power pcorr — Power analysis for a partial-correlation test in a multiple linear regression

Description

power pcorr computes sample size, power, or target squared partial correlation for a partial-correlation test in a multiple linear regression. A partial-correlation test is an $F$ test of the squared partial multiple correlation that is used to test the significance of a subset of coefficients in a regression model. By default, power pcorr computes sample size given power and the squared partial correlation. Alternatively, it computes power given sample size and the squared partial correlation, or it computes the squared partial correlation given sample size and power. See [PSS-2] power for a general introduction to the power command using hypothesis tests.

Quick start

Sample size for a test of $H_0: \rho_p^2 = 0$ versus $H_a: \rho_p^2 \neq 0$ given squared partial correlation of 0.1, 3 tested covariates, and 5 control covariates using default power of 0.8 and significance level $\alpha = 0.05$

```
  power pcorr 0.1, ntested(3) ncontrol(5)
```

Power for sample size of 100

```
  power pcorr 0.1, ntested(3) ncontrol(5) n(100)
```

As above, but for sample sizes of 45, 60, 75, and 90

```
  power pcorr 0.1, ntested(3) ncontrol(5) n(45(15)90)
```

As above, but display results in a graph of power versus sample size

```
  power pcorr 0.1, ntested(3) ncontrol(5) n(45(15)90) graph
```

Effect size and target squared partial correlation for sample size of 100 with power of 0.8

```
  power pcorr, ntested(3) ncontrol(5) n(100) power(0.8)
```

Menu

```
Statistics > Power, precision, and sample size
```
**Syntax**

*Compute sample size*

```plaintext
power pcorr rho2_p [ , power(numlist) options ]
```

*Compute power*

```plaintext
power pcorr rho2_p, n(numlist) [ options ]
```

*Compute effect size and target squared partial correlation*

```plaintext
power pcorr, n(numlist) power(numlist) [ options ]
```

where `rho2_p` is the hypothesized squared partial correlation in a multiple linear regression. `rho2_p` may be specified either as one number or as a list of values in parentheses (see [U] 11.1.8 numlist).
<table>
<thead>
<tr>
<th>options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main</strong></td>
<td></td>
</tr>
<tr>
<td>*alpha(numlist)</td>
<td>significance level; default is alpha(0.05)</td>
</tr>
<tr>
<td>*power(numlist)</td>
<td>power; default is power(0.8)</td>
</tr>
<tr>
<td>*beta(numlist)</td>
<td>probability of type II error; default is beta(0.2)</td>
</tr>
<tr>
<td>*n(numlist)</td>
<td>sample size; required to compute power or effect size</td>
</tr>
<tr>
<td>nfrational</td>
<td>allow fractional sample size</td>
</tr>
<tr>
<td>*ntested(numlist)</td>
<td>number of tested covariates; default is ntested(1)</td>
</tr>
<tr>
<td>*ncontrol(numlist)</td>
<td>number of control covariates; default is ncontrol(1)</td>
</tr>
<tr>
<td>parallel</td>
<td>treat number lists in starred options as parallel when</td>
</tr>
<tr>
<td></td>
<td>multiple values per option are specified (do not enumerate all possible</td>
</tr>
<tr>
<td></td>
<td>combinations of values)</td>
</tr>
<tr>
<td>Table</td>
<td></td>
</tr>
<tr>
<td>[no]table [ (tablespec) ]</td>
<td>suppress table or display results as a table;</td>
</tr>
<tr>
<td></td>
<td>see [PSS-2] power, table</td>
</tr>
<tr>
<td>saving (filename [, replace])</td>
<td>save the table data to filename; use replace to overwrite existing filename</td>
</tr>
<tr>
<td>Graph</td>
<td></td>
</tr>
<tr>
<td>graph [ (graphopts) ]</td>
<td>graph results; see [PSS-2] power, graph</td>
</tr>
<tr>
<td>Iteration</td>
<td></td>
</tr>
<tr>
<td>init(#)</td>
<td>initial value for sample size or squared partial correlation</td>
</tr>
<tr>
<td>iterate(#)</td>
<td>maximum number of iterations; default is iterate(500)</td>
</tr>
<tr>
<td>tolerance(#)</td>
<td>parameter tolerance; default is tolerance(1e-12)</td>
</tr>
<tr>
<td>ftolerance(#)</td>
<td>function tolerance; default is ftolerance(1e-12)</td>
</tr>
<tr>
<td>[no]log</td>
<td>suppress or display iteration log</td>
</tr>
<tr>
<td>[no]dots</td>
<td>suppress or display iterations as dots</td>
</tr>
<tr>
<td>notitle</td>
<td>suppress the title</td>
</tr>
</tbody>
</table>

*Specifying a list of values in at least two starred options, or at least two command arguments, or at least one starred option and one argument results in computations for all possible combinations of the values; see [U] 11.1.8 numlist. Also see the parallel option.

notitle does not appear in the dialog box.

where tablespec is

```
column[ :label] [column[ :label] [ ...]] [ , tableopts]
```

column is one of the columns defined below, and label is a column label (may contain quotes and compound quotes).
<table>
<thead>
<tr>
<th>column</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
<td>significance level</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>power</td>
<td>power</td>
<td>$1 - \beta$</td>
</tr>
<tr>
<td>beta</td>
<td>type II error probability</td>
<td>$\beta$</td>
</tr>
<tr>
<td>N</td>
<td>number of subjects</td>
<td>$N$</td>
</tr>
<tr>
<td>delta</td>
<td>effect size</td>
<td>$\delta$</td>
</tr>
<tr>
<td>rho2_p</td>
<td>squared partial multiple correlation</td>
<td>$\rho_p^2$</td>
</tr>
<tr>
<td>ntested</td>
<td>number of tested covariates</td>
<td>$N_T$</td>
</tr>
<tr>
<td>ncontrol</td>
<td>number of control covariates</td>
<td>$N_C$</td>
</tr>
<tr>
<td>target</td>
<td>target parameter; synonym for rho2_p</td>
<td></td>
</tr>
<tr>
<td>_all</td>
<td>display all supported columns</td>
<td></td>
</tr>
</tbody>
</table>

Column beta is shown in the default table in place of column power if specified.

**Options**

- **Main**
  - `alpha()`, `power()`, `beta()`, `n()`, `nfractional`; see [PSS-2] power. The nfractional option is allowed only for sample-size determination.
  - `ntested(numlist)` specifies the number of tested covariates. The default is `ntested(1)`.
  - `ncontrol(numlist)` specifies the number of control covariates or the number of covariates in the reduced model. The default is `ncontrol(1)`.
  - `parallel`; see [PSS-2] power.

- **Table**
  - `table`, `table()`, `notable`; see [PSS-2] power, table.
  - `saving()`; see [PSS-2] power.

- **Graph**
  - `graph`, `graph()`; see [PSS-2] power, graph. Also see the column table for a list of symbols used by the graphs.

- **Iteration**
  - `init(#)` specifies the initial value of the sample size for the sample-size determination or the initial value of the squared partial correlation for the effect-size determination. The default is to use a bisection search method to compute an initial value.
  - `iterate()`, `tolerance()`, `ftolerance()`, `log`, `nolog`, `dots`, `nodots`; see [PSS-2] power.

The following option is available with `power pcorr` but is not shown in the dialog box:
- `notitle`; see [PSS-2] power.
power pcorr — Power analysis for a partial-correlation test in a multiple linear regression

Remarks and examples

Remarks are presented under the following headings:

Introduction
Using power pcorr
Computing sample size
Computing power
Computing effect size and target squared partial correlation
Performing hypothesis tests on the partial correlation

power pcorr computes sample size, power, and the target squared partial correlation for a multiple regression partial-correlation test. See [PSS-2] Intro (power) for a general introduction to power and sample-size analysis, and see [PSS-2] power for a general introduction to the power command using hypothesis tests.

Introduction

In contrast to a simple linear regression, a multiple regression framework allows researchers to control for additional predictors that may add information to better predict or explain the variation in the dependent variable of interest. There are several scenarios in which we may be interested in knowing whether additional predictors improve the model. Consider an example where a researcher is interested in whether variables such as experience, \( x_{\text{exp}} \), and location, \( x_{\text{loc}} \), add additional information in explaining variation in wage, \( y_{\text{wage}} \), after controlling for education, \( x_{\text{edu}} \), and gender, \( x_{\text{gender}} \).

\[
y_{\text{wage}} = \beta_0 + \beta_{\text{exp}} x_{\text{exp}} + \beta_{\text{loc}} x_{\text{loc}} + \beta_{\text{edu}} x_{\text{edu}} + \beta_{\text{gender}} x_{\text{gender}} + \varepsilon
\]

where \( \varepsilon \) is an independently and normally distributed error term with mean zero and constant standard deviation \( \sigma \). One way to test whether experience and location add additional information is to test the joint significance of the coefficients on experience, \( \beta_{\text{exp}} \), and location, \( \beta_{\text{loc}} \). In this case, we perform an F test using the null hypothesis \( H_0: \beta_{\text{exp}} = \beta_{\text{loc}} = 0 \). \( x_{\text{exp}} \) and \( x_{\text{loc}} \) are the tested covariates; \( x_{\text{edu}} \) and \( x_{\text{gender}} \) are the control covariates.

An equivalent test, what we refer to as a partial-correlation test, can be constructed based on the squared partial correlation, \( \rho_p^2 \). The null hypothesis is \( H_0: \rho_p^2 = 0 \) and, as in the joint test of coefficients, can be tested with an F test. \( \rho_p^2 \) is a function of the coefficient of determination, \( R^2 \), which is a measure of the variation of the dependent variable explained by the model that is used to assess overall model fit for a linear regression. Specifically,

\[
\rho_p^2 = (R_F^2 - R_R^2)/(1 - R_R^2).
\]

\( R_F^2 \) is the \( R^2 \) of the full model that includes the tested and control covariates, and \( R_R^2 \) is the \( R^2 \) of the reduced model that includes only the control covariates.

The power pcorr command provides power and sample-size analysis for a partial-correlation test. For power analysis for an \( R^2 \) test in a multiple linear regression, see [PSS-2] power rsquared. For power analysis for a slope test in a simple linear regression, see [PSS-2] power oneslope.

Using power pcorr

power pcorr computes sample size, power, or the target squared partial correlation \( \rho_{p2,p} \) for a partial-correlation test in a multiple linear regression. By default, all computations are performed at the significance level of 0.05. You may change the significance level by specifying the alpha() option.

By default, the numbers of tested covariates and of control covariates are set to 1. You may change the respective values with the ntested() and ncontrol() options.
To compute sample size, you must specify the squared partial correlation \( \rho^2_p \) and, optionally, the power of the test in the `power()` option. The default power is set to 0.8.

To compute power, you must specify the sample size in the `n()` option and the squared partial correlation \( \rho^2_p \).

To compute the target partial correlation and effect size, which is defined in terms of the partial correlation as \( \delta = \frac{\rho^2_p}{1 - \rho^2_p} \), you must specify the sample size in the `n()` option and the power in the `power()` option.

By default, the computed sample size is rounded up. You can specify the `nfractional` option to see the corresponding fractional sample size; see Fractional sample sizes in [PSS-4] Unbalanced designs for an example. The `nfractional` option is allowed only for sample-size determination.

`power pcorr`’s computations of sample size and effect size require iteration because the denominator degrees of freedom of the noncentral \( F \) distribution depends on the sample size, and the noncentrality parameter depends on the sample size and effect size. The default initial values are obtained using a bisection search method. You may use the `init()` option to specify your own value. The initial value of the sample size must be greater than the number of parameters in the multiple regression model. See [PSS-2] power for the descriptions of other options that control the iteration procedure.

### Computing sample size

To compute sample size, you must specify the squared partial correlation \( \rho^2_p \) and, optionally, the power of the test in the `power()` option. A default power of 0.8 is assumed if `power()` is not specified.

#### Example 1: Sample size for a partial-correlation test

Consider an example from Cohen (1988, 436) where a psychologist investigates a selection procedure based on job candidates’ demographic characteristics used to predict success in a sales position. Suppose we want to conduct a similar study. Data for age, education, and prior experience, our control covariates, are readily available. However, data on verbal aptitude and extraversion, the tested covariates, are costly to obtain, and we decide that these variables are worth including only if the squared partial correlation with the dependent variable is at least 0.0426.

We will conduct a test-case study to determine the minimum sample size required to detect a squared partial correlation of 0.0426 between the dependent variable and the two extra variables, verbal aptitude and extraversion. We compute the minimum sample size required with 80% power at a 5% significance level:
We find that a sample of 220 subjects is required to detect a squared partial correlation of 0.0426 associated with the two extra variables, verbal aptitude and extraversion, with 80% power using a 5% level test. The effect size \( \Delta \) is calculated using the given information about the hypothesized squared partial correlation; see Methods and formulas in \([PSS-2]\) power rsquared for details.

As we mentioned in Using power pcorr, sample-size computation requires iteration. The iteration log is suppressed by default, but you can display it by specifying the log option.

### Computing power

To compute power, you must specify the sample size in the \( \text{n}(\cdot) \) option and the squared partial correlation \( \rho^2_p \).

#### Example 2: Power for a partial-correlation test

Continuing with example 1, suppose that we are designing a new study and anticipate a sample of 200 subjects. Given the study parameters from example 1, we compute the power by specifying the sample size of 200 in the \( \text{n}(\cdot) \) option:

```
   . power pcorr 0.0426, ntested(2) ncontrol(3) n(200)
```

Estimated power for multiple linear regression
F test for partial correlation
Ho: \( \rho^2_p = 0 \) versus Ha: \( \rho^2_p \neq 0 \)

Study parameters:
\[\begin{array}{l}
\text{alpha} = 0.0500 \\
\text{power} = 0.8000 \\
\text{delta} = 0.0445 \\
\text{rho2_p} = 0.0426 \\
\text{ncontrol} = 3 \\
\text{ntested} = 2 \\
\end{array}\]

Estimated power:
\[\text{power} = 0.7588\]

With this smaller sample size, the power of the test decreases to about 76%.
Example 3: Multiple values of study parameters

Continuing with example 2, we want to see the effect of sample size on power. We specify a list of sample sizes in the \texttt{n()} option:

\begin{verbatim}
. power pcorr 0.0426, n(50 100 200 400 800) ntested(2) ncontrol(3)
\end{verbatim}

Estimated power for multiple linear regression
F test for partial correlation
Ho: rho2\_p = 0 versus Ha: rho2\_p \neq 0

<table>
<thead>
<tr>
<th>Study parameters:</th>
<th>Estimated effect size and squared partial correlation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha = 0.0500,</td>
<td>delta = 0.0489, rho2_p = 0.0466</td>
</tr>
<tr>
<td>power = 0.8000,</td>
<td></td>
</tr>
<tr>
<td>N = 200,</td>
<td></td>
</tr>
<tr>
<td>ncontrol = 3,</td>
<td></td>
</tr>
<tr>
<td>ntested = 2</td>
<td></td>
</tr>
</tbody>
</table>

As expected, when the sample size increases, the power tends to get closer to 1.

For multiple values of parameters, the results are automatically displayed in a table, as we see above. For more examples of tables, see \cite{PSS-2} power, table. If you wish to produce a power plot, see \cite{PSS-2} power, graph.

Computing effect size and target squared partial correlation

Effect size $\delta$ for a multiple regression, defined in terms of the partial correlation, is $\delta = \rho_p^2 / (1 - \rho_p^2)$. To compute the effect size and target squared partial correlation, you must specify the sample size in the \texttt{n()} option and the power in the \texttt{power()} option.

Example 4: Minimum detectable squared partial correlation

Continuing with example 2, we may also be interested in finding the minimum value of the squared partial correlation that can be detected with a sample of 200 subjects and 80\% power. To compute this, we specify \texttt{n(200)} and \texttt{power(0.8)}. As before, we use 2 tested covariates and 3 control covariates.

\begin{verbatim}
. power pcorr, n(200) power(0.8) ntested(2) ncontrol(3)
Performing iteration ...
\end{verbatim}

Estimated squared partial correlation for multiple linear regression
F test for partial correlation
Ho: rho2\_p = 0 versus Ha: rho2\_p \neq 0

Study parameters:

<table>
<thead>
<tr>
<th>Study parameters:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha = 0.0500,</td>
<td>delta = 0.0489</td>
</tr>
<tr>
<td>power = 0.8000,</td>
<td>rho2_p = 0.0466</td>
</tr>
<tr>
<td>N = 200,</td>
<td></td>
</tr>
</tbody>
</table>
Performing hypothesis tests on the partial correlation

In this section, we briefly demonstrate the use of the `pcorr` command for estimating partial correlations.

Example 5: Partial-correlation test for one variable

Suppose that our study goal is to investigate whether the mileage of a car (mpg) has an effect on its price (price) after controlling for headroom (headroom) and trunk space (trunk). We compute the partial correlation by using `pcorr` on data about cars from 1978 in `auto.dta`. This preliminary investigation will tell us how many modern cars we should select for our study.

```stata
. use https://www.stata-press.com/data/r16/auto
(1978 Automobile Data)
. pcorr price mpg headroom trunk
(obs=74)
Partial and semipartial correlations of price with

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>mpg</td>
<td>-0.3800</td>
<td>-0.3576</td>
<td>0.1444</td>
<td>0.1279</td>
<td>0.0010</td>
</tr>
<tr>
<td>headroom</td>
<td>-0.1606</td>
<td>-0.1416</td>
<td>0.0258</td>
<td>0.0201</td>
<td>0.1779</td>
</tr>
<tr>
<td>trunk</td>
<td>0.1397</td>
<td>0.1228</td>
<td>0.0195</td>
<td>0.0151</td>
<td>0.2418</td>
</tr>
</tbody>
</table>
```

We obtain a squared partial correlation of around 0.14 for mpg. The significance value is less than 0.05.

Suppose we wish to design a new similar study. We use the estimated squared partial correlation from this study to perform a sample-size analysis. `pcorr` computes the partial correlation for only a single variable, not a group of variables, so our power calculation here uses only one tested covariate; see [R] `pcorr` for details. We use the default value of 1 for `ntested()` and specify 2 control covariates.

```stata
. power pcorr 0.14, ntested(1) ncontrol(2)
Performing iteration ... 
Estimated sample size for multiple linear regression
F test for partial correlation
Ho: rho2_p = 0 versus Ha: rho2_p != 0
Study parameters:
    alpha =  0.0500
    power =  0.8000
    delta =  0.1628
    rho2_p =  0.1400
    ncontrol =  2
    ntested =  1
Estimated sample size:
    N = 51
```

We find that a sample size of 51 is required to detect a squared partial correlation of 0.14 with 80% power using a 5%-level test.
power pcorr stores the following in r():

Scalars
- r(alpha)  significance level
- r(power)   power
- r(beta)    probability of a type II error
- r(delta)   effect size
- r(N)       sample size
- r(nfractional)  1 if nfractional is specified, 0 otherwise
- r(rho2_p)  squared partial correlation
- r(ntested) number of tested covariates
- r(ncontrol) number of control covariates
- r(separator) number of lines between separator lines in the table
- r(divider)  1 if divider is requested in the table, 0 otherwise
- r(init)    initial value for sample size or for squared partial correlation
- r(maxiter) maximum number of iterations
- r(iter)    number of iterations performed
- r(tolerance) requested parameter tolerance
- r(deltax)  final parameter tolerance achieved
- r(ftolerance) requested distance of the objective function from zero
- r(function) final distance of the objective function from zero
- r(converged) 1 if iteration algorithm converged, 0 otherwise

Macros
- r(type) test
- r(method) pcorr
- r(columns) displayed table columns
- r(labels)  table column labels
- r(widths)  table column widths
- r(formats) table column formats

Matrices
- r(pss_table) table of results

Methods and formulas


Reference


Also see

[PSS-2] power — Power and sample-size analysis for hypothesis tests
[PSS-2] power oneslope — Power analysis for a slope test in a simple linear regression
[PSS-2] power rsquared — Power analysis for an R^2 test in a multiple linear regression
[PSS-2] power, graph — Graph results from the power command
[PSS-2] power, table — Produce table of results from the power command
[PSS-5] Glossary
[R] pcorr — Partial and semipartial correlation coefficients