Title

mvtest covariances — Multivariate tests of covariances

Description

`mvtest covariances` performs one-sample and multiple-sample multivariate tests on covariances. These tests assume multivariate normality.

See [MV] **mvtest** for other multivariate tests. See [R] **sdtest** for univariate tests of standard deviations.

Quick start

Test that the covariance matrix of `v1`, `v2`, `v3`, and `v4` is diagonal

```
mvtest covariances v1 v2 v3 v4
```

Test that the covariance matrix is spherical

```
mvtest covariances v1 v2 v3 v4, spherical
```

Test that the covariance matrix is compound symmetric

```
mvtest covariances v1 v2 v3 v4, compound
```

Test that the covariance matrix of the variables equals matrix `mymat`

```
mvtest covariances v1 v2 v3 v4, equals(mymat)
```

Test that the covariance matrix is block diagonal with `v1`, `v2`, and `v3` as block 1, `v4` as block 2, and `v5` and `v6` as block 3

```
mvtest cov v1 v2 v3 v4 v5 v6, block(v1 v2 v3 || v4 || v5 v6)
```

Box's M test that the covariance matrix of `v1`, `v2`, and `v3` is the same across the groups defined by `catvar`

```
mvtest cov v1 v2 v3, by(catvar)
```

Menu

Statistics > Multivariate analysis > MANOVA, multivariate regression, and related > Multivariate test of means, covariances, and normality

Syntax

Multiple-sample tests

`mvtest covariances` *varlist* [*if*] [*in*] [*weight*], `by(`*groupvars*`)` [*multisample_options*]

One-sample tests

`mvtest covariances` *varlist* [*if*] [*in*] [*weight*] [, *one-sample_options*]

multisample_options	Description
Model	
* `by(`*groupvars*`)`	compare subsamples with same values in *groupvars*
`missing`	treat missing values in *groupvars* as ordinary values

* `by(`*groupvars*`)` is required.

one-sample_options	Description
Options	
`diagonal`	test that covariance matrix is diagonal; the default
`spherical`	test that covariance matrix is spherical
`compound`	test that covariance matrix is compound symmetric
`equals(`*C*`)`	test that covariance matrix equals matrix *C*
* `block(`*varlist*$_1$ [`||` ...]`)`	test that covariance matrix is block diagonal with blocks corresponding to *varlist#*

* The full specification is `block(`*varlist*$_1$ [`||` *varlist*$_2$ [`||` ...]]`)`.

`bootstrap`, `by`, `collect`, `jackknife`, `rolling`, and `statsby` are allowed; see **[U] 11.1.10 Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see **[R] bootstrap**.

`aweight`s are not allowed with the `jackknife` prefix; see **[R] jackknife**.

`aweight`s and `fweight`s are allowed; see **[U] 11.1.6 weight**.

Options for multiple-sample tests

Model

`by(`*groupvars*`)` is required with the multiple-sample version of the test. Observations with the same values in *groupvars* form a sample. Observations with missing values in *groupvars* are ignored, unless the `missing` option is specified.

A modified likelihood-ratio statistic testing the equality of covariance matrices for the multiple independent samples defined by `by()` is presented along with an F and χ^2 approximation due to Box (1949). This test is also known as Box's M test.

`missing` specifies that missing values in *groupvars* are treated like ordinary values.

Options for one-sample tests

Options

`diagonal`, the default, tests the hypothesis that the covariance matrix is diagonal, that is, that the variables in *varlist* are independent. A likelihood-ratio test with first-order Bartlett correction is displayed.

`spherical` tests the hypothesis that the covariance matrix is diagonal with constant diagonal values, that is, that the variables in *varlist* are homoskedastic and independent. A likelihood-ratio test with first-order Bartlett correction is displayed.

`compound` tests the hypothesis that the covariance matrix is compound symmetric, that is, that the variables in *varlist* are homoskedastic and that every pair of two variables has the same covariance. A likelihood-ratio test with first-order Bartlett correction is displayed.

`equals(`C`)` specifies that the hypothesized covariance matrix for the k variables in *varlist* is C. The matrix C must be $k \times k$, symmetric, and positive definite. The row and column names of C are ignored. A likelihood-ratio test with first-order Bartlett correction is displayed.

`block(`*varlist*$_1$ [`||` *varlist*$_2$ [`||` ...]]`)` tests the hypothesis that the covariance matrix is block diagonal with blocks *varlist*$_1$, *varlist*$_2$, etc. Variables in *varlist* not included in *varlist*$_1$, *varlist*$_2$, etc., are treated as an additional block. With this pattern, variables in different blocks are independent, but no assumptions are made on the within-block covariance structure. A likelihood-ratio test with first-order Bartlett correction is displayed.

Remarks and examples

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Remarks are presented under the following headings:

One-sample tests for covariance matrices

One-sample and multiple-sample tests for covariance matrices are provided by the `mvtest covariances` command. One-sample tests include the test that the covariance matrix of *varlist* is diagonal, spherical, compound symmetric, block diagonal, or equal to a given matrix.

▷ Example 1

The gasoline-powered milk-truck dataset introduced in example 1 of [MV] **mvtest means** has price per mile for fuel, repair, and capital. We test if the covariance matrix for these three variables has any special structure.

```
. use https://www.stata-press.com/data/r17/milktruck
(Milk transportation costs for 25 gasoline trucks (Johnson and Wichern 2007))

. mvtest covariances fuel repair capital, diagonal

Test that covariance matrix is diagonal

    Adjusted LR chi2(3) =     17.91
            Prob > chi2 =    0.0005

. mvtest covariances fuel repair capital, spherical

Test that covariance matrix is spherical

    Adjusted LR chi2(5) =     21.53
            Prob > chi2 =    0.0006
```

```
. mvtest covariances fuel repair capital, compound
Test that covariance matrix is compound symmetric
   Adjusted LR chi2(4) =     11.29
           Prob > chi2 =    0.0235
```

We reject the hypotheses that the covariance is diagonal, spherical, or compound symmetric.

We now test whether there is covariance between `fuel` and `repair`, with `capital` not covarying with these two variables. Thus we hypothesize a block diagonal structure of the form

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22}^2 & 0 \\ 0 & 0 & \sigma_{33}^2 \end{pmatrix}$$

for the covariance matrix. The `block()` option of `mvtest covariances` provides the test:

```
. mvtest covariances fuel repair capital, block(fuel repair || capital)
Test that covariance matrix is block diagonal
   Adjusted LR chi2(2) =      3.52
           Prob > chi2 =    0.1722
```

We fail to reject the null hypothesis. The covariance matrix might have the block diagonal structure we hypothesized.

The same p-value could have been obtained from Stata's canonical correlation command:

```
. canon (fuel repair) (capital)
  (output omitted)
```

See [MV] **canon**.

Now, in addition to hypothesizing that the covariance is block diagonal, we specifically hypothesize that the variance for `capital` is 10, the variance of `fuel` is three times that of `capital`, the variance of `repair` is two times that of `capital`, and that there is no covariance between `capital` and the other two variables, while there is a covariance of 15 between `fuel` and `repair`. We test that hypothesis by using the `equals()` option.

```
. mat B = (30, 15, 0 \ 15, 20, 0 \ 0, 0, 10)
. matrix list B
symmetric B[3,3]
    c1  c2  c3
r1  30
r2  15  20
r3   0   0  10
. mvtest covariances fuel repair capital, equals(B)
Test that covariance matrix equals matrix B
   Adjusted LR chi2(6) =      5.48
           Prob > chi2 =    0.4837
```

We fail to reject the null hypothesis; the covariance might follow the structure hypothesized.

◁

❑ Technical note

If each block comprises a single variable, the test of independent subvectors reduces to a test that the covariance matrix is diagonal. Thus the following two commands are equivalent:

```
mvtest covariances x1 x2 x3 x4 x5, block(x1 || x2 || x3 || x4 || x5)
```

and

```
mvtest covariances x1 x2 x3 x4 x5, diagonal
```

❑

A multiple-sample test for covariance matrices

The `by()` option of `mvtest covariances` provides a modified likelihood-ratio statistic testing the equality of covariance matrices for the multiple independent samples defined by `by()`. This test is also known as Box's M test. There are both F and χ^2 approximations for the null distribution of the test.

▷ Example 2

We illustrate the multiple-sample test of equality of covariance matrices by using four psychological test scores on 32 men and 32 women (Rencher and Christensen 2012; Beall 1945).

```
. use https://www.stata-press.com/data/r17/genderpsych
(Four psychological test scores, Rencher and Christensen (2012))
. mvtest covariances y1 y2 y3 y4, by(gender)
Test of equality of covariance matrices across 2 samples
       Modified LR chi2 =   14.5606
      Box F(10,18377.7) =      1.35       Prob > F =  0.1950
           Box chi2(10) =     13.55    Prob > chi2 =  0.1945
```

Both the F and the χ^2 approximations indicate that we cannot reject the null hypothesis that the covariance matrices for males and females are equal (Rencher and Christensen 2012, 269).

◁

Equality of group covariance matrices is an assumption of multivariate analysis of variance (see [MV] **manova**) and linear discriminant analysis (see [MV] **discrim lda**). Box's M test, produced by `mvtest covariances` with the `by()` option, is often recommended for testing this assumption.

Stored results

`mvtest covariances` stores the following in `r()`:

Scalars

`r(chi2)`	χ^2
`r(df)`	degrees of freedom for χ^2 test
`r(p_chi2)`	p-value for χ^2 test
`r(F_Box)`	F statistic for Box test (`by()` only)
`r(df_m_Box)`	model degrees of freedom for Box test (`by()` only)
`r(df_r_Box)`	residual degrees of freedom for Box test (`by()` only)
`r(p_F_Box)`	p-value for Box's F test (`by()` only)

Macros

`r(chi2type)`	type of model χ^2 test

Methods and formulas

When comparing the formulas in this section with those found in some multivariate texts, be aware of whether they define the sample covariance matrix with a divisor of N or $N-1$. We use N. The formulas for several of the statistics are presented differently depending on your choice of divisor (but are still equivalent).

Methods and formulas are presented under the following headings:

One-sample tests for covariance matrices
A multiple-sample test for covariance matrices

One-sample tests for covariance matrices

Let the sample consist of N i.i.d. observations, $\mathbf{x}_i$, $i = 1, \ldots, N$, from a k-variate multivariate normal distribution, $\mathrm{MVN}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, with sample mean $\overline{\mathbf{x}} = 1/N \sum_{i=1}^N \mathbf{x}_i$, sample covariance matrix $\mathbf{S} = 1/N \sum_{i=1}^N (\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x}_i - \overline{\mathbf{x}})'$, and sample correlation matrix $\mathbf{R}$.

To test that a covariance matrix equals a given matrix, $H_0 : \boldsymbol{\Sigma} = \boldsymbol{\Sigma}_0$, `mvtest covariances` computes a likelihood-ratio test with Bartlett correction (Rencher and Christensen 2012, 260–261):

$$\chi^2_{\mathrm{ovf}} = (N-1)\left\{1 - \frac{1}{6(N-1)-1}\left(2k+1-\frac{2}{k+1}\right)\right\} \times \left\{\ln|\boldsymbol{\Sigma}_0| - \ln\left|\frac{N}{N-1}\mathbf{S}\right| + \mathrm{trace}\left(\frac{N}{N-1}\mathbf{S}\boldsymbol{\Sigma}_0^{-1}\right) - k\right\}$$

which is approximately χ^2 distributed with $k(k+1)/2$ degrees of freedom.

To test for a spherical covariance matrix, $H_0\colon \boldsymbol{\Sigma} = \sigma^2\mathbf{I}$, `mvtest covariances` computes a likelihood-ratio test with Bartlett correction (Rencher and Christensen 2012, 261–262):

$$\chi^2_{\mathrm{ovs}} = \left\{(N-1) - \frac{2k^2+k+2}{6k}\right\}\left[k\ln\{\mathrm{trace}(\mathbf{S})\} - \ln|\mathbf{S}| - k\ln(k)\right]$$

which is approximately χ^2 distributed with $k(k+1)/2 - 1$ degrees of freedom.

To test for a diagonal covariance matrix, $H_0\colon \boldsymbol{\Sigma}_{ij} = 0$ for $i \neq j$, `mvtest covariances` computes a likelihood-ratio test with first-order Bartlett correction (Rencher and Christensen 2012, 275):

$$\chi^2_{\mathrm{ovd}} = -\left(N - 1 - \frac{2k+5}{6}\right)\ln|\mathbf{R}|$$

which is approximately χ^2 distributed with $k(k-1)/2$ degrees of freedom.

To test for a compound-symmetric covariance matrix, $H_0\colon \boldsymbol{\Sigma} = \sigma^2\{(1-\rho)\mathbf{I} + \rho\mathbf{1}\mathbf{1}'\}$, that is, a covariance matrix with common variance σ^2 and common correlation ρ, `mvtest covariances` computes a likelihood-ratio test with first-order Bartlett correction (Rencher and Christensen 2012, 263–264):

$$\chi^2_{\mathrm{ovc}} = \left\{N - 1 - \frac{k(k+1)^2(2k-3)}{6(k-1)(k^2+k-4)}\right\} \times \left[k\ln\left(s^2\right) + (k-1)\ln(1-r) + \ln\{1+(k-1)r\} - \ln|\mathbf{S}|\right]$$

where

$$s^2 = \frac{1}{k}\sum_{j=1}^{k} s_{jj} \qquad \text{and} \qquad r = \frac{1}{k(k-1)s^2}\sum_{j=1}^{k}\sum_{h=1,h\neq j}^{k} s_{jh}$$

where s_{jh} is the (j,h) element of $\mathbf{S}$. χ^2_{ovc} is approximately χ^2 distributed with $k(k+1)/2-2$ degrees of freedom.

To test that a covariance matrix is block diagonal with b diagonal blocks and with k_j variables in block j, `mvtest covariances` computes a likelihood-ratio test with first-order Bartlett correction (Rencher and Christensen 2012, 271–272). Thus variables in different blocks are hypothesized to be independent.

$$\chi^2_{\text{ovb}} = \left(N - 1 - \frac{2a_3 + 3a_2}{6a_2}\right)\left(\sum_{j=1}^{b} \ln|\mathbf{S}_j| - \ln|\mathbf{S}|\right)$$

where $a_2 = k^2 - \sum_{j=1}^{b} k_j^2$, $a_3 = k^3 - \sum_{j=1}^{b} k_j^3$, and $\mathbf{S}_j$ is the covariance matrix for the jth block. χ^2_{ovb} is approximately χ^2 distributed with $a_2/2$ degrees of freedom.

A multiple-sample test for covariance matrices

Let there be $m \geq 2$ independent samples with the jth sample containing N_j i.i.d. observations, $\mathbf{x}_{ji}$, $i = 1,\ldots,N_j$, from a k-variate multivariate normal distribution $\text{MVN}_k(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$. The observed jth sample mean is $\overline{\mathbf{x}}_j = 1/N_j \sum_{i=1}^{N_j} \mathbf{x}_{ji}$ and covariance is $\mathbf{S}_j = 1/N_j \sum_{i=1}^{N_j} (\mathbf{x}_{ji} - \overline{\mathbf{x}}_j)(\mathbf{x}_{ji} - \overline{\mathbf{x}}_j)'$. Let $N = \sum_{j=1}^{m} N_j$.

To test the equality of covariance matrices in m independent samples, $H_0\colon \boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \cdots = \boldsymbol{\Sigma}_m$, `mvtest covariances` computes a modified likelihood-ratio statistic, which is an unbiased variant of the likelihood-ratio statistic (Rencher and Christensen 2012, 266–268):

$$-2\ln(M) = (N-m)\ln\left|\mathbf{S}_{\text{pooled}}\right| - \sum_{j=1}^{m}\left\{(N_j - 1)\ln\left|\frac{N_j}{N_j - 1}\mathbf{S}_j\right|\right\}$$

where $\mathbf{S}_{\text{pooled}} = \sum_{j=1}^{m}(N_j\mathbf{S}_j)/(N-m)$. Asymptotically, $-2\ln(M)$ is χ^2 distributed. Box (1949, 1950) derived more accurate χ^2 and F approximations (Rencher and Christensen 2012, 267–268).

Box's χ^2 approximation is given by

$$\chi^2_{\text{mv}} = -2(1 - c_1)\ln(M)$$

which is approximately χ^2 distributed with $(m-1)k(k+1)/2$ degrees of freedom.

Box's F approximation is given by

$$F_{\text{mv}} = \begin{cases} -2b_1 \ln(M) & \text{if } c_2 > c_1^2 \\[2ex] \dfrac{2a_2 b_2 \ln(M)}{a_1\{1 + 2b_2\ln(M)\}} & \text{otherwise} \end{cases}$$

which is approximately F distributed with a_1 and a_2 degrees of freedom.

In the χ^2 and F approximations, we have

$$c_1 = \left\{ \sum_{j=1}^{m} (N_j - 1)^{-1} - (N - m)^{-1} \right\} \frac{2k^2 + 3k - 1}{6(k+1)(m-1)}$$

$$c_2 = \left\{ \sum_{j=1}^{m} (N_j - 1)^{-2} - (N - m)^{-2} \right\} \frac{(k-1)(k+2)}{6(m-1)}$$

$a_1 = (m-1)k(k+1)/2$, $a_2 = (a_1 + 2)/|c_2 - c_1^2|$, $b_1 = (1 - c_1 - a_1/a_2)/a_1$, and $b_2 = (1 - c_1 + 2/a_2)/a_2$.

References

Beall, G. 1945. Approximate methods in calculating discriminant functions. *Psychometrika* 10: 205–217. https://doi.org/10.1007/BF02310469.

Box, G. E. P. 1949. A general distribution theory for a class of likelihood criteria. *Biometrika* 36: 317–346. https://doi.org/10.2307/2332671.

——. 1950. Problems in the analysis of growth and wear curves. *Biometrics* 6: 362–389. https://doi.org/10.2307/3001781.

Johnson, R. A., and D. W. Wichern. 2007. *Applied Multivariate Statistical Analysis*. 6th ed. Englewood Cliffs, NJ: Prentice Hall.

Rencher, A. C., and W. F. Christensen. 2012. *Methods of Multivariate Analysis*. 3rd ed. Hoboken, NJ: Wiley.

Also see

[MV] **candisc** — Canonical linear discriminant analysis

[MV] **canon** — Canonical correlations

[MV] **discrim lda** — Linear discriminant analysis

[MV] **manova** — Multivariate analysis of variance and covariance

[R] **correlate** — Correlations of variables

[R] **sdtest** — Variance-comparison tests