**hotelling — Hotelling’s T-squared generalized means test**

### Description

hotelling performs Hotelling’s $T$-squared test of whether a set of means is zero or, alternatively, equal between two groups.

See [MV] *mvtest means* for generalizations of Hotelling’s one-sample test with more general hypotheses, two-sample tests that do not assume that the covariance matrices are the same in the two groups, and tests with more than two groups.

### Quick start

Hotelling’s $T$-squared test that the means of v1, v2, and v3 are jointly zero

```
hotelling v1 v2 v3
```

As above, but suppress the table of means

```
hotelling v1 v2 v3, notable
```

Hotelling’s $T$-squared test that the means of v1, v2, and v3 are the same in the two groups defined by catvar

```
hotelling v1 v2 v3, by(catvar)
```

### Menu

Statistics > Multivariate analysis > MANOVA, multivariate regression, and related > Hotelling’s generalized means test
Syntax

```
hotelling varlist [if] [in] [weight] [, by(varname) notable]
```

Aweights and fweights are allowed; see [U] 11.1.6 weight.

Note: hotel is a synonym for hotelling.

Options

- **by(varname)** specifies a variable identifying two groups; the test of equality of means between groups is performed. If by( ) is not specified, a test of means being jointly zero is performed.
- **notable** suppresses printing a table of the means being compared.

Remarks and examples

hotelling performs Hotelling’s $T$-squared test of whether a set of means is zero or two sets of means are equal. It is a multivariate test that reduces to a standard $t$ test if only one variable is specified.

Example 1

You wish to test whether a new fuel additive improves gas mileage in both stop-and-go and highway situations. Taking 12 cars, you fill them with gas and run them on a highway-style track, recording their gas mileage. You then refill them and run them on a stop-and-go style track. Finally, you repeat the two runs, but this time you use fuel with the additive. Your dataset is

```
. use https://www.stata-press.com/data/r16/gasexp
. describe
```

Contains data from https://www.stata-press.com/data/r16/gasexp.dta

```
ox: 12
vars: 5 21 Nov 2018 12:56

<table>
<thead>
<tr>
<th>variable name</th>
<th>storage</th>
<th>display</th>
<th>value label</th>
</tr>
</thead>
<tbody>
<tr>
<td>id</td>
<td>byte</td>
<td>%9.0g</td>
<td>car ID</td>
</tr>
<tr>
<td>bmpg1</td>
<td>byte</td>
<td>%9.0g</td>
<td>track 1 before additive</td>
</tr>
<tr>
<td>ampg1</td>
<td>byte</td>
<td>%9.0g</td>
<td>track 1 after additive</td>
</tr>
<tr>
<td>bmpg2</td>
<td>byte</td>
<td>%9.0g</td>
<td>track 2 before additive</td>
</tr>
<tr>
<td>ampg2</td>
<td>float</td>
<td>%9.0g</td>
<td>track 2 after additive</td>
</tr>
</tbody>
</table>
```

Sorted by:
To perform the statistical test, you jointly test whether the differences in before-and-after results are zero:

```
. generate diff1 = ampg1 - bmpg1
. generate diff2 = ampg2 - bmpg2
. hotelling diff1 diff2
```

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff1</td>
<td>12</td>
<td>1.75</td>
<td>2.70101</td>
<td>-3</td>
<td>5</td>
</tr>
<tr>
<td>diff2</td>
<td>12</td>
<td>2.083333</td>
<td>2.906367</td>
<td>-3.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

1-group Hotelling's T-squared = 9.6980676
F test statistic: ((12-2)/(12-1)(2)) x 9.6980676 = 4.4082126
H0: Vector of means is equal to a vector of zeros
F(2,10) = 4.4082
Prob > F(2,10) = 0.0424

The means are different at the 4.24% significance level.

## Technical note

We used Hotelling’s $T^2$-squared test because we were testing two differences jointly. Had there been only one difference, we could have used a standard $t$ test, which would have yielded the same results as Hotelling’s test:

```
. ttest ampg1 = bmpg1
```

Paired t test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Err.</th>
<th>Std. Dev.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ampg1</td>
<td>12</td>
<td>22.75</td>
<td>.9384465</td>
<td>3.250874</td>
<td>20.68449 24.81551</td>
</tr>
<tr>
<td>bmpg1</td>
<td>12</td>
<td>21</td>
<td>.7881701</td>
<td>2.730301</td>
<td>19.26525 22.73475</td>
</tr>
<tr>
<td>diff</td>
<td>12</td>
<td>1.75</td>
<td>.7797144</td>
<td>2.70101</td>
<td>.0338602 3.46614</td>
</tr>
</tbody>
</table>

mean(diff) = mean(ampg1 - bmpg1)  $t = 2.2444$
Ho: mean(diff) = 0  degrees of freedom = 11
Ha: mean(diff) < 0  Ha: mean(diff) != 0  Ha: mean(diff) > 0
Pr(T < t) = 0.9768  Pr(|T| > |t|) = 0.0463  Pr(T > t) = 0.0232

. ttest diff1 = 0
One-sample t test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Err.</th>
<th>Std. Dev.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff1</td>
<td>12</td>
<td>1.75</td>
<td>.7797144</td>
<td>2.70101</td>
<td>.0338602 3.46614</td>
</tr>
</tbody>
</table>

mean = mean(diff1)  $t = 2.2444$
Ho: mean = 0  degrees of freedom = 11
Ha: mean < 0  Ha: mean != 0  Ha: mean > 0
Pr(T < t) = 0.9768  Pr(|T| > |t|) = 0.0463  Pr(T > t) = 0.0232
hotelling — Hotelling’s T-squared generalized means test

```
. hotelling diff1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff1</td>
<td>12</td>
<td>1.75</td>
<td>2.70101</td>
<td>-3</td>
<td>5</td>
</tr>
</tbody>
</table>
```

1-group Hotelling’s T-squared = 5.0373832
F test statistic: ((12-1)/(12-1)(1)) x 5.0373832 = 5.0373832

H0: Vector of means is equal to a vector of zeros
F(1,11) = 5.0374
Prob > F(1,11) = 0.0463

Example 2

Now consider a variation on the experiment: rather than using 12 cars and running each car with and without the fuel additive, you run 24 cars, 12 with the additive and 12 without. You have the following dataset:

```-use https://www.stata-press.com/data/r16/gasexp2, clear-
.describe
Contains data from https://www.stata-press.com/data/r16/gasexp2.dta
obs: 24
vars: 4
  21 Nov 2018 13:01
storage  display value
variable name type format label variable label
id    byte %9.0g   car ID
mpg1  byte %9.0g   track 1
mpg2  float %9.0g  track 2
additive byte %9.0g yesno additive?
```

Sorted by:
```
.tabulate additive
```

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>12</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>yes</td>
<td>12</td>
<td>50.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Total 24 100.00

This is an unpaired experiment because there is no natural pairing of the cars; you want to test that the means of mpg1 are equal for the two groups specified by additive, as are the means of mpg2:
. hotelling mpg1 mpg2, by(additive)

<table>
<thead>
<tr>
<th>-&gt; additive = no</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>mpg1</td>
<td>12</td>
<td>21</td>
<td>2.730301</td>
<td>17</td>
<td>25</td>
</tr>
<tr>
<td>mpg2</td>
<td>12</td>
<td>19.92</td>
<td>2.644319</td>
<td>16</td>
<td>24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>-&gt; additive = yes</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>mpg1</td>
<td>12</td>
<td>22.75</td>
<td>3.250874</td>
<td>17</td>
<td>28</td>
</tr>
<tr>
<td>mpg2</td>
<td>12</td>
<td>22</td>
<td>3.316625</td>
<td>16.5</td>
<td>27.5</td>
</tr>
</tbody>
</table>

2-group Hotelling’s T-squared = 7.1347584  
F test statistic: ((24-2-1)/(24-2)(2)) x 7.1347584 = 3.4052256  
H0: Vectors of means are equal for the two groups  
F(2,21) = 3.4052  
Prob > F(2,21) = 0.0524

Technical note
As in the paired experiment, had there been only one test track, the $t$ test would have yielded the same results as Hotelling’s test:

. hotelling mpg1, by(additive)

<table>
<thead>
<tr>
<th>-&gt; additive = no</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>mpg1</td>
<td>12</td>
<td>21</td>
<td>2.730301</td>
<td>17</td>
<td>25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>-&gt; additive = yes</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>mpg1</td>
<td>12</td>
<td>22.75</td>
<td>3.250874</td>
<td>17</td>
<td>28</td>
</tr>
</tbody>
</table>

2-group Hotelling’s T-squared = 2.0390921  
F test statistic: ((24-1-1)/(24-2)(1)) x 2.0390921 = 2.0390921  
H0: Vectors of means are equal for the two groups  
F(1,22) = 2.0391  
Prob > F(1,22) = 0.1673

. ttest mpg1, by(additive)  
Two-sample $t$ test with equal variances

<table>
<thead>
<tr>
<th>Group</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Err.</th>
<th>Std. Dev.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>12</td>
<td>21</td>
<td>.7881701</td>
<td>2.730301</td>
<td>19.26525 22.73475</td>
</tr>
<tr>
<td>yes</td>
<td>12</td>
<td>22.75</td>
<td>.9384465</td>
<td>3.250874</td>
<td>20.68449 24.81551</td>
</tr>
<tr>
<td>combined</td>
<td>24</td>
<td>21.875</td>
<td>.6264476</td>
<td>3.068954</td>
<td>20.57909 23.17091</td>
</tr>
<tr>
<td>diff</td>
<td></td>
<td>-1.75</td>
<td>1.225518</td>
<td>-4.291568 0.7915684</td>
<td></td>
</tr>
</tbody>
</table>

diff = mean(no) - mean(yes)  
t = -1.4280  
Ho: diff = 0  
Ha: diff < 0  
Ha: diff != 0  
Ha: diff > 0  
Pr(T < t) = 0.0837  
Pr(|T| > |t|) = 0.1673  
Pr(T > t) = 0.9163
With more than one pair of means, however, there is no \( t \) test equivalent to Hotelling’s test, although there are other logically (but not practically) equivalent solutions. One is the discriminant function: if the means of \( mpg1 \) and \( mpg2 \) are different, the discriminant function should separate the groups along that dimension.

```
. regress additive mpg1 mpg2
```

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1.46932917</td>
<td>2</td>
<td>.734664585</td>
<td>F(2, 21) = 3.41</td>
</tr>
<tr>
<td>Residual</td>
<td>4.53067083</td>
<td>21</td>
<td>.21574623</td>
<td>Prob &gt; F = 0.0524</td>
</tr>
<tr>
<td>Total</td>
<td>6.00000000</td>
<td>23</td>
<td>.260869565</td>
<td>R-squared = 0.2449</td>
</tr>
</tbody>
</table>

Additive Coef. Std. Err. t P>|t| [95% Conf. Interval]

- \( mpg1 \): -.4570407 .2416657 -1.89 0.072 -.959612 .0455306
- \( mpg2 \): .5014605 .2376762 2.11 0.047 .0071859 .9957352
- \( _cons \): -.0120115 .7437049 -0.02 0.987 -1.55863 1.534607

This test would declare the means different at the 5.24% level. You could also have fit this model by using logistic regression:

```
. logit additive mpg1 mpg2
```

Iteration 0: log likelihood = -16.635532
Iteration 1: log likelihood = -13.395178
Iteration 2: log likelihood = -13.371201
Iteration 3: log likelihood = -13.371143
Iteration 4: log likelihood = -13.371143

Logistic regression Number of obs = 24
LR chi2(2) = 6.53
Prob > chi2 = 0.0382
Log likelihood = -13.371143 Pseudo R2 = 0.1962

| additive | Coef. Std. Err. z P>|z| [95% Conf. Interval] |
|----------|--------------|--------|--------|-------------------|
| \( mpg1 \): -2.306844 1.36139 -1.69 0.090 -4.975119 .3614307
| \( mpg2 \): 2.524477 1.367373 1.85 0.065 -.1555257 5.20448
| \( _cons \): -2.446527 3.689821 -0.66 0.507 -9.678443 4.78539

This test would have declared the means different at the 3.82% level.

Are the means different? Hotelling’s \( T \)-squared and the discriminant function reject equality at the 5.24% level. The logistic regression rejects equality at the 3.82% level.

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**Stored results**

`hotelling` stores the following in \( r() \):

**Scalars**
- \( r(N) \) number of observations
- \( r(k) \) number of variables
- \( r(T2) \) Hotelling’s \( T \)-squared
- \( r(df) \) degrees of freedom
Methods and formulas

See Wilks (1962, 556–561) for a general discussion. The original formulation was by Hotelling (1931) and Mahalanobis (1930, 1936).

For the test that the means of \( k \) variables are 0, let \( \bar{x} \) be a \( 1 \times k \) matrix of the means and \( S \) be the estimated covariance matrix. Then \( T^2 = \bar{x}S^{-1}\bar{x}' \).

For two groups, the test of equality is \( T^2 = (\bar{x}_1 - \bar{x}_2)S^{-1}(\bar{x}_1 - \bar{x}_2)' \).

Harold Hotelling (1895–1973) was an American economist and statistician who made many important contributions to mathematical economics, multivariate analysis, and statistical inference. After obtaining degrees in journalism and mathematics, he taught and researched at Stanford, Columbia, and the University of North Carolina. His work generalizing Student’s \( t \) ratio and on principal components, canonical correlation, multivariate analysis of variance, and correlation continues to be widely used.

Prasanta Chandra Mahalanobis (1893–1972) studied physics and mathematics at Calcutta and Cambridge. He became interested in statistics and on his return to India worked on applications in anthropology, meteorology, hydrology, and agriculture. Mahalanobis became the leader in Indian statistics, specializing in multivariate problems (including what is now called the Mahalanobis distance), the design of large-scale sample surveys, and the contribution of statistics to national planning.

References


Also see

[MV] manova — Multivariate analysis of variance and covariance

[MV] mvtest means — Multivariate tests of means

[R] regress — Linear regression

[R] ttest — \( t \) tests (mean-comparison tests)