

Intro (ciwidth) — Introduction to precision and sample-size analysis for confidence intervals[Description](#)[Remarks and examples](#)[References](#)[Also see](#)

Description

Precision and sample-size (PrSS) analysis is essential for designing a statistical study that uses confidence intervals (CIs) for inference. It investigates the optimal allocation of study resources to increase the likelihood of the successful achievement of a study objective. PrSS analysis provides an estimate of the sample size required to achieve the desired [precision of a CI](#) in a future study.

For power and sample-size analysis for hypothesis tests, see [\[PSS-2\] Intro \(power\)](#).

Remarks and examples

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Remarks are presented under the following headings:

*Precision and sample-size analysis**Confidence intervals**Components of PrSS analysis**Confidence level**CI width**Probability of CI width**Sample size**One-sided versus two-sided CIs**Sensitivity analysis**An example of PrSS analysis in Stata*

This entry describes the statistical methodology for PrSS and its terminology that will be used throughout the manual. For a list of supported PrSS methods and the description of the software, see [\[PSS-3\] ciwidth](#). For more information about PrSS analysis, see [Meeker, Hahn, and Escobar \(2017\)](#), [Dixon and Massey \(1983\)](#), [Zar \(2010\)](#), and [Chow et al. \(2018\)](#), to name a few.

For power and sample-size analysis for hypothesis tests, see [\[PSS-2\] Intro \(power\)](#).

Precision and sample-size analysis

Precision and sample-size (PrSS) analysis is a key component in designing a statistical study that uses confidence intervals (CIs) for inference. It investigates the optimal allocation of study resources to increase the likelihood of the successful achievement of a study objective.

How many subjects do we need in a study to achieve a CI of a desired width? A study with too few subjects may have a CI that is too wide to be useful in practice. A study with too many subjects may offer little gain and will thus waste time and resources. Given limited resources, what is the largest CI width that can be expected for an anticipated number of subjects? PrSS analysis helps answer these questions and more. In what follows, when we refer to PrSS analysis, we imply any of these goals.

We consider PrSS analysis of a future study as opposed to a study that has already happened.

In the context of PrSS analysis, a CI is the inferential method used to evaluate research objectives of a study. PrSS analysis is provided for CIs for one mean, the difference between two means, and more. See [\[PSS-3\] ciwidth](#) for a full list of methods.

Before we discuss the components of PrSS analysis, let us first revisit the basics of CIs.

Confidence intervals

Consider a study in which the research objective is to estimate a population parameter of interest θ . For example, suppose we want to estimate the mean lifetime of all the bulbs in a factory. The entire population is rarely available in a study. Instead, a sample $\mathbf{x} = (x_1, x_2, \dots, x_n)$ of a particular size n is randomly selected from a population, and the parameter of interest is estimated from this sample. In our bulb lifetime example, we can measure only the lifetime of a few bulbs to make inferences about the mean lifetime of all the bulbs in the factory. Continuing with our example, we can use the sample mean $\hat{\theta}$ as an estimate of the mean lifetime of all bulbs. But what if we select a different random sample? Would we obtain the same sample-mean estimate? Given the random nature of the sample, our new estimate will most likely be different. But how much different? In other words, we want to know how precise our sample-mean estimate is given the stochastic nature of a random sample.

In contrast to a single point estimate such as a sample mean, a CI provides an interval estimate for θ that incorporates the precision of the single estimate. Specifically, a $100(1 - \alpha)\%$ CI for θ is formed as an interval $[ll(\mathbf{x}), ul(\mathbf{x})]$ such that $\Pr\{ll(\mathbf{x}) \leq \theta \leq ul(\mathbf{x})\} \geq 1 - \alpha$, for $0 \leq \alpha \leq 1$. $1 - \alpha$, or expressed as a percentage $100(1 - \alpha)\%$, is known as a confidence coefficient or confidence level. $ll = ll(\mathbf{x})$ and $ul = ul(\mathbf{x})$ are confidence limits or confidence bounds, and they depend on the observed data \mathbf{x} . A $100(1 - \alpha)\%$ CI is constructed such that, in a repeated independent sampling, it is guaranteed to contain the true parameter value $100(1 - \alpha)\%$ of the times. Note that for the observed sample \mathbf{x} , the probability that the observed CI $[ll, ul]$ contains the true parameter value is either 0 or 1, and we do not know which. Therefore, a CI is often interpreted as a plausible range of values for θ .

CIs can be two sided or one sided. A two-sided CI has finite confidence limits. For a one-sided CI, one of the confidence limits is infinite. Specifically, an upper one-sided CI has $ll = -\infty$ and is of the form $(-\infty, ul]$. A lower one-sided CI has $ul = \infty$ and is of the form $[ll, \infty)$.

CIs are formed around a point estimate of the parameter of interest. For instance, a normal-based $100(1 - \alpha)\%$ two-sided CI for one population mean has the form

$$\left[\bar{\theta} - z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{\theta} + z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}} \right]$$

where $\bar{\theta}$ is the sample mean, σ is the population standard deviation, and $z_{1-\alpha/2}$ is the $(1 - \alpha/2)$ th quantile of the standard normal distribution. The width of the CI can be used to measure the degree of precision of the point estimate.

For a two-sided CI, the width w is defined as the difference between the upper and lower limits, $w = ul - ll$. For an upper one-sided CI, the width is defined as the difference between the upper confidence limit and the point estimate, $w = ul - \hat{\theta}$. For a lower one-sided CI, the width is defined as the difference between the point estimate and the lower confidence limit, $w = \hat{\theta} - ll$. For instance, for the above normal-based mean CI, $w = 2z_{\alpha/2}\sigma/\sqrt{n}$ for a two-sided CI and $w = z_{\alpha/2}\sigma/\sqrt{n}$ for a one-sided CI. A half-width $w/2$, also known as a margin of error, is also used as a measure of precision for a symmetric CI. In our PrSS analysis, we will use the CI width as a measure of CI precision and will use the two terms (CI precision and CI width) interchangeably.

Typically, CIs depend on the sample size, confidence level, point estimate of the parameter of interest, and potentially on other method-dependent parameters such as the population standard deviation in our normal-based CI. As the sample size increases, the CI width becomes smaller. The CI becomes narrower and thus has a higher precision. As the confidence level increases, the CI width becomes larger and the CI becomes wider.

To compute CIs in Stata, use `ci`, `mean`, `proportion`, `cc`, to name a few.

There is a strong correspondence between CIs and [hypothesis tests](#). A $100(1 - \alpha)\%$ CI can be obtained by inverting the acceptance region of the corresponding level α test. In other words, a $100(1 - \alpha)\%$ CI provides the entire range of hypothetical values for a parameter of interest that cannot be rejected by the test at a significance level of α . Despite the strong correspondence between CIs and hypothesis tests, PrSS analysis and PSS analysis will not necessarily lead to the same requirements for the sample size. A hypothesis test compares the parameter of interest with a single value, whereas a CI provides a range of plausible values. Thus, for the same significance level, the sample-size requirements for the CI will generally be larger than for the hypothesis test.

Next, we review concepts specific to PrSS analysis.

Components of PrSS analysis

The main goal of PrSS analysis is to help plan a study such that the chosen CI method has a desired precision. When the CI precision depends on unknown parameters that need to be estimated from the data, its value will vary from one random sample to another. To ensure that a CI in a future sample achieves at least the desired precision, PrSS analysis must account for the sampling variability of the CI precision. To address this, PrSS incorporates the probability of CI width. This is the probability that a future CI will have a width no larger than the target width. In what follows, we will use the CI width to measure the CI precision.

You can think of PrSS analysis analogously to [power and sample-size analysis](#) with the confidence level playing the role of the significance level, the probability of CI width playing the role of power, and the CI width playing the role of the effect size.

The goal of PrSS analysis can be achieved in several ways. You can

- compute sample size directly given the specified confidence level, CI width, probability of CI width, and other study parameters;
- evaluate the CI width that can be achieved for a specific sample size given a confidence level, probability of CI width, and other study parameters;
- evaluate the probability of CI width that can be achieved for a specific sample size and CI width given a confidence level and other study parameters; or
- evaluate the sensitivity of the sample-size requirements to various study parameters.

The main components of PrSS analysis are

- confidence level, $100(1 - \alpha)$;
- CI width, w ;
- probability of CI width, $\Pr(w)$; and
- sample size, n .

Below we describe each of the main components of PrSS analysis in more detail.

Confidence level

We denote the confidence level by $100(1 - \alpha)$, where α is between 0 and 1, inclusively. You can think of α as the significance level for the corresponding hypothesis test. Researchers typically set the significance level to a small value such as 0.01 or 0.05 to protect the null hypothesis, which usually represents a state for which an incorrect decision is more costly. Similarly, the typical confidence levels are 90%, 95%, etc. The most frequently used level is 95%, and it is also the default confidence level for the `ciwidth` command.

CI width

For a two-sided CI, the CI width is the distance from the upper limit to the lower limit. For an upper one-sided CI, the CI width is the distance from the upper limit to the point estimate. For a lower one-sided CI, the CI width is the distance from the point estimate to the lower limit. CI width is commonly used as a measure of the CI precision. The larger the CI width, the wider and less precise the CI.

For some methods, the lower and upper limits of a two-sided CI are symmetric around the point estimate. In this case, the half-width, also called the margin of error, is used to measure the CI precision.

The CI width is an increasing function of the confidence level. So, as we increase the confidence level, the CI becomes wider. The CI width is generally a decreasing function of the sample size.

Probability of CI width

One of the main goals of PrSS analysis is to estimate the required sample size such that, in a new study, the estimated CI will have a certain width. During PrSS analysis, a researcher specifies the desired CI width, and the sample size is computed based on this width for a given confidence level. The CI width often depends on other study parameters such as a standard deviation in a CI for one mean. Unless we know the values of all study parameters that affect the CI width, the width will generally vary from one sample to another.

For example, the population standard deviation is used to define the width of a one-mean CI. When the population standard deviation is unknown, its estimate, a sample standard deviation, is used in the computations. This estimate will be different between different random samples. Thus, the CI width will be different. To obtain a reliable estimate of the required sample size, the sampling variability of the CI width must be accounted for in the computations.

[Kupper and Hafner \(1989\)](#) recommend that PrSS analysis incorporates what we call a probability of CI width, denoted as $\Pr(w)$. This is a probability that the width of a CI in a future study will not exceed a prespecified target value. You can think of the probability of CI width as the analog of power in power and sample-size analysis. As with power, the values of probability of CI width close to 1 such as 0.9 and 0.95 are commonly used in PrSS analysis.

See [Kupper and Hafner \(1989\)](#) and [Meeker, Hahn, and Escobar \(2017\)](#) for more information.

Sample size

Sample size is usually the main component of interest in PrSS analysis. The sample size required to successfully achieve the objective of a study is determined given a specified confidence level, CI width, probability of CI width, and other study parameters. The higher the desired confidence level, the larger the sample size required, with everything else being equal. To achieve a narrower CI, a larger sample size is required, with everything else being equal. The higher the probability of CI width, the larger the required sample size.

When you compute sample size, the actual probability of CI width (probability of CI width corresponding to the obtained sample size) will most likely be different from the probability of CI width you requested because sample size is an integer. In the computation, the resulting fractional sample size that corresponds to the requested probability of CI width is usually rounded to the nearest integer. To be conservative and to ensure that the actual probability of CI width is at least as large as the requested one, the sample size is rounded up. For sample-size computations that do not use probability of CI width, such as the case of a known population standard deviation for a one-mean CI, the above applies to the specified CI width.

For two-sample designs, fractional sample sizes may arise when you specify a given sample size to compute CI precision. For example, to accommodate an odd total sample size of, say, 51 in a balanced two-sample design, each individual sample size must be 25.5. To be conservative, sample sizes are rounded down on input. The actual sample sizes in our example would be 25 and 25 for a total of 50. See *Fractional sample sizes* in [PSS-4] **Unbalanced designs** for details about sample-size rounding.

For two samples, the allocation of subjects between groups also affects the estimates of CI width and probability of CI width.

One-sided versus two-sided CIs

Two-sided CIs are commonly used in analysis. But there are scenarios where the major interest is only in the lower or upper limit. For example, we may want to know the lower limit for the mean lifetime of bulbs to ensure that, with high certainty, customers will not get bulbs that last less than a year on average, given a 95% confidence level.

For a symmetric two-sided CI, the true value of the parameter of interest has an equal probability of being located outside the lower and upper limits. In this case, by simply changing the confidence level, you can get two-sided confidence limits when computing a one-sided CI and vice versa. For example, the 90% upper confidence limit for a two-sided CI is the upper limit for the 95% one-sided CI.

Sensitivity analysis

Because of limited resources, it may not always be feasible to conduct a study under the original ideal specification. In this case, you may vary study parameters to find an appropriate balance between the desired CI width, probability of CI width, sample size, available resources, and the objective of the study. For example, a researcher may decide to lower the desired confidence level or probability of CI width to decrease the required sample size. In some situations, it may not be possible to reduce the required sample size, in which case more resources must be acquired before the study can be conducted.

CI precision is a function of all the components we described in the previous section—none of the components can be viewed in isolation. For this reason, it is important to perform sensitivity analysis, which investigates CI precision for various specifications of study parameters, and refine the sample-size requirements based on the findings prior to conducting a study. Tables of sample-size values (see [PSS-3] **ciwidth, table**) and graphs of sample-size and CI precision curves (see [PSS-3] **ciwidth, graph**) may be useful for this purpose.

An example of PrSS analysis in Stata

Consider a study of math scores from the SAT exam, similar to the one we discussed in [PSS-2] **Intro (power)**. Suppose that now the investigators would like to measure the average SAT math score. Prior to conducting the study, investigators would like to estimate the sample size required for the two-sided 95% CI for the average SAT score to have a width no larger than 20 points. We can use the `ciwidth onemean` command to estimate the sample size for this study; see [PSS-3] **ciwidth onemean** for more examples.

The investigators do not know the actual value of the standard deviation of the scores, but they do not anticipate it to be larger than the national value of 117 points. As we mentioned in *Probability of CI width*, the CI width of a one-mean CI depends on the standard deviation. Because we assume

an unknown standard deviation, the CI width may vary from one study to another. We must account for this sampling variability when estimating the sample size. To ensure that the CI will have a width of at most 20 points, we must also specify the probability of achieving the target CI width.

Below we demonstrate PrSS analysis of this example interactively by typing the commands; see [PSS-3] GUI (ciwidth) for point-and-click analysis of this example.

We specify the values of the standard deviation, probability of CI width, and CI width in the corresponding `sd()`, `probwidth()`, and `width()` options. `ciwidth onemean` assumes a 95%-level two-sided CI, so we do not need to specify any additional options.

```
. ciwidth onemean, probwidth(.9) width(20) sd(117)
Performing iteration ...
Estimated sample size for a one-mean CI
Student's t two-sided CI
Study parameters:
    level =    95.00
    Pr_width =  0.9000
    width =   20.0000
    sd =   117.0000
Estimated sample size:
    N =    569
```

We find that a sample of 569 subjects is required to obtain a two-sided 95% CI for the mean SAT score that with a 90% probability will have a width no larger than 20 points.

Suppose investigators can enroll 600 subjects, and they would like to estimate the corresponding probability of obtaining the same target CI width as before. To compute the probability of CI width, we need to specify the sample size, so we modify the command above by replacing the `probwidth(.9)` option with the `n(600)` option.

```
. ciwidth onemean, n(600) width(20) sd(117)
Estimated probability of width for a one-mean CI
Student's t two-sided CI
Study parameters:
    level =    95.00
    N =    600
    width =   20.0000
    sd =   117.0000
Estimated probability of width:
    Pr_width =  0.9887
```

For a larger sample of 600 subjects, the probability of obtaining a CI width no larger than 20 increases to 99%.

Investigators would also like to estimate how narrow a CI width they can obtain with a 90% certainty, given a sample of 600 subjects. To compute the CI width, we specify both the probability of CI width in the `probwidth()` option and the sample size in the `n()` option.

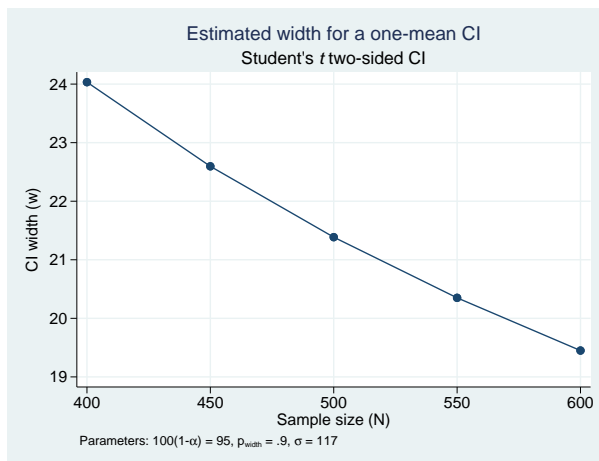
```
. ciwidth onemean, probwidth(.9) n(600) sd(117)
Estimated width for a one-mean CI
Student's t two-sided CI
Study parameters:
    level =    95.00
      N =    600
Pr_width =    0.9000
    sd = 117.0000
Estimated width:
    width =    19.4499
```

We obtain a CI width of about 19 points.

Continuing their analysis, investigators want to assess the impact of different sample sizes on CI width. They wish to estimate CI width for a range of sample sizes between 400 and 600, in increments of 50.

To display the results graphically, we specify the `graph` option. We abbreviate the range of sample sizes as *numlist* in the `n()` option.

```
. ciwidth onemean, probwidth(.9) n(400(50)600) sd(117) graph
```



The default graph plots the estimated CI width on the *y* axis and the requested sample size on the *x* axis. Note how CI width decreases as the sample size increases.

For multiple values of parameters, the results are automatically displayed in a table. However, when we specify the `graph` option, as we did above, the tabular output is suppressed. We can choose to display the results in a table as well by specifying the `table` option:

```
. ciwidth onemean, probwidth(.9) n(400(50)600) sd(117) graph table
(output omitted)
```

The `ciwidth` command performs PrSS analysis for a number of CIs; see [PSS-3] `ciwidth` and method-specific entries for more examples. You can also add your own methods to the `ciwidth` command as described in [PSS-3] `ciwidth usermethod`.

References

- Chow, S.-C., J. Shao, H. Wang, and Y. Lohknygina. 2018. *Sample Size Calculations in Clinical Research*. 3rd ed. Boca Raton, FL: CRC Press.
- Dixon, W. J., and F. J. Massey, Jr. 1983. *Introduction to Statistical Analysis*. 4th ed. New York: McGraw-Hill.
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- Meeker, W. Q., G. J. Hahn, and L. A. Escobar. 2017. *Statistical Intervals: A Guide for Practitioners and Researchers*. 2nd ed. Hoboken, NJ: Wiley.
- Zar, J. H. 2010. *Biostatistical Analysis*. 5th ed. Upper Saddle River, NJ: Pearson.

Also see

- [PSS-3] **ciwidth** — Precision and sample-size analysis for CIs
- [PSS-3] **ciwidth, table** — Produce table of results from the ciwidth command
- [PSS-3] **ciwidth, graph** — Graph results from the ciwidth command
- [PSS-3] **GUI (ciwidth)** — Graphical user interface for precision and sample-size analysis
- [PSS-2] **Intro (power)** — Introduction to power and sample-size analysis for hypothesis tests
- [PSS-4] **Unbalanced designs** — Specifications for unbalanced designs
- [PSS-5] **Glossary**