

## matrix eigenvalues — Eigenvalues of nonsymmetric matrices

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## Description

`matrix eigenvalues` returns the real part of the eigenvalues in the  $1 \times n$  row vector  $\mathbf{r}$  and the imaginary part of the eigenvalues in the  $1 \times n$  row vector  $\mathbf{c}$ . Thus the  $j$ th eigenvalue is  $\mathbf{r}[1,j] + i * \mathbf{c}[1,j]$ .

The eigenvalues are sorted by their moduli;  $\mathbf{r}[1,1] + i * \mathbf{c}[1,1]$  has the largest modulus, and  $\mathbf{r}[1,n] + i * \mathbf{c}[1,n]$  has the smallest modulus.

If you want the eigenvalues for a symmetric matrix, see [P] [matrix symeigen](#).

Also see [M-5] [eigensystem\(\)](#) for alternative routines for obtaining eigenvectors and eigenvalues.

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## Syntax

```
matrix eigenvalues  $\mathbf{r}$   $\mathbf{c}$  =  $\mathbf{A}$ 
```

where  $\mathbf{A}$  is an  $n \times n$  nonsymmetric, real matrix.

## Remarks and examples

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Typing `matrix eigenvalues  $\mathbf{r}$   $\mathbf{c}$  =  $\mathbf{A}$`  for  $\mathbf{A}$   $n \times n$  returns

$$\mathbf{r} = (r_1, r_2, \dots, r_n)$$

$$\mathbf{c} = (c_1, c_2, \dots, c_n)$$

where  $\mathbf{r}_j$  is the real part and  $\mathbf{c}_j$  the imaginary part of the  $j$ th eigenvalue. The eigenvalues are part of the solution to the problem

$$\mathbf{A}\mathbf{x}_j = \lambda_j\mathbf{x}_j$$

and, in particular,

$$\lambda_j = \mathbf{r}_j + i * \mathbf{c}_j$$

The corresponding eigenvectors,  $\mathbf{x}_j$ , are not saved by `matrix eigenvalues`. The returned  $\mathbf{r}$  and  $\mathbf{c}$  are ordered so that  $|\lambda_1| \geq |\lambda_2| \geq \dots \geq |\lambda_n|$ , where  $|\lambda_j| = \sqrt{\mathbf{r}_j^2 + \mathbf{c}_j^2}$ .

## ▷ Example 1

In time-series analysis, researchers often use eigenvalues to verify the stability of the fitted model.

Suppose that we have fit a univariate time-series model and that the stability condition requires the moduli of all the eigenvalues of a “companion” matrix **A** to be less than 1. (See [Hamilton \[1994\]](#) for a discussion of these models and conditions.)

First, we form the companion matrix.

```
. matrix A = (0.66151492, .2551595, .35603325, -0.15403902, -.12734386)
. matrix A = A \ (I(4), J(4,1,0))
. matrix list A
A[5,5]
      c1          c2          c3          c4          c5
r1   .66151492   .2551595   .35603325  -.15403902  -.12734386
r1         1         0         0         0         0
r2         0         1         0         0         0
r3         0         0         1         0         0
r4         0         0         0         1         0
```

Next we use `matrix eigenvalues` to obtain the eigenvalues, which we will then list:

```
. matrix eigenvalues re im = A
. matrix list re
re[1,5]
      c1          c2          c3          c4          c5
real   .99121823   .66060006  -.29686008  -.29686008  -.3965832
. matrix list im
im[1,5]
      c1          c2          c3          c4          c5
complex 0         0         .63423776  -.63423776  0
```

Finally, we compute and list the moduli, which are all less than 1, although the first is close:

```
. forvalues i = 1/5 {
2.     display sqrt(re[1,'i']^2 + im[1,'i']^2)
3. }
.99121823
.66060006
.70027384
.70027384
.3965832
```

◀

## Methods and formulas

Stata’s internal eigenvalue extraction routine for nonsymmetric matrices is based on the public domain LAPACK routine DGEEV. [Anderson et al. \(1999\)](#) provide an excellent introduction to these routines. Stata’s internal routine also uses, with permission, **f2c** (©1990–1997 by AT&T, Lucent Technologies, and Bellcore).

## References

Anderson, E., Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen. 1999. *LAPACK Users’ Guide*. 3rd ed. Philadelphia: Society for Industrial and Applied Mathematics.

- Gould, W. W. 2011a. Understanding matrices intuitively, part 1. *The Stata Blog: Not Elsewhere Classified*. <http://blog.stata.com/2011/03/03/understanding-matrices-intuitively-part-1/>.
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- Hamilton, J. D. 1994. *Time Series Analysis*. Princeton, NJ: Princeton University Press.

## Also see

- [P] **matrix** — Introduction to matrix commands
- [P] **matrix symeigen** — Eigenvalues and eigenvectors of symmetric matrices
- [M-4] **matrix** — Matrix functions
- [U] **14 Matrix expressions**