	F	P>F					
agegrp								
(linear)	1	139.11	0.0000					
(quadratic)	1	0.15	0.6962					
(cubic)	1	0.37	0.5448					
(quartic)	1	0.43	0.5153					
Joint	4	35.02	0.0000					
Denominator	70							

The row labeled (linear) tests the linear effect of agegrp, the only effect that appears to be significant in this case.

The labels for our agegrp variable show the age ranges that correspond to each level.

. label list ages ages: 1 10-19 2 20-29 3 30-39 4 40-59 5 60-79

Notice that these groups do not have equal widths. Now, let's refit our model using the agemidpt variable. The values of agemidpt indicate the midpoint of each age group that was defined by the agegrp variable and are, therefore, not equally spaced.

 -9F -					
-	Number of obs = Root MSE =	75 10.329			0.6668 0.6477
Source	Partial SS	df	MS	F	Prob>F
Model	14943.4	4	3735.8499	35.02	0.0000
agemidpt	14943.4	4	3735.8499	35.02	0.0000
Residual	7468.2197	70	106.68885		
Total	22411.619	74	302.85972		

. anova chol agemidpt

Now if we use the q. operator, we will obtain the same results as above because the level indices of agemidpt are equivalent to the values of agegrp.

. contrast q.agemidpt, noeffects									
Contrasts of marginal linear predictions									
Margins: asbal	Margins: asbalanced								
	df	F	P>F						
agemidpt									
(linear)	1	139.11	0.0000						
(quadratic)	1	0.15	0.6962						
(cubic)	1	0.37	0.5448						
(quartic)	1	0.43	0.5153						
Joint	4	35.02	0.0000						
Denominator	70								

However, if we use the p. operator, we will instead fit an orthogonal polynomial to the midpoint values.

. contrast p.agemidpt, noeffects Contrasts of marginal linear predictions Margins: asbalanced

	df	F	P>F
agemidpt			
(linear)	1	133.45	0.0000
(quadratic)	1	5.40	0.0230
(cubic)	1	0.05	0.8198
(quartic)	1	1.16	0.2850
Joint	4	35.02	0.0000
Denominator	70		

Using the values of the midpoints, the quadratic effect is also significant at the 5% level.

### Technical note

We used the noeffects option when working with orthogonal polynomial contrasts. Apart from perhaps the sign of the contrast, the values of the individual contrasts are not meaningful for orthogonal polynomial contrasts. In addition, many textbooks provide tables with contrast coefficients that can be used to compute orthogonal polynomial contrasts where the levels of a factor are equally spaced. If we use these coefficients and calculate the contrasts manually with user-defined contrasts, as described below, the Wald tests for the polynomial terms will be equivalent, but the values of the individual contrasts will not necessarily match those that we obtain when using the polynomial contrast operator. When we use one of these contrast operators, an algorithm is used to calculate the coefficients of the polynomial contrast that will allow for unequal spacing in the levels of the factor as well as in the weights for the cell frequencies (when using pw. or qw.), as described in *Methods and formulas*.

## **User-defined contrasts**

In the previous examples, we performed tests using contrast operators. When there is not a contrast operator available to calculate the contrast in which we are interested, we can specify custom contrasts.

Here we fit a one-way model for cholesterol on the factor race, which has three levels:

```
. label list race
race:
1 Black
2 White
3 Other
. anova chol race
```

τ.	ace					
	-	Number of obs = Root MSE =	75 17.3775			0.0299
	1	TOOL HOE =	11.5//5	Auj n-sy	uareu -	0.0029
	Source	Partial SS	df	MS	F	Prob>F
	Model	669.27823	2	334.63912	1.11	0.3357
	race	669.27823	2	334.63912	1.11	0.3357
	Residual	21742.341	72	301.97696		
	Total	22411.619	74	302.85972		

margins calculates the estimated cell mean cholesterol level for each race:

. margins race								
Adjusted pred	Number o	f obs = 75						
Expression: Linear prediction, predict()								
	Delta-method							
	Margin	std. err.	t	P> t	[95% conf.	interval]		
race								
Black	204.4279	3.475497	58.82	0.000	197.4996	211.3562		
White	197.6132	3.475497	56.86	0.000	190.6849	204.5415		
Other	198.7127	3.475497	57.18	0.000	191.7844	205.6409		

Suppose we want to test the following linear combination:

$$\sum_{i=1}^{3} c_i \mu_i$$

where  $\mu_i$  is the cell mean of chol when race is equal to its *i*th level (the means estimated using margins above). Assuming the  $c_i$  elements sum to zero, this linear combination is a contrast. We can specify this type of custom contrast by using the following syntax:

 $\{ {\tt race} \; c_1 \; c_2 \; c_3 \}$ 

The null hypothesis for the test of the main effects of race is

$$H_{0_{\text{race}}}: \mu_1 = \mu_2 = \mu_3$$

Although  $H_{0_{\text{mee}}}$  can be tested using any of several different contrasts on the cell means, we will test it by comparing the second and third cell means with the first. To test that the cell means for blacks and whites are equal,  $\mu_1 = \mu_2$ , we can specify the contrast

To test that the cell means for blacks and other races are equal,  $\mu_1 = \mu_3$ , we can specify the contrast

{race -1 0 1}

We can use both in a single call to contrast.

```
. contrast {race -1 1 0} {race -1 0 1}
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F	
race				
(1)	1	1.92	0.1699	
(2)	1	1.35	0.2488	
Joint	2	1.11	0.3357	
Denominator	72			
	Contrast	Std. err.	[95% conf	. interval]
race (1) (2)	-6.814717 -5.715261	4.915095 4.915095	-16.61278 -15.51332	2.983345 4.082801

The row labeled (1) is the test for  $\mu_1 = \mu_2$ , the first specified contrast. The row labeled (2) is the test for  $\mu_1 = \mu_3$ , the second specified contrast. The row labeled Joint is the overall test for the main effects of race.

Now, let's fit a model with two factors, race and age group:

. anova chol race##agegrp

	Number of obs Root MSE	= =	7 9.6178	75 R-squa 35 Adj R-	red = squared =	0.7524 0.6946
Source	Partial SS		df	MS	F	Prob>F
Model	16861.438		14	1204.3884	13.02	0.0000
race agegrp	669.27823 14943.4		2 4	334.63912 3735.8499	3.62 40.39	0.0329 0.0000
race#agegrp	1248.7601		8	156.09501	1.69	0.1201
Residual	5550.1814		60	92.503024		
Total	22411.619		74	302.85972		

The null hypothesis for the test of the main effects of race is now

$$H_{0_{\text{race}}}: \mu_{1.} = \mu_{2.} = \mu_{3.}$$

where  $\mu_{i}$  is the marginal mean of chol when race is equal to its *i*th level.

We can use the same syntax as above to perform this test by specifying contrasts on the marginal means of race:

```
. contrast {race -1 1 0} {race -1 0 1}
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race			
(1)	1	6.28	0.0150
(2)	1	4.41	0.0399
Joint	2	3.62	0.0329
Denominator	60		
	Contrast	Std. err.	[95% conf. interval
race (1) (2)	-6.814717 -5.715261	2.720339 2.720339	-12.2562 -1.3732 -11.15675273773

Custom contrasts may be specified on the cell means of interactions, too. Here we use margins to calculate the mean of chol for each cell in the interaction of race and agegrp:

```
. margins race#agegrp
Adjusted predictions
```

```
Expression: Linear prediction, predict()
```

```
Number of obs = 75
```

-	-	-				
	Delta-method					
	Margin	std. err.	t	P> t	[95% conf.	interval]
race#agegrp						
Black#10-19	179.2309	4.301233	41.67	0.000	170.6271	187.8346
Black#20-29	196.4777	4.301233	45.68	0.000	187.874	205.0814
Black#30-39	210.6694	4.301233	48.98	0.000	202.0656	219.2731
Black#40-59	214.097	4.301233	49.78	0.000	205.4933	222.7008
Black#60-79	221.6646	4.301233	51.54	0.000	213.0609	230.2684
White#10-19	186.0727	4.301233	43.26	0.000	177.469	194.6765
White#20-29	184.6714	4.301233	42.93	0.000	176.0676	193.2751
White#30-39	196.2633	4.301233	45.63	0.000	187.6595	204.867
White#40-59	209.9953	4.301233	48.82	0.000	201.3916	218.5991
White#60-79	211.0633	4.301233	49.07	0.000	202.4595	219.667
Other#10-19	176.2556	4.301233	40.98	0.000	167.6519	184.8594
Other#20-29	185.0209	4.301233	43.02	0.000	176.4172	193.6247
Other#30-39	199.2498	4.301233	46.32	0.000	190.646	207.8535
Other#40-59	207.9189	4.301233	48.34	0.000	199.3152	216.5227
Other#60-79	225.118	4.301233	52.34	0.000	216.5143	233.7218

Now, we are interested in testing the following linear combination of these cell means:

$$\sum_{i=1}^{3} \sum_{j=1}^{5} c_{ij} \mu_{ij}$$

We can specify this type of custom contrast using the following syntax:

```
{race#agegrp c_{11} c_{12} \dots c_{15} c_{21} c_{22} \dots c_{25} c_{31} c_{32} \dots c_{35}}
```

Because the marginal means of chol for each level of race are linear combinations of the cell means, we can compose the test for the main effects of race in terms of the cell means directly. The constraint that the marginal means for blacks and whites are equal,  $\mu_{1.} = \mu_{2.}$ , translates to the following constraint on the cell means:

$$\frac{1}{5}(\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15})=\frac{1}{5}(\mu_{21}+\mu_{22}+\mu_{23}+\mu_{24}+\mu_{25})$$

Ignoring the common factor, we can specify this contrast as

```
{race#agegrp -1 -1 -1 -1 -1 1 1 1 1 0 0 0 0 0}
```

contrast will fill in the trailing zeros for us if we neglect to specify them, so

{race#agegrp -1 -1 -1 -1 -1 1 1 1 1 1}

is also allowed. The other constraint,  $\mu_{1} = \mu_{3}$ , translates to

$$\frac{1}{5}(\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15})=\frac{1}{5}(\mu_{31}+\mu_{32}+\mu_{33}+\mu_{34}+\mu_{35})$$

This can be specified to contrast as

The following call to contrast yields the same test results as above.

```
. contrast {race#agegrp -1 -1 -1 -1 -1 1 1 1 1 1}
> {race#agegrp -1 -1 -1 -1 -1 0 0 0 0 0 1 1 1 1 1}, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race#agegrp (1) (1) (2) (2) Joint	1 1 2	6.28 4.41 3.62	0.0150 0.0399 0.0329
Denominator	60		

The row labeled (1) (1) is the test for

$$\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{21} + \mu_{22} + \mu_{23} + \mu_{24} + \mu_{25}$$

It was the first specified contrast. The row labeled (2) (2) is the test for

$$\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35}$$

It was the second specified contrast. The row labeled Joint tests (1) (1) and (2) (2) simultaneously.

We used the noeffects option above to suppress the table of contrasts. We can omit the 1/5 from the equations for  $\mu_{1.} = \mu_{2.}$  and  $\mu_{1.} = \mu_{3.}$  and still obtain the appropriate tests. However, if we want to calculate the differences in the marginal means, we must include the 1/5 = 0.2 on each of the contrast coefficients as follows:

. contrast {race#agegrp -0.2 -0.2 -0.2 -0.2 -0.2 /// 0.2 0.2 0.2 0.2 0.2 0.2} /// {race#agegrp -0.2 -0.2 -0.2 -0.2 -0.2 /// 0 0 0 0 0 /// 0.2 0.2 0.2 0.2 0.2}

So far, we have reproduced the reference category contrasts by specifying user-defined contrasts on the marginal means and then on the cell means. For this test, it would have been easier to use the r. contrast operator:

. contrast r.race, noeffects Contrasts of marginal linear predictions Margins: asbalanced

	df	F	P>F
race (White vs Black)	1	6.28	0.0150
(Other vs Black)	1	4.41	0.0399
Joint	2	3.62	0.0329
Denominator	60		

In most cases, we can use contrast operators to perform tests. However, if we want to compare, for instance, the second and third age groups with the fourth and fifth age groups with the test

$$\frac{1}{2}(\mu_{\cdot 2}+\mu_{\cdot 3})=\frac{1}{2}(\mu_{\cdot 4}+\mu_{\cdot 5})$$

there is not a contrast operator that corresponds to this particular contrast. A custom contrast is necessary.

. contrast {agegrp 0 -0.5 -0.5 0.5 0.5}						
Contrasts of marginal linear predictions						
Margins: asbal	lanced					
	df	F	P>F			
agegrp	1	62.19	0.0000			
Denominator	60					
	Contrast	Std. err.	[95% conf	. interval]		
agegrp (1)	19.58413	2.483318	14.61675	24.5515		

# **Empty cells**

An empty cell is a combination of the levels of factor variables that is not observed in the estimation sample. In the previous examples, we have seen data with three levels of race, five levels of agegrp, and all level combinations of race and agegrp present. Suppose there are no observations for white individuals in the second age group (ages 20-29).

note: 2.races	.agegrp Ident	iiies no o	DServations	s in the	sampie.	
Source	SS	df	MS		CI OI ODD	= 70
					,,	= 13.51
Model	15751.6113	13	1211.6624:			= 0.0000
Residual	5022.71559	56	89.6913498	1	uurou	= 0.7582
					n oquur ou	= 0.7021
Total	20774.3269	69	301.077202	1 Root	MSE =	9.4706
chol	Coefficient	Std. err.	t	P> t	[95% conf	interval]
race						
White	12.84185	5.989703	2.14	0.036	.8430383	24.84067
Other	167627	5.989703	-0.03	0.978	-12.16644	11.83119
agegrp						
20-29	17.24681	5.989703	2.88	0.006	5.247991	29.24562
30-39	31.43847	5.989703	5.25	0.000	19.43966	43.43729
40-59	34.86613	5.989703	5.82	0.000	22.86732	46.86495
60-79	44.43374	5.989703	7.42	0.000	32.43492	56.43256
race#agegrp						
White#20-29	0	(empty)				
White#30-39	-22.83983	8.470719	-2.70	0.009	-39.80872	-5.870939
White#40-59	-14.67558	8.470719	-1.73	0.089	-31.64447	2.293306
White#60-79	-10.51115	8.470719	-1.24	0.220	-27.48004	6.457735
Other#20-29	-6.054425	8.470719	-0.71	0.478	-23.02331	10.91446
Other#30-39	-11.48083	8.470719	-1.36	0.181	-28.44971	5.488063
Other#40-59	6796112	8.470719	-0.08	0.936	-17.6485	16.28928
Other#60-79	-1.578052	8.470719	-0.19	0.853	-18.54694	15.39084
_cons	175.2309	4.235359	41.37	0.000	166.7464	183.7153

. regress chol race##agegrp note: 2.race#2.agegrp identifies no observations in the sample.

Now, let's use contrast to test the main effects of race:

. contrast race					
Contrasts of m	narginal linear pr	edictions			
Margins: asbal	Lanced				
	df	F	P>F		
race	(not testable)				
Denominator	56				

By "not testable", contrast means that it cannot form a test for the main effects of race based on estimable functions of the model coefficients. agegrp has five levels, so contrast constructs an estimate of the *i*th margin for race as

$$\hat{\mu}_{i\cdot} = \frac{1}{5} \sum_{j=1}^{5} \hat{\mu}_{ij} = \hat{\mu}_0 + \hat{\alpha}_i + \frac{1}{5} \sum_{j=1}^{5} \left\{ \hat{\beta}_j + (\widehat{\alpha \beta})_{ij} \right\}$$

but  $(\widehat{\alpha\beta})_{22}$  was constrained to zero because of the empty cell, so  $\hat{\mu}_{2}$  is not an estimable function of the model coefficients.

See Estimable functions in Methods and formulas of [R] margins for a technical description of estimable functions. The emptycells(reweight) option causes contrast to estimate  $\mu_2$  by

$$\hat{\mu}_{2.} = \frac{\hat{\mu}_{21} + \hat{\mu}_{23} + \hat{\mu}_{24} + \hat{\mu}_{25}}{4}$$

which is an estimable function of the model coefficients.

. contrast rad	ce, empty	cells()	reweight)		
Contrasts of m	marginal	linear	predictions		
Margins: asbalanced Empty cells: reweight					
		df	F	P>F	
race		2	3.17	0.0498	

56

We can reconstruct the effect of the emptycells (reweight) option by using custom contrasts.

	df	F	P>F
race#agegrp (1) (1) (2) (2) Joint	1 1 2	1.06 2.37 3.17	0.3080 0.1291 0.0498
Denominator	56		

The row labeled (1) (1) is the test for

Denominator

$$\frac{1}{5}(\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15})=\frac{1}{4}(\mu_{21}+\mu_{23}+\mu_{24}+\mu_{25})$$

It was the first specified contrast. The row labeled (2) (2) is the test for

 $\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35}$ 

It was the second specified contrast. The row labeled Joint is the overall test of the main effects of race.

# Empty cells, ANOVA style

Let's refit the linear model from the previous example with anova to compare with contrast's test for the main effects of race.

. anova chol race##agegrp

-	Number of obs = Root MSE =	7 9.4705		red = squared =	0.7582 0.7021
Source	Partial SS	df	MS	F	Prob>F
Model	15751.611	13	1211.6624	13.51	0.0000
race agegrp race#agegrp	305.49046 14387.856 795.80757	2 4 7	152.74523 3596.964 113.6868	1.70 40.10 1.27	0.1914 0.0000 0.2831
Residual	5022.7156	56	89.69135		
Total	20774.327	69	301.0772		

contrast and anova handled the empty cell differently; the F statistic reported by contrast was 3.17, but anova reported 1.70. To see how they differ, consider the following table of the cell means and margins for our situation.

	agegrp					
	1	2	3	4	5	
1	$\mu_{11}$	$\mu_{12}$	$\mu_{13}$	$\mu_{14}$	$\mu_{15}$	$\mu_{1.}$
2	$\mu_{21}$		$\mu_{23}$	$\mu_{24}$	$\mu_{25}$	
3	$\mu_{31}$	$\mu_{32}$	$\mu_{33}$	$\mu_{34}$	$\mu_{35}$	$\mu_{3\cdot}$
	$\mu_{\cdot 1}$		$\mu_{\cdot 3}$	$\mu_{.4}$	$\mu_{.5}$	
	1 2 3	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	3 $\mu_{31}$ $\mu_{32}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

For testing the main effects of race, we know that we will be testing the equality of the marginal means for rows 1 and 3, that is,  $\mu_1 = \mu_3$ . This translates into the following constraint:

$$\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35}$$

Because row 2 contains an empty cell in column 2, anova dropped column 2 and tested the equality of the marginal mean for row 2 with the average of the marginal means from rows 1 and 3, using only the remaining cell means. This translates into the following constraint:

$$2(\mu_{21} + \mu_{23} + \mu_{24} + \mu_{25}) = \mu_{11} + \mu_{13} + \mu_{14} + \mu_{15} + \mu_{31} + \mu_{33} + \mu_{34} + \mu_{35}$$
(1)

Now that we know the constraints that anova used to test for the main effects of race, we can use custom contrasts to reproduce the anova test result.

```
. contrast {race#agegrp -1 -1 -1 -1 -1 0 0 0 0 1 1 1 1 1}
>
           {race#agegrp 1 0 1 1 1 -2 0 -2 -2 -2 1 0 1 1 1}, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
                                    F
                       df
                                             P>F
 race#agegrp
    (1) (1)
                                 2.37
                                          0.1291
                        1
    (2) (2)
                        1
                                 1.03
                                          0.3138
                        2
      Joint
                                 1.70
                                          0.1914
 Denominator
                       56
```

The row labeled (1) (1) is the test for  $\mu_{1.} = \mu_{3.}$ ; it was the first specified contrast. The row labeled (2) (2) is the test for the constraint in (1); it was the second specified contrast. The row labeled Joint is an overall test for the main effects of race.

### **Nested effects**

contrast has the | operator for computing simple effects when the levels of one factor are nested within the levels of another. Here is a fictional example where we are interested in the effect of five methods of teaching algebra on students' scores for the math portion of the SAT. Suppose three algebra classes are randomly sampled from classes using each of the five methods so that class is nested in method as demonstrated in the following tabulation.

```
. use https://www.stata-press.com/data/r19/sat (Fictional SAT data)
```

```
. tabulate class method
```

		Five metho	ds of teach	ning algebra	L	
Class ID	1	2	3	4	5	Total
1	5	0	0	0	0	5
2	5	0	0	0	0	5
3	5	0	0	0	0	5
4	0	5	0	0	0	5
5	0	5	0	0	0	5
6	0	5	0	0	0	5
7	0	0	5	0	0	5
8	0	0	5	0	0	5
9	0	0	5	0	0	5
10	0	0	0	5	0	5
11	0	0	0	5	0	5
12	0	0	0	5	0	5
13	0	0	0	0	5	5
14	0	0	0	0	5	5
15	0	0	0	0	5	5
Total	15	15	15	15	15	75

We will consider method as fixed and class nested in method as random. To use class nested in method as the error term for method, we can specify the following anova model:

	-	Number of obs Root MSE	s = =	71.85	-	ared = R-squared =	0.7599 0.7039
Sour	ce	Partial SS		df	MS	F	Prob>F
Mod	lel	980312		14	70022.286	3 13.56	0.0000
meth class meth		905872 74440		4 10	226468 7444		0.0000
class meth	ıod	74440		10	7444	1.44	0.1845
Residu	al	309760		60	5162.6667	7	
Tot	al	1290072		74	17433.405	5	

Like anova, contrast allows the | operator, which specifies that one variable is nested in the levels of another. We can use contrast to test the main effects of method and the simple effects of class within method.

```
. contrast method class|method
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
method	(not testable)		
class method			
1	2	2.80	0.0687
2	2	0.91	0.4089
3	2	1.10	0.3390
4	2	0.22	0.8025
5	2	2.18	0.1221
Joint	10	1.44	0.1845
Denominator	60		

Although contrast was able to perform the individual tests for the simple effects of class within method, empty cells in the interaction between method and class prevented contrast from testing for a main effect of method. Here we add the emptycells(reweight) option so that contrast can take the empty cells into account when computing the marginal means for method.

```
. contrast method class|method, emptycells(reweight)
Contrasts of marginal linear predictions
Margins: asbalanced
Empty cells: reweight
```

	df	F	P>F
method	4	43.87	0.0000
class method			
1	2	2.80	0.0687
2	2	0.91	0.4089
3	2	1.10	0.3390
4	2	0.22	0.8025
5	2	2.18	0.1221
Joint	10	1.44	0.1845
Denominator	60		

Now, contrast does report a test for the main effects of method. However, if we compare this with the anova results, we will see that the results are different. They are different because contrast uses the residual error term to compute the F test by default. Using notation similar to anova, we can use the / operator to specify a different error term for the test. Therefore, we can reproduce the test of main effects from our anova command by typing

```
. contrast method / class|method /, emptycells(reweight)
Contrasts of marginal linear predictions
Margins: asbalanced
Empty cells: reweight
```

	df	F	P>F
method	4	30.42	0.0000
class method	10	(denominator)	
class   method			
1	2	2.80	0.0687
2	2	0.91	0.4089
3	2	1.10	0.3390
4	2	0.22	0.8025
5	2	2.18	0.1221
Joint	10	1.44	0.1845
Denominator	60		

# **Multiple comparisons**

We have seen that contrast can report the individual linear combinations that make up the requested effects. Depending upon the specified option, contrast will report confidence intervals, *p*-values, or both in the effects table. By default, the reported confidence intervals and *p*-values are not adjusted for multiple comparisons. Use the mcompare() option to adjust the confidence intervals and *p*-values for multiple comparisons of the individual effects.

Let's compute the grand mean effects of race using the g. operator. We also specify the mcompare(bonferroni) option to compute *p*-values and confidence intervals using Bonferroni's adjustment.

```
. use https://www.stata-press.com/data/r19/cholesterol
(Artificial cholesterol data)
. anova chol race##agegrp
(output omitted)
. contrast g.race, mcompare(bonferroni)
```

Contrasts of marginal linear predictions

Margins: asbalanced

			В	onferroni
	df	F	P>F	P>F
race				
(Black vs mean)	1	7.07	0.0100	0.0301
(White vs mean)	1	2.82	0.0982	0.2947
(Other vs mean)	1	0.96	0.3312	0.9936
Joint	2	3.62	0.0329	
Denominator	60			

Note: Bonferroni-adjusted p-values are reported for tests on individual contrasts only.

	Number of comparisons
race	3

	Contrast	Std. err.	Bonferroni [95% conf. interval]	
race				
(Black vs mean)	4.17666	1.570588	.3083743	8.044945
(White vs mean)	-2.638058	1.570588	-6.506343	1.230227
(Other vs mean)	-1.538602	1.570588	-5.406887	2.329684

The last table reports a Bonferroni-adjusted confidence interval for each individual contrast. (Use the effects option to add p-values to the last table.) The first table includes a Bonferroni-adjusted p-value for each test that is not a joint test.

Joint tests are never adjusted for multiple comparisons. For example,

. contrast race@agegrp, mcompare(bonferroni) Contrasts of marginal linear predictions Margins: asbalanced

	df	F	P>F
race@agegrp			
10-19	2	1.37	0.2620
20-29	2	2.44	0.0958
30-39	2	3.12	0.0512
40-59	2	0.53	0.5889
60-79	2	2.90	0.0628
Joint	10	2.07	0.0409
Denominator	60		

Note: Bonferroni-adjusted p-values are reported for tests on individual contrasts only.

	Number of comparisons
race@agegrp	10

			Bonfe	rroni
	Contrast	Std. err.	[95% conf.	interval]
race@agegrp				
(White vs base) 10-19	6.841855	6.082862	-10.88697	24.57068
(White vs base) 20-29	-11.80631	6.082862	-29.53513	5.922513
(White vs base) 30-39	-14.40607	6.082862	-32.13489	3.322751
(White vs base) 40-59	-4.101691	6.082862	-21.83051	13.62713
(White vs base) 60-79	-10.60137	6.082862	-28.33019	7.127448
(Other vs base) 10-19	-2.975244	6.082862	-20.70407	14.75358
(Other vs base) 20-29	-11.45679	6.082862	-29.18561	6.272031
(Other vs base) 30-39	-11.41958	6.082862	-29.1484	6.309244
(Other vs base) 40-59	-6.17807	6.082862	-23.90689	11.55075
(Other vs base) 60-79	3.453375	6.082862	-14.27545	21.1822

Here we have five tests of simple effects with two degrees of freedom each. No Bonferroni-adjusted *p*-values are available for these tests, but the confidence intervals for the individual contrasts are adjusted.

18

24

25

67

# Unbalanced data

By default, contrast treats all factors as balanced when computing marginal means. By balanced, we mean that contrast assumes an equal number of observations in each level of each factor and an equal number of observations in each cell of each interaction. If our data are balanced, there is no issue. If, however, our data are not balanced, we might prefer that contrast use the actual cell frequencies from our data in computing marginal means. We instruct contrast to use observed frequencies by adding the asobserved option.

Even if our data are unbalanced, we might still want contrast to compute balanced marginal means. It depends on what we want to test and what our data represent. If we have data from a designed experiment that started with an equal number of males and females but the data became unbalanced because the data from a few males were unusable, we might still want our margins computed as though the data were balanced. If, however, we have a representative sample of individuals from Los Angeles with 40% of European descent, 34% African-American, 25% Hispanic, and 1% Australian, we probably want our margins computed using these representative frequencies. We do not want Australians receiving the same weight as Europeans.

The following examples will use an unbalanced version of our dataset.

```
. use https://www.stata-press.com/data/r19/cholesterol3
(Artificial cholesterol data, unbalanced)
. tab race agegrp
                                       Age group
                  10 - 19
                               20-29
                                                                    60-79
      Race
                                           30-39
                                                        40-59
                                                                                 Total
     Black
                       1
                                   5
                                                5
                                                            4
                                                                         3
                                   5
                                                7
     White
                       4
                                                            4
                                                                        4
     Other
                       3
                                   7
                                                6
                                                            5
                                                                         4
     Total
                       8
                                  17
                                               18
                                                           13
                                                                       11
```

The row labeled Total gives observed cell frequencies for age group. These can be obtained by summing frequencies from the cells in the corresponding column. In this respect, we can also refer to them as marginal frequencies. We use the terms marginal frequencies and cell frequencies interchangeably below.

We begin by fitting the two-factor model with an interaction.

```
. anova chol race##agegrp
```

I	Number of obs Root MSE	= =	8.374	j	-squared =	0.8179 0.7689
Source	Partial SS		df	MS	F	Prob>F
Model	16379.993		14	1169.9995	16.68	0.0000
race	230.7544		2	115.3772	1.64	0.2029
agegrp	13857.988		4	3464.4969	49.39	0.0000
race#agegrp	857.81521		8	107.2269	1.53	0.1701
Residual	3647.2774		52	70.13995		
Total	20027.27		66	303.44349	1	

### Using observed cell frequencies

Recall that the marginal means are computed from the cell means. Treating the factors as balanced yields the following marginal means for race:

$$\begin{split} \eta_{1.} &= \frac{1}{5}(\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}) \\ \eta_{2.} &= \frac{1}{5}(\mu_{21} + \mu_{22} + \mu_{23} + \mu_{24} + \mu_{25}) \\ \eta_{3.} &= \frac{1}{5}(\mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35}) \end{split}$$

If we have a fixed population and unbalanced cells, then the  $\eta_i$  do not represent population means. If, however, our data are representative of the population, we can use the frequencies from our estimation sample to estimate the population marginal means, denoted  $\mu_i$ .

Here are the results of testing for a main effect of race, treating all the factors as balanced.

. contrast r.race Contrasts of marginal linear predictions Margins: asbalanced

	df	F	P>F	
race				
(White vs Black)	1	3.28	0.0757	
(Other vs Black)	1	1.50	0.2263	
Joint	2	1.64	0.2029	
Denominator	52			
	Contrast	Std. err.	[95% conf	. interval]
race				
(White vs Black)	-5.324254	2.93778	-11.21934	.5708338
(Other vs Black)	-3.596867	2.93778	-9.491955	2.298221

The row labeled (White vs Black) is the test for  $\eta_{2.} = \eta_{1.}$ . The row labeled (Other vs Black) is the test for  $\eta_{3.} = \eta_{1.}$ .

If the observed marginal frequencies are representative of the distribution of the levels of agegrp, we can use them to form the marginal means of chol for each of the levels of race from the cell means.

$$\begin{split} \mu_{1\cdot} &= \frac{1}{67} (8\mu_{11} + 17\mu_{12} + 18\mu_{13} + 13\mu_{14} + 11\mu_{15}) \\ \mu_{2\cdot} &= \frac{1}{67} (8\mu_{21} + 17\mu_{22} + 18\mu_{23} + 13\mu_{24} + 11\mu_{25}) \\ \mu_{3\cdot} &= \frac{1}{67} (8\mu_{31} + 17\mu_{32} + 18\mu_{33} + 13\mu_{34} + 11\mu_{35}) \end{split}$$

Here are the results of testing for the main effects of race, using the observed marginal frequencies:

```
. contrast r.race, asobserved
Contrasts of marginal linear predictions
Margins: asobserved
```

	df	F	P>F	
race				
(White vs Black)	1	7.25	0.0095	
(Other vs Black)	1	3.89	0.0538	
Joint	2	3.74	0.0304	
Denominator	52			
	Contrast	Std. err.	[95% conf	. interva
race				
(White vs Black) (Other vs Black)	-7.232433 -5.231198	2.686089 2.651203	-12.62246 -10.55123	-1.8424 .08882

The row labeled (White vs Black) is the test for  $\mu_{2.} = \mu_{1.}$ . The row labeled (Other vs Black) is the test for  $\mu_{3.} = \mu_{1.}$ . Both tests were insignificant when we tested the cell means resulting from balanced frequencies; however, when we tested the cell means from observed frequencies, the first test is significant beyond the 5% level (and the second test is nearly so).

Here we reproduce the results of the asobserved option with custom contrasts. Because we are modifying the way that the marginal means are constructed from the cell means, we will specify the contrasts on the predicted cell means. We use macro expansion, =exp, to evaluate the fractions instead of approximating them with decimals. Macro expansion guarantees that the contrast coefficients sum to zero. For more information, see *Macro expansion operators and function* in [P] macro.

. contrast {race#agegrp - `=8/67' - `=17/67' - `=18/67' - `=13/67' - `=11/67' > `=8/67' `=17/67' `=18/67' `=13/67' `=11/67'} > {race#agegrp - `=8/67' - `=17/67' - `=18/67' - `=13/67' - `=11/67'} > 0 0 0 0 0 0 > `=8/67' `=17/67' `=18/67' `=13/67' `=11/67'} Contrasts of marginal linear predictions Margins: asbalanced

	df	F	P>F	
race#agegrp (1) (1) (2) (2)	1	7.25 3.89	0.0095	
Joint	2	3.74	0.0304	
Denominator	52			
	I			
	Contrast	Std. err.	[95% conf	. interval]
race#agegrp (1) (1) (2) (2)	-7.232433 -5.231198	2.686089 2.651203	-12.62246 -10.55123	-1.842402 .0888295

#### Weighted contrast operators

contrast provides observation-weighted versions of five of the contrast operators—gw., hw., jw., pw., and qw.. The first three of these operators perform comparisons of means across cells, and like the marginal means just discussed, these means can be computed in two ways: 1) as though the cell frequencies were equal or 2) using the observed cell frequencies from the estimation sample. The weighted operators provide versions of the standard (as balanced) operators that weight these means by their cell frequencies. The two orthogonal polynomial operators involve similar adjustments for weighting.

Let's examine what this means by using the gw. operator. The gw. operator is a weighted version of the g. operator. The gw. operator computes the grand mean using the cell frequencies for race obtained from the model fit.

Here we test the effects of race, comparing each level with the weighted grand mean but otherwise treating the factors as balanced in the marginal mean calculations.

. contrast gw.race	Э			
Contrasts of margi	inal linear p	redictions		
Margins: asbalance	ed			
	df	F	P>F	
race				
(Black vs mean)	1	2.78	0.1014	
(White vs mean)	1	2.06	0.1573	
(Other vs mean)	1	0.06	0.8068	
Joint	2	1.64	0.2029	
Denominator	52			
	r			
	Contrast	Std. err.	[95% conf.	interval]
race				
(Black vs mean)	3.24931	1.948468	6605779	7.159198
(White vs mean)	-2.074944	1.44618	-4.976915	.8270276
(Other vs mean)	347557	1.414182	-3.18532	2.490206

The observed marginal frequencies of race are 18, 24, and 25. Thus, the row labeled (Black vs Mean) tests  $\eta_1 = (18\eta_1 + 24\eta_2 + 25\eta_3)/67$ ; the row labeled (White vs Mean) tests  $\eta_2 = (18\eta_1 + 24\eta_2 + 25\eta_3)/67$ ; and the row labeled (Other vs Mean) tests  $\eta_3 = (18\eta_1 + 24\eta_2 + 25\eta_3)/67$ .

Now, we reproduce the above results using custom contrasts. We are weighting the calculation of the grand mean from the marginal means for each of the races, but we are not weighting the calculation of the marginal means themselves. Therefore, we can specify the custom contrast on the marginal means for race instead of on the cell means.

```
. contrast {race '=49/67' - '=24/67' - '=25/67'}
> {race - '=18/67' '=43/67' - '=25/67'}
> {race - '=18/67' - '=24/67' '=42/67'}
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F	
race				
(1)	1	2.78	0.1014	
(2)	1	2.06	0.1573	
(3)	1	0.06	0.8068	
Joint	2	1.64	0.2029	
Denominator	52			
	I			
	Contrast	Std. err.	[95% conf.	interval]
race				
(1)	3.24931	1.948468	6605779	7.159198
(2)	-2.074944	1.44618	-4.976915	.8270276
(3)	347557	1.414182	-3.18532	2.490206

Now, we will test for each race the difference between the marginal mean and the weighted grand mean, treating the factors as observed in the marginal mean calculations.

Contrasts of margin	inal linear p	redictions		
Margins: asobserve	ed			
	df	F	P>F	
race		0.04	0.0110	
(Black vs mean) (White vs mean)	1	6.81 3.74	0.0118 0.0587	
(Other vs mean)	1	0.26		
Joint	2	3.74		
Denominator	52			
	Contrast	Std. err.	[95% conf	. interval]
race				
(Black vs mean)	4.542662	1.740331	1.050432	8.034891
(White vs mean)	-2.689771			
(Other vs mean)	6885363	1.341261	-3.379973	2.002901

. contrast gw.race, asobserved wald ci

The row labeled (Black vs Mean) tests  $\mu_{1.} = (18\mu_{1.} + 24\mu_{2.} + 25\mu_{3.})/67$ ; the row labeled (White vs Mean) tests  $\mu_{2.} = (18\mu_{1.} + 24\mu_{2.} + 25\mu_{3.})/67$ ; and the row labeled (Other vs Mean) tests  $\mu_{3.} = (18\mu_{1.} + 24\mu_{2.} + 25\mu_{3.})/67$ .

Here we use a custom contrast to reproduce the above result testing  $\mu_{1.} = (18\mu_{1.} + 24\mu_{2.} + 25\mu_{3.})/67$ . Because both the calculation of the marginal means and the calculation of the grand mean are adjusted, we specify the custom contrast on the cell means.

```
. contrast {race#agegrp
                         '=49/67*8/67' '=49/67*17/67' '=49/67*18/67'
                          '=49/67*13/67' '=49/67*11/67'
>
>
                         -'=24/67*8/67' -'=24/67*17/67' -'=24/67*18/67'
>
                         -'=24/67*13/67' -'=24/67*11/67'
>
                         - '=25/67*8/67' - '=25/67*17/67' - '=25/67*18/67'
>
                         -'=25/67*13/67' -'=25/67*11/67'}, nowald
Contrasts of marginal linear predictions
Margins: asbalanced
                 Contrast
                             Std. err.
                                           [95% conf. interval]
 race#agegrp
    (1) (1)
                 4.542662
                             1.740331
                                           1.050432
                                                        8.034891
```

The Helmert and reverse Helmert contrasts also involve calculating averages of the marginal means; therefore, weighted versions of these parameters are available as well. The hw. operator is a weighted version of the h. operator that computes the mean of the subsequent levels using the cell frequencies obtained from the model fit. The jw. operator is a weighted version of the j. operator that computes the mean of the previous levels using the cell frequencies obtained from the model fit.

For orthogonal polynomials, we can use the pw. and qw. operators, which are the weighted versions of the p. and q. operators. In this case, the cell frequencies from the model fit are used in the calculation of the orthogonal polynomial contrast coefficients.

### Testing factor effects on slopes

For linear models where the independent variables are all factor variables, the linear prediction at fixed levels of the factor variables turns out to be a cell mean. With these models, contrast computes and tests the effects of the factor variables on the expected mean of the dependent variable. When factor variables are interacted with continuous variables, contrast distinguishes factor effects on the intercept from factor effects on the slope.

Here we have 1980 census data including information on the birthrate (brate), the median age (medage), and the region of the country (region) for each of the 50 states. We can fit an ANCOVA model for brate using main effects of the factor variable region and the continuous variable medage.

```
. use https://www.stata-press.com/data/r19/census3
(1980 Census data by state)
. label list cenreg
cenreg:
           1 NE
           2 NCentral
           3 South
           4 West
. anova brate i.region c.medage
                          Number of obs =
                                                   50
                                                         R-squared
                                                                        =
                                                                           0.8264
                          Root MSE
                                        =
                                              12.7575
                                                         Adj R-squared =
                                                                           0.8110
                   Source
                            Partial SS
                                                df
                                                           MS
                                                                      F
                                                                           Prob>F
                   Model
                             34872.859
                                                     8718.2147
                                                                    53.57 0.0000
                                                 4
                  region
                             2197.7545
                                                 3
                                                     732.58484
                                                                     4.50 0.0076
                  medage
                             15327.423
                                                     15327.423
                                                                    94.18 0.0000
                                                 1
                Residual
                             7323.9611
                                                45
                                                     162.75469
                              42196.82
                                                49
                                                     861.15959
                   Total
```

For those more comfortable with linear regression, this is equivalent to the regression model

```
. regress brate i.region c.medage
```

You may use either.

We can use contrast to compute reference category effects for region. These contrasts compare the adjusted means of NCentral, South, and West regions with the adjusted mean of the NE region.

. contrast	r	region		
Contrasts	of	marginal	linear	predictions
Margins: a	asba	alanced		

	df	F	P>F	
region				
(NCentral vs NE)	1	2.24	0.1417	
(South vs NE)	1	0.78	0.3805	
(West vs NE)	1	10.33	0.0024	
Joint	3	4.50	0.0076	
Denominator	45			
	Contrast	Std. err.	[95% conf	. interval]
region				
(NCentral vs NE)	9.061063	6.057484	-3.139337	21.26146
(South vs NE)	5.06991	5.72396	-6.458738	16.59856
(West vs NE)	21.71328	6.755616	8.106774	35.31979

Let's add the interaction between region and medage to the model.

. anova brace region##c.medage		anova	brate	region##c.medage
--------------------------------	--	-------	-------	------------------

	Number of obs = Root MSE =	5 10.024		red = squared =	0.9000 0.8833
Source	Partial SS	df	MS	F	Prob>F
Model	37976.315	7	5425.1878	53.99	0.0000
region	3405.0704	3	1135.0235	11.30	0.0000
medage	5279.7145	1	5279.7145	52.54	0.0000
region#medage	3103.456	3	1034.4853	10.29	0.0000
Residual	4220.5051	42	100.48822		
Total	42196.82	49	861.15959		

The parameterization for the expected value of brate as a function of region and medage is given by

$$E(\texttt{brate}|\texttt{region} = i, \texttt{medage}) = \alpha_0 + \alpha_i + \beta_0 \texttt{medage} + \beta_i \texttt{medage}$$

where  $\alpha_0$  is the intercept and  $\beta_0$  is the slope of medage. We are modeling the effects of region in two different ways. The  $\alpha_i$  parameters measure the effect of region on the intercept, and the  $\beta_i$  parameters measure the effect of region on the slope of medage.

contrast computes and tests effects on slopes separately from effects on intercepts. First, we will compute the reference category effects of region on the intercept:

. contrast r.region Contrasts of marginal linear predictions Margins: asbalanced

	df	F	P>F	
region				
(NCentral vs NE)	1	0.09	0.7691	
(South vs NE)	1	0.01	0.9389	
(West vs NE)	1	8.50	0.0057	
Joint	3	11.30	0.0000	
Denominator	42			
	L			
	Contrast	Std. err.	[95% conf.	interval]
region				
(NCentral vs NE)	-49.38396	167.1281	-386.6622	287.8942
(South vs NE)	-9.058983	117.424	-246.0302	227.9123
(West vs NE)	343.0024	117.6547	105.5656	580.4393

Now, we will compute the reference category effects of region on the slope of medage:

. contrast r.region#c.medage Contrasts of marginal linear predictions Margins: asbalanced

	df	F	P>F	
region#c.medage				
(NCentral vs NE)	1	0.16	0.6917	
(South vs NE)	1	0.03	0.8558	
(West vs NE)	1	8.18	0.0066	
Joint	3	10.29	0.0000	
Denominator	42			
	Contrast	Std. err.	[95% conf.	interval]
region#c.medage				
(NCentral vs NE)	2.208539	5.530981	-8.953432	13.37051
(South vs NE)	.6928008	3.788735	-6.953175	8.338777
(West vs NE)	-10.94649	3.827357	-18.67041	-3.22257

At the 5% level, the slope of medage for the West region differs from that of the NE region, but at that level of significance, we cannot say that the slope for the NCentral or the South region differs from that of the NE region.

This model is simple enough that the reference category contrasts reproduce the coefficients for region and for the interactions in an equivalent model fit by regress.

. 1081000 014	1081011101	o u u B o					
Source	SS	df	MS		er of obs	=	50
Model Residual	37976.3149 4220.5051	7 42	5425.18784 100.488217	4 Prob	42) > F juared	=	53.99 0.0000 0.9000
					R-squared	=	0.8833
Total	42196.82	49	861.159592	2 Root	MSE	=	10.024
brate	Coefficient	Std. err.	t	P> t	[95% con	nf.	interval]
region NCentral	-49.38396	167.1281	-0.30	0.769	-386.6622	2	287.8942
South	-9.058983	117.424	-0.08	0.939	-246.0302		227.9123
West	343.0024	117.6547	2.92	0.006	105.5656		580.4393
medage	-8.802707	3.462865	-2.54	0.015	-15.79108	5	-1.814362
region# c.medage							
NCentral	2.208539	5.530981	0.40	0.692	-8.953432	2	13.37051
South	.6928008	3.788735	0.18	0.856	-6.953175	5	8.338777
West	-10.94649	3.827357	-2.86	0.007	-18.67042	1	-3.22257
_cons	411.8268	108.2084	3.81	0.000	193.4533	3	630.2002

. regress brate region##c.medage

This will not be the case for models that are more complicated.

### Chow tests

Now, let's suppose we are fitting a model for birthrates on median age and marriage rate. We are also interested in whether the regression coefficients differ for states in the east versus states in the west. We use census divisions to create a new variable, west, that indicates which states are in the western half of the United States.

```
. generate west = inlist(division, 4, 7, 8, 9)
```

We fit a model that includes a separate intercept for west as well as an interaction between west and each of the other variables in our model.

regress brate i west##c medage i west##c mrgrate

. regress brat	Se I.West##C.m	edage 1.we	sc##c.mrgra	lle			
Source	SS	df	MS		per of obs	=	50
Model Residual	38516.2172 3680.60281	5 44	7703.24344 83.6500639	Prob R-so	44) > > F quared	= =	92.09 0.0000 0.9128
Total	42196.82	49	861.159592		R-squared : MSE	=	0.9029 9.146
brate	Coefficient	Std. err.	t	P> t	[95% con	f.	interval]
1.west medage	327.8733 -7.532304	58.71793 1.387624	5.58 -5.43	0.000	209.5351 -10.32888		446.2115 -4.735731
west# c.medage 1	-10.11443	1.849103	-5.47	0.000	-13.84105		-6.387808
mrgrate	828.6813	643.3443	1.29	0.204	-467.8939		2125.257
west# c.mrgrate 1	-800.8036	645.488	-1.24	0.221	-2101.699		500.092
cons	366.5325	47.08904	7.78	0.000	271.6308		461.4343

We can test the effects of west on the intercept and on the slopes of medage and mrgrate. We will specify all of these effects in a single contrast command and include the overall option to obtain a joint test of effects, that is, a test that the coefficients for eastern states and for western states are equal.

```
. contrast west west#c.medage west#c.mrgrate, overall
Contrasts of marginal linear predictions
Margins: asbalanced
                          df
                                       F
                                                 P>F
                                   31.18
                                              0.0000
          west
                           1
                                   29.92
                                              0.0000
west#c.medage
                           1
west#c.mrgrate
                           1
                                    1.54
                                              0.2213
       Overall
                           3
                                   22.82
                                              0.0000
```

44

Denominator

This overall test is referred to as a Chow test in econometrics (Chow 1960).

## **Beyond linear models**

contrast may be used after almost any estimation command, with the added benefit that contrast provides direct support for testing main and interaction effects that is not available in most estimation commands. To illustrate, we will use contrast with results from a logistic regression. Stata's logit command fits logistic regression models, reporting the fitted regression coefficients. The logistic command fits the same models but reports odds ratios. Although contrast can report odds ratios for the computed effects, the tests are all computed from linear combinations of the model coefficients regardless of which estimation command we used.

Suppose we have data on patient satisfaction for three hospitals in a city. Let's begin by fitting a model for satisfied, whether the patient was satisfied with his or her treatment, using the main effects of hospital:

```
. use https://www.stata-press.com/data/r19/hospital, clear
(Artificial hospital satisfaction data)
. logit satisfied i.hospital
Iteration 0: Log likelihood = -393.72216
Iteration 1: Log likelihood = -387.55736
Iteration 2: Log likelihood = -387.4768
Iteration 3: Log likelihood = -387.47679
Logistic regression
                                                         Number of obs =
                                                                            802
                                                         LR chi2(2)
                                                                       = 12.49
                                                         Prob > chi2
                                                                       = 0.0019
Log likelihood = -387.47679
                                                         Pseudo R2
                                                                       = 0.0159
                                                 P>|z|
                                                           [95% conf. interval]
   satisfied
               Coefficient
                            Std. err.
                                            z
   hospital
          2
                 .5348129
                            .2136021
                                          2.50
                                                 0.012
                                                           .1161604
                                                                        .9534654
          3
                 .7354519
                            .2221929
                                          3.31
                                                 0.001
                                                           .2999618
                                                                       1.170942
       _cons
                 1.034708
                            .1391469
                                          7.44
                                                 0.000
                                                           .7619855
                                                                       1.307431
```

Because there are no other independent variables in this model, the reference category effects of hospital computed by contrast will match the fitted model coefficients, assuming a common reference level.

```
. contrast r.hospital
```

Contrasts of marginal linear predictions

Margins: asbalanced

	df	chi2	P>chi2
hospital (2 vs 1) (3 vs 1) Joint	1 1 2	6.27 10.96 12.55	0.0123 0.0009 0.0019

	Contrast	Std. err.	[95% conf. interval]
hospital (2 vs 1) (3 vs 1)	.5348129 .7354519	.2136021 .2221929	.1161604 .9534654 .2999618 1.170942

Number of obs =

Pseudo R2

LR chi2(14) =

Prob > chi2 = 0.0004

802

38.51

= 0.0489

We see that the reference category effects are equal to the fitted coefficients. They also have the same interpretation, the difference in log odds from the reference category. The top table also provides a joint test of these effects, a test of the main effects of hospital.

We also have information on the condition for which each patient is being treated in the variable illness. Here we fit a logistic regression using a two-way crossed model of hospital and illness.

Log likelihood = -374.46865

satisfied	Odds ratio	Std. err.	Z	P> z	[95% conf.	interval]
hospital						
- 2	1.226496	.5492177	0.46	0.648	.509921	2.950049
3	1.711111	.8061016	1.14	0.254	.6796395	4.308021
illness						
Stroke	1.328704	.6044214	0.62	0.532	.544779	3.240678
Pneumonia	.7993827	.3408305	-0.53	0.599	.3466015	1.843653
Lung dise	1.231481	.5627958	0.46	0.649	.5028318	3.016012
Kidney fa	1.25	.5489438	0.51	0.611	.5285676	2.956102
hospital# illness						
2#Stroke	2.434061	1.768427	1.22	0.221	.5860099	10.11016
2#Pneumonia 2 #	4.045805	2.868559	1.97	0.049	1.008058	16.23769
Lung dise 2 #	.54713	.3469342	-0.95	0.342	.1578866	1.89599
Kidney fa	1.594425	1.081104	0.69	0.491	.4221288	6.022312
3#Stroke	.5416535	.3590089	-0.93	0.355	.1477555	1.985635
3#Pneumonia 3 #	1.579502	1.042504	0.69	0.489	.4332209	5.758783
Lung dise 3 #	3.137388	2.595748	1.38	0.167	.6198955	15.87881
Kidney fa	1.672727	1.226149	0.70	0.483	.3976256	7.036812
_cons	2.571429	.8099239	3.00	0.003	1.386983	4.767358

Note: \_cons estimates baseline odds.

Using contrast, we can obtain an ANOVA-style table of tests for the main effects and interaction effects of hospital and illness.

```
. contrast hospital##illness
Contrasts of marginal linear predictions
Margins: asbalanced
                            df
                                       chi2
                                                 P>chi2
                                      14.92
                                                 0.0006
        hospital
                             2
         illness
                             4
                                       4.09
                                                 0.3937
hospital#illness
                             8
                                      20.45
                                                 0.0088
```

Our interaction effect is significant, so we decide to evaluate the simple reference category effects of hospital within illness. We are particularly interested in patient satisfaction when being treated for a heart attack or stroke, so we will use the i. operator to limit our output to simple effects within the first two illnesses.

```
. contrast r.hospital@i(1 2).illness, nowald
Contrasts of marginal linear predictions
Margins: asbalanced
```

	Contrast	Std. err.	[95% conf. interval]
hospital@illness (2 vs 1) Heart attack (2 vs 1) Stroke (3 vs 1) Heart attack (3 vs 1) Stroke	.2041611 1.093722 .5371429 0759859	.4477942 .5721288 .4710983 .4662325	6734995 1.081822 0276296 2.215074 3861928 1.460479 9897847 .8378129

The row labeled (2 vs 1) heart attack estimates simple effects on the log odds when comparing hospital 2 with hospital 1 for patients having heart attacks. These effects are differences in the cell means of the linear predictions.

We can add the or option to report an odds ratio for each of these simple effects:

```
. contrast r.hospital@i(1 2).illness, nowald or
Contrasts of marginal linear predictions
Margins: asbalanced
```

	Odds ratio	Std. err.	[95% conf. interval]	]
hospital@illness (2 vs 1) Heart attack (2 vs 1) Stroke (3 vs 1) Heart attack (3 vs 1) Stroke	1.226496 2.985366 1.711111 .9268293	.5492177 1.708014 .8061016 .4321179	.509921 2.95004 .9727486 9.16208 .6796395 4.30802 .3716567 2.31130	9 1

These odds ratios are just the exponentiated version of the contrasts in the previous table.

For contrasts of the margins of nonlinear predictions, such as predicted probabilities, see [R] margins, contrast.

# **Multiple equations**

contrast works with models containing multiple equations. Commands such as intreg and gnbreg allow their ancillary parameters to be modeled as functions of independent variables, and contrast can compute and test effects within these equations. In addition, contrast allows a special pseudofactor for equation—called \_eqns—when working with results from manova, mvreg, mlogit, and mprobit.

In example 4 of [MV] **manova**, we fit a two-way MANOVA model using data from Woodard (1931). Here we will fit this model using mvreg. The data represent patients with jaw fractures. y1 is the patient's age, y2 is blood lymphocytes, and y3 is blood polymorphonuclears. Two factor variables, gender and fracture, are used as independent variables.

Equ							
	ation	Obs	Parms	RMSE	"R-sq"	F	P>F
y1		27	6	10.21777	0.4086	2.902124	0.0382
y2		27	6	5.268768	0.4743	3.78967	0.0133
yЗ		27	6	4.993647	0.4518	3.460938	0.0195
		Coefficient	Std. er:	r. t	P> t	[95% conf	. interval]
y1							
	2.gender fracture	-17.5	11.0364	5 -1.59	0.128	-40.45156	5.451555
	2	-12.625	5.51822	5 -2.29	0.033	-24.10078	-1.149222
	3	5.666667	5.89923	1 0.96	0.348	-6.601456	17.93479
	gender# fracture						
	22	21.375	12.68678	3 1.68	0.107	-5.008595	47.75859
	23	8.833333	13.8349	2 0.64	0.530	-19.93796	37.60463
	_cons	39.5	4.17138	9.47	0.000	30.82513	48.17487
y2							
	2.gender fracture	20.5	5.6909	2 3.60	0.002	8.665083	32.33492
	2	-3.125	2.8454	5 -1.10	0.285	-9.042458	2.792458
	3	.6666667	3.04192	5 0.22	0.829	-5.659362	6.992696
	gender#						
	fracture	10,005	C F4400	7 0 00	0 007	22 00064	6 00000
	22 23	-19.625 -23.66667	6.54190		0.007 0.003	-33.22964	-6.02036
	-	-23.00007 35.5	7.13394 2.15096		0.003	-38.50252 31.02682	-8.830813 39.97318
	_cons		2.150900		0.000	51.02082	
yЗ							
	2.gender	-18.16667	5.39375	5 -3.37	0.003	-29.38359	-6.949739
	fracture	4 000000	0 00007	7 0 40	0 000	4 50540	6 604705
	2	1.083333	2.69687		0.692	-4.52513	6.691797
	3 gender#	-3	2.88308	3 -1.04	0.310	-8.9957	2.9957
	gender# fracture						
		19.91667	6.20030	5 3.21	0.004	7.022426	32.81091
	22	23.5	6.7614		0.004	9.438837	37.56116
	20	20.0	0.1014	5 5.40	0.002	3.400001	01.00110

contrast computes Wald tests using the coefficients from the first equation by default.

```
. contrast gender##fracture
Contrasts of marginal linear predictions
Margins: asbalanced
                           df
                                         F
                                                  P>F
y1
                                               0.1569
                                     2.16
         gender
                            1
       fracture
                            2
                                     2.74
                                               0.0880
gender#fracture
                            2
                                      1.69
                                               0.2085
    Denominator
                           21
```

Here we use the equation() option to compute the Wald tests in the y2 equation:

```
    contrast gender##fracture, equation(y2)
    Contrasts of marginal linear predictions
    Margins: asbalanced
```

	df	F	P>F
у2			
gender	1	5.41	0.0301
fracture	2	7.97	0.0027
gender#fracture	2	5.97	0.0088
Denominator	21		

Here we use the equation index to compute the Wald tests in the third equation:

```
. contrast gender##fracture, equation(#3)
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
уЗ			
gender	1	2.23	0.1502
fracture	2	6.36	0.0069
gender#fracture	2	6.66	0.0058
Denominator	21		

Here we use the atequations option to compute Wald tests for each equation in the model. We also use the vsquish option to suppress the extra blank lines between terms.

. contrast gender##fracture, atequations vsquish Contrasts of marginal linear predictions Margins: asbalanced

	• · · · · · · · · · · · · · · · · · · ·		
	df	F	P>F
y1			
gender	1	2.16	0.1569
fracture	2	2.74	0.0880
gender#fracture	2	1.69	0.2085
y2			
gender	1	5.41	0.0301
fracture	2	7.97	0.0027
gender#fracture	2	5.97	0.0088
y3			
gender	1	2.23	0.1502
fracture	2	6.36	0.0069
gender#fracture	2	6.66	0.0058
Denominator	21		

Because we are investigating the results from mvreg, we can use the special \_eqns factor to test for a marginal effect on the means among the dependent variables:

	df	F	P>F
_eqns	2	49.19	0.0000
Denominator	21		

Here we test whether the main effects of gender differ among the dependent variables:

```
. contrast gender#_eqns
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
gender#_eqns	2	3.61	0.0448
Denominator	21		

Although it is not terribly interesting in this case, we can even calculate contrasts across equations:

```
. contrast gender#r._eqns
Contrasts of marginal linear predictions
Margins: asbalanced
                              df
                                            F
                                                     P>F
     gender# eqns
(joint) (2 vs 1)
                                        5.82
                                                  0.0251
                               1
                                        0.40
                                                  0.5352
(joint) (3 vs 1)
                               1
           Joint
                               2
                                        3.61
                                                  0.0448
      Denominator
                              21
```

### Video example

Introduction to contrasts in Stata: One-way ANOVA

# Stored results

contrast stores the following in r():

```
Scalars
    r(df_r)
                                  variance degrees of freedom
                                  number of terms in termlist
    r(k_terms)
    r(level)
                                  confidence level of confidence intervals
Macros
    r(cmd)
                                 contrast
    r(cmdline)
                                  command as typed
    r(est_cmd)
                                  e(cmd) from original estimation results
    r(est_cmdline)
                                  e(cmdline) from original estimation results
    r(title)
                                  title in output
    r(overall)
                                  overall or empty
    r(emptycells)
                                 empspec from emptycells()
    r(mcmethod)
                                 method from mcompare()
    r(mctitle)
                                 title for method from mcompare()
    r(mcadjustall)
                                  adjustall or empty
    r(margin_method)
                                  asbalanced or asobserved
Matrices
    r(b)
                                 contrast estimates
    r(V)
                                 variance-covariance matrix of the contrast estimates
    r(error)
                                 contrast estimability codes;
                                    0 means estimable.
                                    8 means not estimable
    r(L)
                                  matrix of contrasts applied to the model coefficients
    r(table)
                                  matrix containing the contrasts with their standard errors,
                                  test statistics, p-values, and confidence intervals
    r(F)
                                  vector of F statistics; r(df_r) present
    r(chi2)
                                  vector of \chi^2 statistics; r(df_r) not present
    r(p)
                                  vector of p-values corresponding to r(F) or r(chi2)
    r(df)
                                  vector of degrees of freedom corresponding to r(p)
    r(df2)
                                  vector of denominator degrees of freedom corresponding to r(F)
```

contrast with the post option stores the following in e():

Scalars	
e(df_r)	variance degrees of freedom
e(k_terms)	number of terms in termlist
Macros	
e(cmd)	contrast
e(cmdline)	command as typed
e(properties)	b V
e(est_cmd)	e(cmd) from original estimation results
e(est_cmdline)	e(cmdline) from original estimation results
e(title)	title in output
e(overall)	overall or empty
e(emptycells)	<pre>empspec from emptycells()</pre>
e(margin_method)	asbalanced or asobserved
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	contrast estimates
e(V)	variance-covariance matrix of the contrast estimates
e(error)	contrast estimability codes;
	0 means estimable,
	8 means not estimable
e(L)	matrix of contrasts applied to the model coefficients
e(F)	vector of unadjusted F statistics; e(df_r) present
e(chi2)	vector of $\chi^2$ statistics; e(df_r) not present
e(p)	vector of unadjusted p-values corresponding to e(F) or e(chi2)
e(df)	vector of degrees of freedom corresponding to e(p)
e(df2)	vector of denominator degrees of freedom corresponding to e(F)

# Methods and formulas

Methods and formulas are presented under the following headings:

Marginal linear predictions Contrast operators Reference level contrasts Adjacent contrasts Grand mean contrasts Helmert contrasts Reverse Helmert contrasts Orthogonal polynomial contrasts Contrasts within interactions Multiple comparisons

# Marginal linear predictions

contrast treats intercept effects separately from slope effects. To illustrate, consider the following parameterization for a quadratic regression of y on x that also models the effects of two factor variables A and B, where the levels of A are indexed by  $i = 1, \ldots, k_a$  and the levels of B are indexed by  $j = 1, \ldots, k_b$ .

$$\begin{split} E(y|A = i, B = j, x) &= \eta_{0ij} + \eta_{1ij}x + \eta_{2ij}x^2 \\ \eta_{0ij} &= \eta_0 + \alpha_{0i} + \beta_{0j} + (\alpha\beta)_{0ij} \\ \eta_{1ij} &= \eta_1 + \alpha_{1i} + \beta_{1j} + (\alpha\beta)_{1ij} \\ \eta_{2ij} &= \eta_2 + \alpha_{2i} + \beta_{2j} + (\alpha\beta)_{2ij} \end{split}$$

We have partitioned the coefficients into three groups of parameters:  $\eta_{0ij}$  is a cell prediction for the intercept,  $\eta_{1ij}$  is a cell prediction for the slope on x, and  $\eta_{2ij}$  is a cell prediction for the slope on  $x^2$ . For the intercept parameters,  $\eta_0$  is the intercept,  $\alpha_{0i}$  represents a main effect for factor A at its *i*th level,  $\beta_{0j}$  represents a main effect for factor B at its *j*th level, and  $(\alpha\beta)_{0ij}$  represents an effect for the interaction of A and B at the *ij*th level. The individual coefficients in  $\eta_{1ij}$  and  $\eta_{2ij}$  have similar interpretations, but the effects are on the slopes of x and  $x^2$ , respectively.

The marginal intercepts for A are given by

$$\eta_{0i.} = \sum_{j=1}^{k_b} f_{ij} \eta_{0ij}$$

where  $f_{ij}$  is a marginal relative frequency of the *j*th level of *B* and is controlled by the asobserved and emptycells(reweight) options according to

$$f_{ij} = \begin{cases} 1/k_b, & \text{default} \\ w_{.j}/w_{..}, & \text{asobserved} \\ 1/(k_b - e_{i.}), & \text{emptycells(reweight)} \\ w_{ij}/w_{i.}, & \text{emptycells(reweight) and asobserved} \end{cases}$$

Above,  $w_{ij}$  is the number of individuals with A at its *i*th level and B at its *j*th,

$$w_{i.} = \sum_{j=1}^{k_b} w_{ij}$$
$$w_{.j} = \sum_{i=1}^{k_a} w_{ij}$$
$$w_{..} = \sum_{i=1}^{k_a} \sum_{j=1}^{k_b} w_{ij}$$

and  $e_i$  is the number of empty cells where A is at its *i*th level. The marginal intercepts for B and marginal slopes on x and  $x^2$  are similarly defined.

Estimates for the cell intercepts and slopes are computed using the corresponding linear combination of the coefficients from the fitted model. For example, the estimated cell intercepts are computed using

$$\hat{\eta}_{0ij} = \hat{\eta}_0 + \hat{\alpha}_{0i} + \hat{\beta}_{0j} + (\widehat{\alpha\beta})_{0ij}$$

and the estimated marginal intercepts for A are computed as

$$\hat{\eta}_{0i.} = \sum_{j=1}^{k_b} f_{ij} \hat{\eta}_{0ij}$$

#### **Contrast operators**

contrast performs Wald tests using linear combinations of marginal linear predictions. For example, the following linear combination can be used to test for a specific effect of factor A on the marginal intercepts.

$$\sum_{i=1}^{k_a} c_i \eta_{0i}.$$

If the  $c_i$  elements sum to zero, the linear combination is called a contrast. If the factor A is represented by a variable named A, then we specify this contrast using the following syntax:

$$\{ A c_1 c_2 \ldots c_{k_a} \}$$

Similarly, the following linear combination can be used to test for a specific interaction effect of factors A and B on the marginal slope of x.

$$\sum_{i=1}^{k_a} \sum_{j=1}^{k_b} c_{ij} \eta_{1ij}$$

If the factor B is represented by a variable named B, then we specify this contrast using the following syntax:

$$\{ A \# B \ c_{11} \ c_{12} \ \dots \ c_{1k_b} \ c_{21} \ \dots \ c_{k_a k_b} \}$$

contrast has variable operators for several commonly used contrasts. Each contrast operator specifies a matrix of linear combinations that yield the requested set of contrasts to be applied to the marginal linear predictions associated with the attached factor variable.

#### **Reference level contrasts**

The r. operator compares each level with a reference level. Let **R** be the corresponding contrast matrix for factor A, and then **R** is a  $(k_a - 1) \times k_a$  matrix with elements

$$\mathbf{R}_{ij} = \begin{cases} -1, & \text{if } j \text{ is the reference level} \\ 1, & \text{if } i = j \text{ and } j \text{ is less than the reference level} \\ 1, & \text{if } i + 1 = j \text{ and } j \text{ is greater than the reference level} \\ 0, & \text{otherwise} \end{cases}$$

If  $k_a = 5$  and the reference level is the third level of A (specified as rb(#3).A), then

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

#### Adjacent contrasts

The a. operator compares each level with the next level. Let A be the corresponding contrast matrix for factor A, and then A is a  $(k_a - 1) \times k_a$  matrix with elements

$$\mathbf{A}_{ij} = \begin{cases} 1, & \text{if } i = j \\ -1, & \text{if } i + 1 = j \\ 0, & \text{otherwise} \end{cases}$$

If  $k_a = 5$ , then

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

The ar. operator compares each level with the previous level. If A is the contrast matrix for the a. operator, then -A is the corresponding contrast matrix for the ar. operator.

#### Grand mean contrasts

The g. operator compares each level with the mean of all the levels. Let G be the corresponding contrast matrix for factor A, and then G is a  $k_a \times k_a$  matrix with elements

$$\mathbf{G}_{ij} = \begin{cases} 1 - 1/k_a, & \text{if } i = j \\ -1/k_a, & \text{if } i \neq j \end{cases}$$

If  $k_a = 5$ , then

$$\mathbf{G} = \begin{pmatrix} 4/5 & -1/5 & -1/5 & -1/5 & -1/5 \\ -1/5 & 4/5 & -1/5 & -1/5 & -1/5 \\ -1/5 & -1/5 & 4/5 & -1/5 & -1/5 \\ -1/5 & -1/5 & -1/5 & 4/5 & -1/5 \\ -1/5 & -1/5 & -1/5 & -1/5 & 4/5 \end{pmatrix}$$

The gw. operator compares each level with the weighted mean of all the levels. The weights are taken from the observed weighted cell frequencies in the estimation sample of the fitted model. Let  $\mathbf{G}_w$  be the corresponding contrast matrix for factor A, and then  $\mathbf{G}_w$  is a  $k_a \times k_a$  matrix with elements

$$\mathbf{G}_{ij} = \begin{cases} 1-w_i/w_{\cdot}, & \text{if } i=j\\ -w_j/w_{\cdot}, & \text{if } i\neq j \end{cases}$$

where  $w_i$  is a marginal weight representing the number of individuals with A at its *i*th level and  $w_i = \sum_i w_i$ .

#### Helmert contrasts

The h. operator compares each level with the mean of the subsequent levels. Let H be the corresponding contrast matrix for factor A, and then H is a  $(k_a - 1) \times k_a$  matrix with elements

$$\mathbf{H}_{ij} = \begin{cases} 1, & \text{if } i = j \\ -1/(k_a - i), & \text{if } i < j \\ 0, & \text{otherwise} \end{cases}$$

If  $k_a = 5$ , then

$$\mathbf{H} = \begin{pmatrix} 1 & -1/4 & -1/4 & -1/4 \\ 0 & 1 & -1/3 & -1/3 \\ 0 & 0 & 1 & -1/2 & -1/2 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

The hw. operator compares each level with the weighted mean of the subsequent levels. Let  $\mathbf{H}_w$  be the corresponding contrast matrix for factor A, and then  $\mathbf{H}_w$  is a  $(k_a - 1) \times k_a$  matrix with elements

$$\mathbf{H}_{wij} = \begin{cases} 1, & \text{if } i = j \\ -w_j / \sum_{l=j}^{k_a} w_l, & \text{if } i < j \\ 0, & \text{otherwise} \end{cases}$$

#### **Reverse Helmert contrasts**

The j. operator compares each level with the mean of the previous levels. Let J be the corresponding contrast matrix for factor A, and then J is a  $(k_a - 1) \times k_a$  matrix with elements

$$\mathbf{J}_{ij} = \begin{cases} 1, & \text{if } i+1=j \\ -1/i, & \text{if } j \leq i \\ 0, & \text{otherwise} \end{cases}$$

If  $k_a = 5$ , then

$$\mathbf{H} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 1 & 0 & 0 \\ -1/3 & -1/3 & -1/3 & 1 & 0 \\ -1/4 & -1/4 & -1/4 & -1/4 & 1 \end{pmatrix}$$

The jw. operator compares each level with the weighted mean of the previous levels. Let  $\mathbf{J}_w$  be the corresponding contrast matrix for factor A, and then  $\mathbf{J}_w$  is a  $(k_a - 1) \times k_a$  matrix with elements

$$\mathbf{J}_{wij} = \begin{cases} 1, & \text{if } i+1=j \\ -w_j/\sum_{l=1}^i w_l, & \text{if } i\leq j \\ 0, & \text{otherwise} \end{cases}$$

#### Orthogonal polynomial contrasts

The p. operator applies orthogonal polynomial contrasts using the level values of the attached factor variable. The q. operator applies orthogonal polynomial contrasts using the level indices of the attached factor variable. These two operators are equivalent when the level values of the attached factor are equally spaced. The pw. and qw. operators are weighted versions of p. and q., where the weights are taken from the observed weighted cell frequencies in the estimation sample of the fitted model. contrast uses the Christoffel–Darboux recurrence formula for computing orthogonal polynomial contrasts (Abramowitz and Stegun 1964). The elements of the contrasts are normalized such that

$$\mathbf{Q}'\mathbf{W}\mathbf{Q} = \frac{1}{w_{\cdot}}\mathbf{I}$$

where **W** is a diagonal matrix of the marginal cell weights  $w_1, w_2, \ldots, w_k$  of the attached factor variable (all 1 for p. and q.), and  $w_i$  is the sum of the weights (the number of levels k for p. and q.).

### Contrasts within interactions

Contrast operators are allowed to be specified on factor variables participating in interactions. In such cases, contrast applies the proper matrix product of the contrast matrices to the cell margins of the interacted factor variables.

For example, consider the contrasts implied by specifying r.A#h.B. Let M be the matrix of estimated cell margins for the levels of A and B, where the rows of M are indexed by the levels of A and the columns are indexed by the levels of B. contrast puts the estimated cell margins in the following vector form:

$$\mathbf{v} = \operatorname{vec}(\mathbf{M}') = \begin{pmatrix} \mathbf{M}_{11} \\ \mathbf{M}_{12} \\ \vdots \\ \mathbf{M}_{1k_b} \\ \mathbf{M}_{21} \\ \mathbf{M}_{22} \\ \vdots \\ \mathbf{M}_{2k_b} \\ \vdots \\ \mathbf{M}_{k_ak_b} \end{pmatrix}$$

The individual contrasts are then given by the elements of

$$(\mathbf{R} \otimes \mathbf{H})\mathbf{v}$$

where  $\otimes$  denotes the Kronecker direct product.

## **Multiple comparisons**

See [R] **pwcompare** for details on the methods and formulas used to adjust *p*-values and confidence intervals for multiple comparisons. The formulas for Bonferroni's method and Šidák's method are presented with m = k(k-1)/2, the number of pairwise comparisons for a factor term with *k* levels. For contrasts, *m* is instead the number of contrasts being performed on the factor term; often, m = k - 1 for a term with *k* levels.

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# Also see

- [R] contrast postestimation Postestimation tools for contrast
- [R] lincom Linear combinations of parameters
- [R] margins Marginal means, predictive margins, and marginal effects
- [R] margins, contrast Contrasts of margins
- [R] pwcompare Pairwise comparisons
- [R] test Test linear hypotheses after estimation

#### [U] 20 Estimation and postestimation commands

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