power onepropotion, cluster — Power analysis for a one-sample proportion test, CRD

Description

power onepropotion, cluster computes the number of clusters, cluster size, power, or target proportion for a one-sample proportion test in a cluster randomized design (CRD). It computes the number of clusters given cluster size, power, and the values of the null and alternative proportions. It also computes cluster size given the number of clusters, power, and the values of the null and alternative proportions. Alternatively, it computes power given the number of clusters, cluster size, and the values of the null and alternative proportions, or it computes the target proportion given the number of clusters, cluster size, power, and the null proportion. See [PSS-2] power onepropotion for a general discussion of power and sample-size analysis for a one-sample proportion test. Also see [PSS-2] power for a general introduction to the power command using hypothesis tests.

Quick start

Compute number of clusters for two-sided test of \( H_0: \pi = 0.2 \) versus \( H_a: \pi \neq 0.2 \) with null proportion \( p_0 = 0.2 \), alternative proportion \( p_a = 0.1 \), and cluster size of 5, using default intraclass correlation of 0.5, power of 0.8, and significance level \( \alpha = 0.05 \)

\[
\text{power onepropotion 0.2 0.1, m(5)}
\]

As above, but with an intraclass correlation of 0.7

\[
\text{power onepropotion 0.2 0.1, m(5) rho(0.7)}
\]

As above, but the cluster size varies with a coefficient of variation of 0.6

\[
\text{power onepropotion 0.2 0.1, m(5) rho(0.7) cvcluster(0.6)}
\]

Compute cluster size when 52 clusters are sampled:

\[
\text{power onepropotion 0.2 0.1, k(52)}
\]

Power for 52 clusters with cluster size of 5

\[
\text{power onepropotion 0.2 0.1, k(52) m(5)}
\]

Power for 20, 30, 40, and 50 clusters

\[
\text{power onepropotion 0.2 0.1, k(20(10)50) m(5)}
\]

As above, but display results in a graph of power versus number of clusters

\[
\text{power onepropotion 0.2 0.1, k(20(10)50) m(5) graph}
\]

Effect size and target proportion for \( p_0 = 0.2 \) with 40 clusters of size 5, power of 0.9, and \( \alpha = 0.01 \), and default direction upper

\[
\text{power onepropotion 0.2, k(40) m(5) power(0.9) alpha(0.01)}
\]
Menu

Statistics > Power, precision, and sample size

Syntax

Compute number of clusters

```
power oneprop p0 p_a, {m(numlist) | n(numlist) | cluster} [options]
```

Compute cluster size

```
power oneprop p0 p_a, k(numlist) [options]
```

Compute power

```
power oneprop p0 p_a, k(numlist) {m(numlist) | n(numlist) | cluster} [options]
```

Compute effect size and target proportion

```
power oneprop p0, k(numlist) {m(numlist) | n(numlist)} [options]
```

where \( p_0 \) is the null (hypothesized) proportion or the value of the proportion under the null hypothesis and \( p_a \) is the alternative (target) proportion or the value of the proportion under the alternative hypothesis. \( p_0 \) and \( p_a \) may each be specified either as one number or as a list of values in parentheses (see [U] 11.1.8 numlist).
Power analysis for a one-sample proportion test, CRD

**options**

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main</strong></td>
</tr>
<tr>
<td><code>cluster</code></td>
</tr>
<tr>
<td><code>*alpha(numlist)</code></td>
</tr>
<tr>
<td><code>*power(numlist)</code></td>
</tr>
<tr>
<td><code>*beta(numlist)</code></td>
</tr>
<tr>
<td><code>*k(numlist)</code></td>
</tr>
<tr>
<td><code>*m(numlist)</code></td>
</tr>
<tr>
<td><code>*n(numlist)</code></td>
</tr>
<tr>
<td><code>nfractional</code></td>
</tr>
<tr>
<td><code>*diff(numlist)</code></td>
</tr>
<tr>
<td><code>*rho(numlist)</code></td>
</tr>
<tr>
<td><code>*cvcluster(numlist)</code></td>
</tr>
<tr>
<td>`direction(upper</td>
</tr>
<tr>
<td><code>onesided</code></td>
</tr>
<tr>
<td><code>parallel</code></td>
</tr>
<tr>
<td><strong>Table</strong></td>
</tr>
<tr>
<td><code>[no]table[(tablespec)]</code></td>
</tr>
<tr>
<td><code>saving(filename[,replace])</code></td>
</tr>
<tr>
<td><strong>Graph</strong></td>
</tr>
<tr>
<td><code>graph[(graphopts)]</code></td>
</tr>
<tr>
<td><strong>Iteration</strong></td>
</tr>
<tr>
<td><code>init(#)</code></td>
</tr>
<tr>
<td><code>iterate(#)</code></td>
</tr>
<tr>
<td><code>tolerance(#)</code></td>
</tr>
<tr>
<td><code>ftolerance(#)</code></td>
</tr>
<tr>
<td><code>[no]log</code></td>
</tr>
<tr>
<td><code>[no]dots</code></td>
</tr>
<tr>
<td><code>notitle</code></td>
</tr>
</tbody>
</table>

*Specifying a list of values in at least two starred options, or at least two command arguments, or at least one starred option and one argument results in computations for all possible combinations of the values; see [U] 11.1.8 `numlist`. Also see the `parallel` option.

`notitle` does not appear in the dialog box.
where `tablespec` is

\[
\text{column}[:\text{label}] [\text{column}[:\text{label}]] [\ldots] [, \text{tableopts}]
\]

`column` is one of the columns defined below, and `label` is a column label (may contain quotes and compound quotes).

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
<td>significance level</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>power</td>
<td>power</td>
<td>(1 - \beta)</td>
</tr>
<tr>
<td>beta</td>
<td>type II error probability</td>
<td>(\beta)</td>
</tr>
<tr>
<td>K</td>
<td>number of clusters</td>
<td>(K)</td>
</tr>
<tr>
<td>M</td>
<td>cluster size</td>
<td>(M)</td>
</tr>
<tr>
<td>N</td>
<td>number of observations</td>
<td>(N)</td>
</tr>
<tr>
<td>delta</td>
<td>effect size</td>
<td>(\delta)</td>
</tr>
<tr>
<td>p0</td>
<td>null proportion</td>
<td>(p_0)</td>
</tr>
<tr>
<td>pa</td>
<td>alternative proportion</td>
<td>(p_a)</td>
</tr>
<tr>
<td>diff</td>
<td>difference between the alternative and null proportions</td>
<td>(p_a - p_0)</td>
</tr>
<tr>
<td>rho</td>
<td>intraclass correlation</td>
<td>(\rho)</td>
</tr>
<tr>
<td>CV_cluster</td>
<td>coefficient of variation for cluster sizes</td>
<td>(CV_{cl})</td>
</tr>
<tr>
<td>target</td>
<td>target parameter; synonym for pa</td>
<td></td>
</tr>
<tr>
<td>_all</td>
<td>display all supported columns</td>
<td></td>
</tr>
</tbody>
</table>

Column `beta` is shown in the default table in place of column `power` if specified. Columns `diff` and `CV_cluster` are shown in the default table if specified.

### Options

- **Main**

  `cluster` specifies that computations should be performed for a CRD. This option is implied when either the `k()` or `m()` option is specified. It is required if the `n()` option is used to compute the number of clusters.

  `alpha()`, `power()`, `beta()`: see [PSS-2] power.

  `k(numlist)` specifies the number of clusters. This option is required to compute the cluster size, power, or effect size.

  `m(numlist)` specifies the cluster size. This option or the `n()` option is required to compute the number of clusters, power, or effect size. `m()` may contain noninteger values. In this case or if the `cvcluster()` option is specified, `m()` represents the average cluster size.

  `n(numlist)` specifies the number of observations. This option or the `m()` option is required to compute the number of clusters, power, or effect size.

  `nfractional`: see [PSS-2] power. The `nfractional` option is allowed when computing the number of clusters and cluster size to display fractional (without rounding) values of the number of clusters, cluster size, and sample size.
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**diff(numlist)** specifies the difference between the alternative proportion and the null proportion, \( p_a - p_0 \). You can specify either the alternative proportion \( p_a \) as a command argument or the difference between the two proportions in **diff()**. If you specify **diff(#)**, the alternative proportion is computed as \( p_a = p_0 + # \). This option is not allowed with the effect-size determination.

**rho(numlist)** specifies the intraclass correlation. The default is \( \rho(0.5) \).

**cvcluster(numlist)** specifies the coefficient of variation for cluster sizes. This option is used with varying cluster sizes.

**direction()**, **onesided**, **parallel**; see [PSS-2] **power**.

**saving();** see [PSS-2] **power, table**.

**graph();** see [PSS-2] **power, graph**. Also see the **column** table for a list of symbols used by the graphs.

**init(#)** specifies the initial value for the number of clusters or cluster size for sample-size determination or the initial value for the proportion for the effect-size determination. The default is to use a closed-form normal approximation to compute an initial value for the estimated parameter.

**iterate()**, **tolerance()**, **ftolerance()**, **log**, **nolog**, **dots**, **nodots**; see [PSS-2] **power**.

The following option is available with **power oneproportion, cluster** but is not shown in the dialog box:

**notitle();** see [PSS-2] **power**.

**Remarks and examples**

Remarks are presented under the following headings:

- Using power oneproportion, cluster
- Computing number of clusters
- Computing cluster size
- Computing power
- Computing effect size and target proportion
- Performing hypothesis tests on proportion in a CRD

**power oneproportion, cluster** requests that computations for the power oneproportion command be done for a CRD. In a CRD, groups of subjects or clusters are randomized instead of individual subjects, so the sample size is determined by the number of clusters and the cluster size. The sample-size determination thus consists of the determination of the number of clusters given cluster size or the determination of cluster size given the number of clusters. For a general discussion of using **power oneproportion**, see [PSS-2] **power oneproportion**. The discussion below is specific to the CRD.
Using power oneproportion, cluster

If you specify the cluster option, include \( k() \) to specify the number of clusters or include \( m() \) to specify the cluster size, the `power oneproportion` command will perform computations for a one-sample proportion test in a CRD. The computations for a CRD are based on the large-sample Wald \( z \) test.

All computations are performed for a two-sided hypothesis test where, by default, the significance level is set to 0.05. You may change the significance level by specifying the `alpha()` option. You can specify the `onesided` option to request a one-sided test.

To compute the number of clusters, you must specify the proportions under the null and alternative hypotheses as command arguments \( p_0 \) and \( p_a \), respectively, and specify the cluster size in the \( m() \) option. Instead of specifying the \( m() \) option, you may specify the sample size in the \( n() \) option and specify the `cluster` option, so that `power onemean` will perform its computation for a cluster randomized design instead of the default individual-level design. You may also specify the power of the test in the `power()` option.

To compute cluster size, you must specify the null proportion \( p_0 \), the alternative proportion \( p_a \), and the number of clusters in the \( k() \) option. You may also specify the power of the test in the `power()` option.

To compute power, you must specify the number of clusters in the \( k() \) option, the cluster size in the \( m() \) option or the sample size in the \( n() \) option, the null proportion \( p_0 \), and the alternative proportion \( p_a \).

Instead of the alternative proportion \( p_a \), you may specify the difference \( p_a - p_0 \) between the alternative proportion and the null proportion in the `diff()` option when computing sample size or power.

The effect size \( \delta \) is defined as the difference between the alternative and null proportions. In a CRD, the effect size \( \delta \) is also adjusted for the cluster design; see `Methods and formulas`.

To compute effect size and the corresponding target proportion, you must specify the number of clusters in the \( k() \) option, the cluster size in the \( m() \) option or the sample size in the \( n() \) option, the power in the `power()` option, and the null proportion \( p_0 \). You may also specify the direction of the effect in the `direction()` option. The direction is upper by default, `direction(upper)`; see `Using power oneproportion` in [PSS-2] `power oneproportion` for other details.

All computations assume an intraclass correlation of 0.5. You can change this by specifying the `rho()` option. Also, all clusters are assumed to be of the same size unless the coefficient of variation for cluster sizes is specified in the `cvcluster()` option.

By default, the computed number of clusters, cluster size, and sample size is rounded up. However, you can specify the `nfractional` option to see the corresponding fractional values; see `Fractional sample sizes` in [PSS-4] `Unbalanced designs` for an example. If the `cvcluster()` option is specified when computing cluster size, then cluster size represents the average cluster size and is thus not rounded. When sample size is specified in the \( n() \) option, fractional cluster size may be reported to accommodate the specified number of clusters and sample size.

Some of `power oneproportion, cluster`'s computations require iteration, such as to compute the number of clusters for a two-sided test; see `Methods and formulas` for details and [PSS-2] `power` for the descriptions of options that control the iteration procedure.
Computing number of clusters

To compute the number of clusters, you must specify the proportions under the null and alternative hypotheses as command arguments \( p_0 \) and \( p_a \), respectively, and specify the cluster size in the \( m() \) option. Instead of specifying the \( m() \) option, you may specify the sample size in the \( n() \) option and specify the \( \text{cluster} \) option, so that \texttt{power oneproportion, cluster} will perform its computation for a cluster randomized design instead of the default individual-level design. You may also specify the power of the test in the \( \text{power}() \) option.

Example 1: Number of clusters for a one-sample proportion test in a CRD, specifying cluster size

Ahn, Heo, and Zhang (2015, 33) demonstrate sample-size computations for a clustered binary outcome by using the data from Hujoel, Moulton, and Loesche (1990) as pilot data. The data recorded positive test results from an enzymatic diagnostic test (EDT) of a specific (target) infection. There were 29 subjects in the study, and each subject had multiple infected sites, as determined by a gold standard test, which were then retested for the presence of the target infection using the EDT. The number of infected sites varied among subjects with an average of 4.897 sites, and observations within a subject were correlated with an intraclass correlation of 0.2. Ahn, Heo, and Zhang (2015) used these estimates to compute the required number of clusters for a new study to test whether the proportion of infected sites detected by the EDT is 0.6, \( H_0: p = 0.6 \), against the alternative \( H_a: p = 0.7 \).

We demonstrate how to use \texttt{power oneproportion, cluster} to compute the required number of clusters.

For simplicity, we assume a constant cluster size across subjects and use an integer cluster size of 5. To detect a proportion of 0.7 against the reference value of 0.6 with 80% power using a 5%-level two-sided test, we type

\[
\texttt{. power oneproportion 0.6 0.7, m(5) rho(0.2)}
\]

Performing iteration ...

Estimated number of clusters for a one-sample proportion test
Cluster randomized design, Wald z test
\( H_0: p = p_0 \) versus \( H_a: p != p_0 \)
Study parameters:
\begin{align*}
\text{alpha} &= 0.0500 \\
\text{power} &= 0.8000 \\
\text{delta} &= 0.1000 \\
p_0 &= 0.6000 \\
p_a &= 0.7000 \\
\end{align*}
Cluster design:
\begin{align*}
M &= 5 \\
rho &= 0.2000 \\
\end{align*}
Estimated number of clusters and sample size:
\begin{align*}
K &= 60 \\
N &= 300 \\
\end{align*}

We find that given 5 sites per subject, 60 subjects and thus a total of 300 infected sites are required to detect a proportion of 0.7 for the infection of interest against a reference proportion of 0.6 with 80% power using a 5%-level two-sided test. The effect size (\texttt{delta}) is calculated as the difference between the alternative and null proportions.
Example 2: Number of clusters for a one-sample proportion test in a CRD, with varying cluster sizes

Unlike the simplified case in example 1, in a practical study, the number of infected sites per subject may vary. We use the average number of infected sites of 4.897 and a coefficient of variation of 0.25. To account for varying cluster sizes, we specify \( m(4.897) \) and \( cv_{\text{cluster}}(0.25) \).

\[
\text{. power oneproportion 0.6 0.7, m(4.897) rho(0.2) cvcluster(0.25)}
\]

Performing iteration ...

Estimated number of clusters for a one-sample proportion test
Cluster randomized design, Wald z test
Ho: \( p = p_0 \) versus Ha: \( p \neq p_0 \)

Study parameters:
- \( \alpha = 0.0500 \)
- \( \text{power} = 0.8000 \)
- \( \delta = 0.1000 \)
- \( p_0 = 0.6000 \)
- \( p_a = 0.7000 \)

Cluster design:
- \( \text{Average M} = 4.8970 \)
- \( \rho = 0.2000 \)
- \( CV_{\text{cl}} = 0.2500 \)

Estimated number of clusters and sample size:
- \( K = 61 \)
- \( N = 299 \)

We now need 61 subjects for a total of 299 sites to achieve the same power.

Computing cluster size

To compute cluster size, you must specify the null proportion \( p_0 \), the alternative proportion \( p_a \), and the number of clusters in the \( k() \) option. You may also specify the power of the test in the \( \text{power}() \) option.

Example 3: Cluster size for a one-sample proportion test in a CRD

Continuing with example 1, suppose that we are designing a new study and would like to recruit 80 subjects in the study. We would like to get an idea of how many infected sites we need to achieve 80% power. Given the study parameters from example 1, we compute the number of infected sites by specifying 80 clusters in the \( k() \) option.
Computing power

To compute power, you must specify the number of clusters in the **k()** option, the cluster size in the **m()** option or the sample size in the **n()** option, the null proportion \( p_0 \), and the alternative proportion \( p_a \).

Example 4: Power for a one-sample proportion test in a CRD

Continuing with example 1, suppose that we have 80 subjects and each subject has 5 infected sites. Given the study parameters from example 1, we compute the power by specifying 80 clusters in the **k()** option and cluster size of 5 in the **m()** option:

```
. power oneproportion 0.6 0.7, k(80) m(5) rho(0.2)
```

Estimated power for a one-sample proportion test
Cluster randomized design, Wald z test
\( H_0: p = p_0 \) versus \( H_a: p \neq p_0 \)

Study parameters:
```
alpha = 0.0500
delta = 0.1000
p0 = 0.6000
pa = 0.7000
```

Cluster design:
```
K = 80
rho = 0.2000
```

Estimated power:
```
power = 0.9020
```

The computed power is about 90%.
To investigate the effect of the number of clusters on power, we can specify a list of numbers in the \( k() \) option:

```
. power oneproportion 0.6 0.7, k(20(20)100) m(5) rho(0.2)
```

Estimated power for a one-sample proportion test

<table>
<thead>
<tr>
<th>alpha</th>
<th>power</th>
<th>K</th>
<th>M</th>
<th>N</th>
<th>delta</th>
<th>p0</th>
<th>pa</th>
<th>rho</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>.3696</td>
<td>20</td>
<td>5</td>
<td>100</td>
<td>.1</td>
<td>.6</td>
<td>.7</td>
<td>.2</td>
</tr>
<tr>
<td>.05</td>
<td>.6332</td>
<td>40</td>
<td>5</td>
<td>200</td>
<td>.1</td>
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<td>5</td>
<td>300</td>
<td>.1</td>
<td>.6</td>
<td>.7</td>
<td>.2</td>
</tr>
<tr>
<td>.05</td>
<td>.902</td>
<td>80</td>
<td>5</td>
<td>400</td>
<td>.1</td>
<td>.6</td>
<td>.7</td>
<td>.2</td>
</tr>
<tr>
<td>.05</td>
<td>.9532</td>
<td>100</td>
<td>5</td>
<td>500</td>
<td>.1</td>
<td>.6</td>
<td>.7</td>
<td>.2</td>
</tr>
</tbody>
</table>

As expected, as the number of clusters increases, the power tends to get closer to 1.

For multiple values of parameters, the results are automatically displayed in a table, as we see above. For more examples of tables, see [PSS-2] power, table. If you wish to produce a power plot, see [PSS-2] power, graph.

### Computing effect size and target proportion

The effect size \( \delta \) is defined as the difference between the alternative and null proportions. In a CRD, the effect size \( \delta \) is also adjusted for the cluster design; see Methods and formulas.

To compute effect size and the corresponding target proportion, you must specify the number of clusters in the \( k() \) option, the cluster size in the \( m() \) option or the sample size in the \( n() \) option, the power in the \( power() \) option, and the null proportion \( p_0 \). You may also specify the direction of the effect in the \( direction() \) option. The direction is upper by default, \( direction(upper) \); see Using power oneproportion in [PSS-2] power oneproportion for other details.

#### Example 6: Effect size for a one-sample proportion test in a CRD

Continuing with example 4, we may also be interested in finding the minimum value of the proportion that can be detected with a sample of 80 subjects, 5 infected sites per subject, and 80% power. To compute this, we specify the null value of 0.6 as the command argument and the required options \( k(80), m(5), \) and \( power(0.8) \) and continue to use \( rho(0.2) \).
. power oneproportion 0.6, k(80) m(5) power(0.8) rho(0.2)

Performing iteration ...

Estimated target proportion for a one-sample proportion test
Cluster randomized design, Wald z test
Ho: p = p₀ versus Ha: p ≠ p₀; pa > p₀

Study parameters:

- alpha = 0.0500
- power = 0.8000
- p₀ = 0.6000

Cluster design:

- K = 80
- M = 5
- N = 400
- rho = 0.2000

Estimated effect size and target proportion:

- delta = 0.0871
- pa = 0.6871

Given the null value of 0.6, the minimum detectable value of the proportion is about 0.69, which is slightly smaller than the alternative proportion of 0.7 used in previous examples, because here we use more subjects than, for instance, in example 1, more sites per subject than in example 3, and lower power than in example 4.

Performing hypothesis tests on proportion in a CRD

power oneproportion, cluster performs PSS computations based on a large-sample test of proportion that accounts for a CRD or for clustered data. We can perform this test by using prtest, cluster(); see [R] prtest. In this section, we briefly demonstrate how to test the hypothesis that the proportion is different from a reference value on the collected clustered data by using prtest.

Example 7: Testing for proportion with clustered data

Ahn, Heo, and Zhang (2015, 33) report the data from Hujoel, Moulton, and Loesche (1990) on positive test results from the EDT; see example 1 for details about the study. Let’s use prtest to test the null hypothesis $H_0: p = 0.6$.

For clustered data, prtest requires that we specify the cluster identifier in the cluster() option and population intraclass correlation in the rho() option. We use the intraclass correlation of 0.2 as in Ahn, Heo, and Zhang (2015, 33).
We do not find any statistical evidence to reject the null hypothesis of \( H_0: p = 0.6 \).

Suppose that we want to design a new similar study and use the estimates from this study to compute the required number of clusters. We are interested in detecting the alternative value of, say, 0.66 with 80% power for a 5%-level two-sided test. To compute the required number of clusters, we use the average cluster size of 4.9 as observed in this study.

We need 178 subjects to detect the 0.06 difference between the alternative and null proportions, given the null proportion of 0.6, with 80% power using a 5%-level two-sided test.
Stored results

power oneproportion, cluster stores the following in r():

Scalars
r(alpha)  significance level
r(power)   power
r(beta)    probability of a type II error
r(delta)   effect size
r(K)       number of clusters
r(M)       cluster size
r(N)       number of subjects
r(nfractional)  1 if nfractional is specified, 0 otherwise
r(onesided)  1 for a one-sided test, 0 otherwise
r(p0)       proportion under the null hypothesis
r(pa)       proportion under the alternative hypothesis
r(diff)     difference between the alternative and null proportions
r(rho)      intraclass correlation
r(CV_cluster) coefficient of variation for cluster sizes
r(divider)  1 if divider is requested in the table, 0 otherwise
r(init)     initial value for estimated parameter
r(maxiter)  maximum number of iterations
r(iter)     number of iterations performed
r(tolerance) requested parameter tolerance
r(deltax)   final parameter tolerance achieved
r(ftolerance) requested distance of the objective function from zero
r(function) final distance of the objective function from zero
r(converged) 1 if iteration algorithm converged, 0 otherwise

Macros
r(type)    test
r(method)  oneproportion
r(design)  CRD
r(test)    wald
r(direction) upper or lower
r(columns)  displayed table columns
r(labels)   table column labels
r(widths)  table column widths
r(formats)  table column formats

Matrices
r(pss_table)  table of results

Methods and formulas

The computation for a CRD is based on the Wald test under the large-sample normal approximation, adjusted for the cluster design; see Large-sample normal approximation under Methods and formulas in [PSS-2] power oneproportion for the common notation for a one-sample proportion test.

Methods and formulas are presented under the following headings:

Equal cluster sizes
Unequal cluster sizes
Equal cluster sizes

In a CRD, let $K$ be the number of clusters, $M$ be the number of observations in each cluster, and $n$ be the total number of subjects, where $n = MK$. Let $x_{ij}$ be the outcome of a Bernoulli trial of the $j$th ($j = 1, 2, \ldots, M$) observation from the $i$th cluster ($i = 1, 2, \ldots, K$). Let $\rho$ be the intraclass correlation and DE be the design effect defined as

$$DE = 1 + \rho(M - 1)$$

Let $P(x_{ij} = 1) = p$ denote the probability of a success in the population. Each individual observation is a Bernoulli trial with a success probability $p$. Let

$$\hat{p} = \frac{1}{n} \sum_{i=1}^{K} \sum_{j=1}^{M} x_{ij}$$

and

$$se(\hat{p}) = \sqrt{\frac{\hat{p}(1 - \hat{p})DE}{n}}$$

denote the sample proportion and its standard error, respectively. Let $p_0$ and $p_a$ denote the respective null and alternative values of the proportion parameters.

For a large sample, the distribution of the sample proportion $\hat{p}$ may be approximated by the normal distribution with proportion $p$ and variance $p(1 - p)DE/n$. The Wald test statistic $z = (\hat{p} - p_0)/\sqrt{\hat{p}(1 - \hat{p})DE/n}$ under the null hypothesis follows a standard normal distribution; see, for example, Ahn, Heo, and Zhang (2015).

Let $\alpha$ be the significance level, $\beta$ be the probability of a type II error, and $z_{1 - \alpha}$ and $z_\beta$ be the $(1 - \alpha)$th and the $\beta$th quantiles of the standard normal distribution. Let

$$p_{std} = \frac{(p_a - p_0)}{\sqrt{p_a(1 - p_a)DE}}$$

(1)

The power $\pi = 1 - \beta$ is computed using

$$\pi = \begin{cases} 
\Phi \left( \sqrt{n}p_{std} - z_{1 - \alpha} \right) & \text{for an upper one-sided test} \\
\Phi \left( -\sqrt{n}p_{std} - z_{1 - \alpha} \right) & \text{for a lower one-sided test} \\
\Phi \left( \sqrt{n}p_{std} - z_{1 - \alpha}/2 \right) + \Phi \left( -\sqrt{n}p_{std} - z_{1 - \alpha}/2 \right) & \text{for a two-sided test}
\end{cases}$$

(2)

where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution.

Given the cluster size $M$, the number of clusters $K$ for a one-sided test is computed by inverting a one-sided power equation from (2),

$$K = \left( \frac{z_{1 - \alpha} - z_\beta}{p_{std}\sqrt{M}} \right)^2$$

(3)

Given the sample size $n$, the number of clusters $K$ for a one-sided test is computed as

$$K = \frac{n (p_a - p_0)^2}{\rho p_a(1 - p_a)(z_{1 - \alpha} - z_\beta)^2} - \frac{1}{\rho} + 1$$

(4)

Given the number of clusters $K$, the cluster size $M$ for a one-sided test is computed by solving (2), after substituting $p_{std}$ from (1),
The number of clusters and cluster size for a two-sided test are computed iteratively using the two-sided power equation from (2). The initial values are obtained from (3), (4), and (5), with \( \alpha/2 \).

The minimum detectable value of the proportion is computed iteratively using the corresponding power equation from (2).

**Unequal cluster sizes**

For unequal cluster sizes, we assume that the cluster sizes are independent and identically distributed and are small relative to the number of clusters; see Ahn, Heo, and Zhang (2015) for details. Let the coefficient of variation of the cluster sizes be \( CV_{cl} \). According to van Breukelen, Candel, and Berger (2007) and Campbell and Walters (2014), to adjust for varying cluster sizes, define the relative efficiency (RE) of unequal versus equal cluster sizes as

\[
RE = 1 - \lambda(1 - \lambda)CV_{cl}^2
\]

where \( \lambda = \rho M / (\rho M + 1 - \rho) \). With unequal cluster sizes, \( p_{std} \) becomes

\[
p_{std} = \frac{(p_a - p_0)}{\sqrt{p_a(1 - p_a)DE/RE}}
\]

With \( p_{std} \) as defined in (6), we can obtain the formula for computing the number of clusters given cluster size for a one-sided test using (3). In all other cases, parameters are computed iteratively using the power equations in (2) with \( p_{std} \) from (6).

**References**


**Also see**

[PSS-2] *power onepropportion* — Power analysis for a one-sample proportion test

[PSS-2] *power* — Power and sample-size analysis for hypothesis tests

[PSS-2] *power, graph* — Graph results from the power command

[PSS-2] *power, table* — Produce table of results from the power command

[PSS-5] *Glossary*

[R] *prtest* — Tests of proportions