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Description

`mvtest covariances` performs one-sample and multiple-sample multivariate tests on covariances. These tests assume multivariate normality.

See [\[MV\] mvtest](#) for other multivariate tests. See [\[R\] sdtest](#) for univariate tests of standard deviations.

Quick start

Test that the covariance matrix of `v1`, `v2`, `v3`, and `v4` is diagonal

```
mvtest covariances v1 v2 v3 v4
```

Test that the covariance matrix is spherical

```
mvtest covariances v1 v2 v3 v4, spherical
```

Test that the covariance matrix is compound symmetric

```
mvtest covariances v1 v2 v3 v4, compound
```

Test that the covariance matrix of the variables equals matrix `mymat`

```
mvtest covariances v1 v2 v3 v4, equals(mymat)
```

Test that the covariance matrix is block diagonal with `v1`, `v2`, and `v3` as block 1, `v4` as block 2, and `v5` and `v6` as block 3

```
mvtest cov v1 v2 v3 v4 v5 v6, block(v1 v2 v3 || v4 || v5 v6)
```

Box's M test that the covariance matrix of `v1`, `v2`, and `v3` is the same across the groups defined by `catvar`

```
mvtest cov v1 v2 v3, by(catvar)
```

Menu

Statistics > Multivariate analysis > MANOVA, multivariate regression, and related > Multivariate test of means, covariances, and normality

Syntax

Multiple-sample tests

```
mvtest covariances varlist [if] [in] [weight], by(groupvars) [multisample_options]
```

One-sample tests

```
mvtest covariances varlist [if] [in] [weight] [, one-sample_options]
```

multisample_options

Description

Model

* **by**(*groupvars*) compare subsamples with same values in *groupvars*
missing treat missing values in *groupvars* as ordinary values

* **by**(*groupvars*) is required.

one-sample_options

Description

Options

diagonal test that covariance matrix is diagonal; the default
spherical test that covariance matrix is spherical
compound test that covariance matrix is compound symmetric
equals(C) test that covariance matrix equals matrix *C*
 * **block**(*varlist*₁ [|| ...]) test that covariance matrix is block diagonal with blocks corresponding to *varlist*#

* The full specification is **block**(*varlist*₁ [|| *varlist*₂ [|| ...]]).

bootstrap, **by**, **jackknife**, **rolling**, and **statsby** are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the **bootstrap** prefix; see [R] **bootstrap**.

aweights are not allowed with the **jackknife** prefix; see [R] **jackknife**.

aweights and **fweight**s are allowed; see [U] 11.1.6 **weight**.

Options for multiple-sample tests

Model

by(*groupvars*) is required with the multiple-sample version of the test. Observations with the same values in *groupvars* form a sample. Observations with missing values in *groupvars* are ignored, unless the **missing** option is specified.

A modified likelihood-ratio statistic testing the equality of covariance matrices for the multiple independent samples defined by **by**() is presented along with an *F* and chi-squared approximation due to Box (1949). This test is also known as Box's *M* test.

missing specifies that missing values in *groupvars* are treated like ordinary values.

Options for one-sample tests

Options

`diagonal`, the default, tests the hypothesis that the covariance matrix is diagonal, that is, that the variables in *varlist* are independent. A likelihood-ratio test with first-order Bartlett correction is displayed.

`spherical` tests the hypothesis that the covariance matrix is diagonal with constant diagonal values, that is, that the variables in *varlist* are homoskedastic and independent. A likelihood-ratio test with first-order Bartlett correction is displayed.

`compound` tests the hypothesis that the covariance matrix is compound symmetric, that is, that the variables in *varlist* are homoskedastic and that every pair of two variables has the same covariance. A likelihood-ratio test with first-order Bartlett correction is displayed.

`equals(C)` specifies that the hypothesized covariance matrix for the k variables in *varlist* is C . The matrix C must be $k \times k$, symmetric, and positive definite. The row and column names of C are ignored. A likelihood-ratio test with first-order Bartlett correction is displayed.

`block(varlist1 [|| varlist2 [|| ...]])` tests the hypothesis that the covariance matrix is block diagonal with blocks *varlist*₁, *varlist*₂, etc. Variables in *varlist* not included in *varlist*₁, *varlist*₂, etc., are treated as an additional block. With this pattern, variables in different blocks are independent, but no assumptions are made on the within-block covariance structure. A likelihood-ratio test with first-order Bartlett correction is displayed.

Remarks and examples

[stata.com](http://www.stata.com)

Remarks are presented under the following headings:

One-sample tests for covariance matrices
A multiple-sample test for covariance matrices

One-sample tests for covariance matrices

One-sample and multiple-sample tests for covariance matrices are provided by the `mvtest covariances` command. One-sample tests include the test that the covariance matrix of *varlist* is diagonal, spherical, compound symmetric, block diagonal, or equal to a given matrix.

► Example 1

The gasoline-powered milk-truck dataset introduced in [example 1](#) of [\[MV\] mvtest means](#) has price per mile for fuel, repair, and capital. We test if the covariance matrix for these three variables has any special structure.

```
. use http://www.stata-press.com/data/r15/milktruck
(Milk transportation costs for 25 gasoline trucks (Johnson and Wichern 2007))
. mvtest covariances fuel repair capital, diagonal
Test that covariance matrix is diagonal
  Adjusted LR chi2(3) =    17.91
  Prob > chi2 =    0.0005
. mvtest covariances fuel repair capital, spherical
Test that covariance matrix is spherical
  Adjusted LR chi2(5) =    21.53
  Prob > chi2 =    0.0006
```

```
. mvtest covariances fuel repair capital, compound
Test that covariance matrix is compound symmetric
Adjusted LR chi2(4) =    11.29
Prob > chi2 =    0.0235
```

We reject the hypotheses that the covariance is diagonal, spherical, or compound symmetric.

We now test whether there is covariance between `fuel` and `repair`, with `capital` not covarying with these two variables. Thus we hypothesize a block diagonal structure of the form

$$\Sigma = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22}^2 & 0 \\ 0 & 0 & \sigma_{33}^2 \end{pmatrix}$$

for the covariance matrix. The `block()` option of `mvtest covariances` provides the test:

```
. mvtest covariances fuel repair capital, block(fuel repair || capital)
Test that covariance matrix is block diagonal
Adjusted LR chi2(2) =    3.52
Prob > chi2 =    0.1722
```

We fail to reject the null hypothesis. The covariance matrix might have the block diagonal structure we hypothesized.

The same p -value could have been obtained from Stata's canonical correlation command:

```
. canon (fuel repair) (capital)
(output omitted)
```

See [MV] **canon**.

Now, in addition to hypothesizing that the covariance is block diagonal, we specifically hypothesize that the variance for `capital` is 10, the variance of `fuel` is three times that of `capital`, the variance of `repair` is two times that of `capital`, and that there is no covariance between `capital` and the other two variables, while there is a covariance of 15 between `fuel` and `repair`. We test that hypothesis by using the `equals()` option.

```
. mat B = (30, 15, 0 \ 15, 20, 0 \ 0, 0, 10)
. matrix list B
symmetric B[3,3]
   c1  c2  c3
r1  30
r2  15  20
r3   0   0  10
. mvtest covariances fuel repair capital, equals(B)
Test that covariance matrix equals matrix B
Adjusted LR chi2(6) =    5.48
Prob > chi2 =    0.4837
```

We fail to reject the null hypothesis; the covariance might follow the structure hypothesized.

□ Technical note

If each block comprises a single variable, the test of independent subvectors reduces to a test that the covariance matrix is diagonal. Thus the following two commands are equivalent:

```
mvtest covariances x1 x2 x3 x4 x5, block(x1 || x2 || x3 || x4 || x5)
```

and

```
mvtest covariances x1 x2 x3 x4 x5, diagonal
```

□

A multiple-sample test for covariance matrices

The `by()` option of `mvtest covariances` provides a modified likelihood-ratio statistic testing the equality of covariance matrices for the multiple independent samples defined by `by()`. This test is also known as Box's M test. There are both F and chi-squared approximations for the null distribution of the test.

▷ Example 2

We illustrate the multiple-sample test of equality of covariance matrices by using four psychological test scores on 32 men and 32 women (Rencher and Christensen 2012; Beall 1945).

```
. use http://www.stata-press.com/data/r15/genderpsych
(Four Psychological Test Scores, Rencher and Christensen (2012))
. mvtest covariances y1 y2 y3 y4, by(gender)
Test of equality of covariance matrices across 2 samples
    Modified LR chi2 =   14.5606
    Box F(10,18377.7) =     1.35      Prob > F =  0.1950
    Box chi2(10) =     13.55      Prob > chi2 =  0.1945
```

Both the F and the chi-squared approximations indicate that we cannot reject the null hypothesis that the covariance matrices for males and females are equal (Rencher and Christensen 2012, 269).

◀

Equality of group covariance matrices is an assumption of multivariate analysis of variance (see [MV] [manova](#)) and linear discriminant analysis (see [MV] [discrim lda](#)). Box's M test, produced by `mvtest covariances` with the `by()` option, is often recommended for testing this assumption.

Stored results

`mvtest covariances` stores the following in `r()`:

Scalars

<code>r(chi2)</code>	chi-squared
<code>r(df)</code>	degrees of freedom for chi-squared test
<code>r(p_chi2)</code>	p -value for χ^2 test
<code>r(F_Box)</code>	F statistic for Box test (<code>by()</code> only)
<code>r(df_m_Box)</code>	model degrees of freedom for Box test (<code>by()</code> only)
<code>r(df_r_Box)</code>	residual degrees of freedom for Box test (<code>by()</code> only)
<code>r(p_F_Box)</code>	p -value for Box's F test (<code>by()</code> only)

Macros

<code>r(chi2type)</code>	type of model chi-squared test
--------------------------	--------------------------------

Methods and formulas

When comparing the formulas in this section with those found in some multivariate texts, be aware of whether they define the sample covariance matrix with a divisor of N or $N - 1$. We use N . The formulas for several of the statistics are presented differently depending on your choice of divisor (but are still equivalent).

Methods and formulas are presented under the following headings:

- One-sample tests for covariance matrices*
- A multiple-sample test for covariance matrices*

One-sample tests for covariance matrices

Let the sample consist of N i.i.d. observations, \mathbf{x}_i , $i = 1, \dots, N$, from a k -variate multivariate normal distribution, $MVN_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, with sample mean $\bar{\mathbf{x}} = 1/N \sum_{i=1}^N \mathbf{x}_i$, sample covariance matrix $\mathbf{S} = 1/N \sum_{i=1}^N (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})'$, and sample correlation matrix \mathbf{R} .

To test that a covariance matrix equals a given matrix, $H_0: \boldsymbol{\Sigma} = \boldsymbol{\Sigma}_0$, `mvtest covariances` computes a likelihood-ratio test with Bartlett correction (Rencher and Christensen 2012, 260–261):

$$\chi_{\text{ovf}}^2 = (N - 1) \left\{ 1 - \frac{1}{6(N - 1) - 1} \left(2k + 1 - \frac{2}{k + 1} \right) \right\} \\ \times \left\{ \ln |\boldsymbol{\Sigma}_0| - \ln \left| \frac{N}{N - 1} \mathbf{S} \right| + \text{trace} \left(\frac{N}{N - 1} \mathbf{S} \boldsymbol{\Sigma}_0^{-1} \right) - k \right\}$$

which is approximately χ^2 distributed with $k(k + 1)/2$ degrees of freedom.

To test for a spherical covariance matrix, $H_0: \boldsymbol{\Sigma} = \sigma^2 \mathbf{I}$, `mvtest covariances` computes a likelihood-ratio test with Bartlett correction (Rencher and Christensen 2012, 261–262):

$$\chi_{\text{ovs}}^2 = \left\{ (N - 1) - \frac{2k^2 + k + 2}{6k} \right\} \left[k \ln \{ \text{trace}(\mathbf{S}) \} - \ln |\mathbf{S}| - k \ln(k) \right]$$

which is approximately χ^2 distributed with $k(k + 1)/2 - 1$ degrees of freedom.

To test for a diagonal covariance matrix, $H_0: \boldsymbol{\Sigma}_{ij} = 0$ for $i \neq j$, `mvtest covariances` computes a likelihood-ratio test with first-order Bartlett correction (Rencher and Christensen 2012, 275):

$$\chi_{\text{ovd}}^2 = - \left(N - 1 - \frac{2k + 5}{6} \right) \ln |\mathbf{R}|$$

which is approximately χ^2 distributed with $k(k - 1)/2$ degrees of freedom.

To test for a compound-symmetric covariance matrix, $H_0: \boldsymbol{\Sigma} = \sigma^2 \{ (1 - \rho) \mathbf{I} + \rho \mathbf{1}\mathbf{1}' \}$, that is, a covariance matrix with common variance σ^2 and common correlation ρ , `mvtest covariances` computes a likelihood-ratio test with first-order Bartlett correction (Rencher and Christensen 2012, 263–264):

$$\chi_{\text{ovc}}^2 = \left\{ N - 1 - \frac{k(k + 1)^2(2k - 3)}{6(k - 1)(k^2 + k - 4)} \right\} \\ \times [k \ln(s^2) + (k - 1) \ln(1 - r) + \ln\{1 + (k - 1)r\} - \ln |\mathbf{S}|]$$

where

$$s^2 = \frac{1}{k} \sum_{j=1}^k s_{jj} \quad \text{and} \quad r = \frac{1}{k(k-1)s^2} \sum_{j=1}^k \sum_{h=1, h \neq j}^k s_{jh}$$

where s_{jh} is the (j, h) element of \mathbf{S} . χ_{ovc}^2 is approximately χ^2 distributed with $k(k+1)/2 - 2$ degrees of freedom.

To test that a covariance matrix is block diagonal with b diagonal blocks and with k_j variables in block j , `mvtest covariances` computes a likelihood-ratio test with first-order Bartlett correction (Rencher and Christensen 2012, 271–272). Thus variables in different blocks are hypothesized to be independent.

$$\chi_{\text{ovb}}^2 = \left(N - 1 - \frac{2a_3 + 3a_2}{6a_2} \right) \left(\sum_{j=1}^b \ln |\mathbf{S}_j| - \ln |\mathbf{S}| \right)$$

where $a_2 = k^2 - \sum_{j=1}^b k_j^2$, $a_3 = k^3 - \sum_{j=1}^b k_j^3$, and \mathbf{S}_j is the covariance matrix for the j th block. χ_{ovb}^2 is approximately χ^2 distributed with $a_2/2$ degrees of freedom.

A multiple-sample test for covariance matrices

Let there be $m \geq 2$ independent samples with the j th sample containing N_j i.i.d. observations, \mathbf{x}_{ji} , $i = 1, \dots, N_j$, from a k -variate multivariate normal distribution $\text{MVN}_k(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$. The observed j th sample mean is $\bar{\mathbf{x}}_j = 1/N_j \sum_{i=1}^{N_j} \mathbf{x}_{ji}$ and covariance is $\mathbf{S}_j = 1/N_j \sum_{i=1}^{N_j} (\mathbf{x}_{ji} - \bar{\mathbf{x}}_j)(\mathbf{x}_{ji} - \bar{\mathbf{x}}_j)'$. Let $N = \sum_{j=1}^m N_j$.

To test the equality of covariance matrices in m independent samples, $H_0: \boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \dots = \boldsymbol{\Sigma}_m$, `mvtest covariances` computes a modified likelihood-ratio statistic, which is an unbiased variant of the likelihood-ratio statistic (Rencher and Christensen 2012, 266–268):

$$-2 \ln(M) = (N - m) \ln |\mathbf{S}_{\text{pooled}}| - \sum_{j=1}^m \left\{ (N_j - 1) \ln \left| \frac{N_j}{N_j - 1} \mathbf{S}_j \right| \right\}$$

where $\mathbf{S}_{\text{pooled}} = \sum_{j=1}^m (N_j \mathbf{S}_j) / (N - m)$. Asymptotically, $-2 \ln(M)$ is χ^2 distributed. Box (1949, 1950) derived more accurate χ^2 and F approximations (Rencher and Christensen 2012, 267–268).

Box's χ^2 approximation is given by

$$\chi_{\text{mv}}^2 = -2(1 - c_1) \ln(M)$$

which is approximately χ^2 distributed with $(m-1)k(k+1)/2$ degrees of freedom.

Box's F approximation is given by

$$F_{\text{mv}} = \begin{cases} -2b_1 \ln(M) & \text{if } c_2 > c_1^2 \\ \frac{2a_2 b_2 \ln(M)}{a_1 \{1 + 2b_2 \ln(M)\}} & \text{otherwise} \end{cases}$$

which is approximately F distributed with a_1 and a_2 degrees of freedom.

In the χ^2 and F approximations, we have

$$c_1 = \left\{ \sum_{j=1}^m (N_j - 1)^{-1} - (N - m)^{-1} \right\} \frac{2k^2 + 3k - 1}{6(k + 1)(m - 1)}$$

$$c_2 = \left\{ \sum_{j=1}^m (N_j - 1)^{-2} - (N - m)^{-2} \right\} \frac{(k - 1)(k + 2)}{6(m - 1)}$$

$a_1 = (m - 1)k(k + 1)/2$, $a_2 = (a_1 + 2)/|c_2 - c_1^2|$, $b_1 = (1 - c_1 - a_1/a_2)/a_1$, and $b_2 = (1 - c_1 + 2/a_2)/a_2$.

References

- Beall, G. 1945. Approximate methods in calculating discriminant functions. *Psychometrika* 10: 205–217.
- Box, G. E. P. 1949. A general distribution theory for a class of likelihood criteria. *Biometrika* 36: 317–346.
- . 1950. Problems in the analysis of growth and wear curves. *Biometrics* 6: 362–389.
- Johnson, R. A., and D. W. Wichern. 2007. *Applied Multivariate Statistical Analysis*. 6th ed. Englewood Cliffs, NJ: Prentice Hall.
- Rencher, A. C., and W. F. Christensen. 2012. *Methods of Multivariate Analysis*. 3rd ed. Hoboken, NJ: Wiley.

Also see

- [MV] **candisc** — Canonical linear discriminant analysis
- [MV] **canon** — Canonical correlations
- [R] **correlate** — Correlations (covariances) of variables or coefficients
- [MV] **discrim lda** — Linear discriminant analysis
- [MV] **manova** — Multivariate analysis of variance and covariance
- [R] **sdtest** — Variance-comparison tests