matrix symeigen — Eigenvalues and eigenvectors of symmetric matrices

Description

`matrix symeigen` returns the eigenvectors in the columns of **X**: $n \times n$ and the corresponding eigenvalues in **v**: $1 \times n$. The eigenvalues are sorted: **v**`[1,1]` contains the largest eigenvalue (and **X**`[1...,1]` its corresponding eigenvector), and **v**`[1,`n`]` contains the smallest eigenvalue (and **X**`[1...,`n`]` its corresponding eigenvector).

If you want the eigenvalues for a nonsymmetric matrix, see [P] **matrix eigenvalues**.

Also see [M-5] **eigensystem( )** for other routines for obtaining eigenvalues and eigenvectors.

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Syntax

`matrix symeigen` **X v** = **A**

where **A** is an $n \times n$ symmetric matrix.

Remarks and examples

Typing `matrix symeigen` **X v** = **A** for **A**: $n \times n$ returns

$$\mathbf{v} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$$
$$\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. Each $\mathbf{x}_i$ and λ_i is a solution to

$$\mathbf{A}\mathbf{x}_i = \lambda_i \mathbf{x}_i$$

or, more compactly,

$$\mathbf{A}\mathbf{X} = \mathbf{X}\,\mathrm{diag}(\mathbf{v})$$

▷ Example 1

Eigenvalues and eigenvectors have many uses. We will demonstrate that `symeigen` returns matrices meeting the definition:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. matrix accum A = weight mpg length, noconstant deviation
(obs=74)
. matrix list A
symmetric A[3,3]
            weight         mpg      length
weight    44094178
   mpg  -264948.11   2443.4595
length   1195077.3  -7483.5135   36192.662
. matrix symeigen X lambda = A
. matrix list lambda
lambda[1,3]
             e1          e2          e3
r1   44128163  3830.4869  820.73955
. matrix list X
X[3,3]
                e1          e2          e3
weight   .99961482  -.02756261   .00324179
   mpg  -.00600667   -.1008305   .99488549
length   .02709477   .99452175   .10095722
. matrix AX = A*X
. matrix XLambda = X*diag(lambda)
. matrix list AX
AX[3,3]
                e1          e2          e3
weight    44111166  -105.57823   2.6606641
   mpg   -265063.5  -386.22991   816.54187
length   1195642.6   3809.5025   82.859585
. matrix list XLambda
XLambda[3,3]
                e1          e2          e3
weight    44111166  -105.57823   2.6606641
   mpg   -265063.5  -386.22991   816.54187
length   1195642.6   3809.5025   82.859585
```

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Methods and formulas

Stata's internal eigenvalue and eigenvector extraction routines are translations of the public domain EISPACK routines, Smith et al. (1976), which are in turn based on Wilkinson and Reinsch (1971). EISPACK was developed under contract for the Office of Scientific and Technical Information, US Department of Energy, by Argonne National Laboratory and supported by funds provided by the Nuclear Regulatory Commission. Stata's use of these routines is by permission of the National Energy Software Center of the Argonne National Laboratory. A brief but excellent introduction to the techniques used by these routines can be found in Press et al. (2007, 563–599).

References

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. 2007. *Numerical Recipes: The Art of Scientific Computing*. 3rd ed. New York: Cambridge University Press.

Smith, B. T., J. M. Boyle, J. J. Dongarra, B. S. Garbow, Y. Ikebe, V. C. Klema, and C. B. Moler. 1976. *Matrix Eigensystem Routines–EISPACK Guide*. Vol. 6 of *Lecture Notes in Computer Science*, 2nd ed. Berlin: Springer.

Wilkinson, J. H., and C. H. Reinsch. 1971. *Linear Algebra*. Vol. 2 of *Handbook for Automatic Computation*. New York: Springer.

Also see

[P] **matrix** — Introduction to matrix commands

[P] **matrix eigenvalues** — Eigenvalues of nonsymmetric matrices

[M-4] **Matrix** — Matrix functions

[U] 14 Matrix expressions