hotelling — Hotelling's T^2 generalized means test

Description

`hotelling` performs Hotelling's T^2 test of whether a set of means is zero or, alternatively, equal between two groups.

See [MV] **mvtest means** for generalizations of Hotelling's one-sample test with more general hypotheses, two-sample tests that do not assume that the covariance matrices are the same in the two groups, and tests with more than two groups.

Quick start

Hotelling's T^2 test that the means of `v1`, `v2`, and `v3` are jointly zero

```
hotelling v1 v2 v3
```

As above, but suppress the table of means

```
hotelling v1 v2 v3, notable
```

Hotelling's T^2 test that the means of `v1`, `v2`, and `v3` are the same in the two groups defined by `catvar`

```
hotelling v1 v2 v3, by(catvar)
```

Menu

Statistics > Multivariate analysis > MANOVA, multivariate regression, and related > Hotelling's generalized means test

Syntax

hotelling *varlist* [*if*] [*in*] [*weight*] [, by(*varname*) notable]

collect is allowed; see [U] 11.1.10 Prefix commands.

aweights and fweights are allowed; see [U] 11.1.6 weight.

Note: hotel is a synonym for hotelling.

Options

Main

by(*varname*) specifies a variable identifying two groups; the test of equality of means between groups is performed. If by() is not specified, a test of means being jointly zero is performed.

notable suppresses printing a table of the means being compared.

Remarks and examples

stata.com

hotelling performs Hotelling's T^2 test of whether a set of means is zero or two sets of means are equal. It is a multivariate test that reduces to a standard t test if only one variable is specified.

▷ Example 1

You wish to test whether a new fuel additive improves gas mileage in both stop-and-go and highway situations. Taking 12 cars, you fill them with gas and run them on a highway-style track, recording their gas mileage. You then refill them and run them on a stop-and-go style track. Finally, you repeat the two runs, but this time you use fuel with the additive. Your dataset is

```
. use https://www.stata-press.com/data/r17/gasexp
. describe
Contains data from https://www.stata-press.com/data/r17/gasexp.dta
 Observations:            12
    Variables:             5                  21 Nov 2020 12:56
--------------------------------------------------------------------------------
Variable      Storage   Display    Value
    name         type    format    label      Variable label
--------------------------------------------------------------------------------
id              byte    %9.0g                 Car ID
bmpg1           byte    %9.0g                 Track 1 before additive
ampg1           byte    %9.0g                 Track 1 after additive
bmpg2           byte    %9.0g                 Track 2 before additive
ampg2           float   %9.0g                 Track 2 after additive
--------------------------------------------------------------------------------
Sorted by:
```

To perform the statistical test, you jointly test whether the differences in before-and-after results are zero:

```
. generate diff1 = ampg1 - bmpg1
. generate diff2 = ampg2 - bmpg2
. hotelling diff1 diff2

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
       diff1 |         12        1.75     2.70101         -3          5
       diff2 |         12    2.083333    2.906367       -3.5        5.5
1-group Hotelling's T-squared = 9.6980676
F test statistic: ((12-2)/(12-1)(2)) x 9.6980676 = 4.4082126
H0: Vector of means is equal to a vector of zeros
              F(2,10) =     4.4082
       Prob > F(2,10) =     0.0424
```

The means are different at the 4.24% significance level.

◁

❑ Technical note

We used Hotelling's T^2 test because we were testing two differences jointly. Had there been only one difference, we could have used a standard t test, which would have yielded the same results as Hotelling's test:

```
. ttest ampg1 = bmpg1
Paired t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
   ampg1 |      12       22.75    .9384465    3.250874    20.68449    24.81551
   bmpg1 |      12          21    .7881701    2.730301    19.26525    22.73475
---------+--------------------------------------------------------------------
    diff |      12        1.75    .7797144     2.70101    .0338602     3.46614
------------------------------------------------------------------------------
     mean(diff) = mean(ampg1 - bmpg1)                             t =   2.2444
 H0: mean(diff) = 0                              Degrees of freedom =       11
 Ha: mean(diff) < 0           Ha: mean(diff) != 0           Ha: mean(diff) > 0
 Pr(T < t) = 0.9768         Pr(|T| > |t|) = 0.0463          Pr(T > t) = 0.0232
. ttest diff1 = 0
One-sample t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
   diff1 |      12        1.75    .7797144     2.70101    .0338602     3.46614
------------------------------------------------------------------------------
    mean = mean(diff1)                                            t =   2.2444
H0: mean = 0                                     Degrees of freedom =       11
    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.9768         Pr(|T| > |t|) = 0.0463          Pr(T > t) = 0.0232
```

```
. hotelling diff1
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
       diff1 |         12        1.75     2.70101         -3          5
1-group Hotelling's T-squared = 5.0373832
F test statistic: ((12-1)/(12-1)(1)) x 5.0373832 = 5.0373832
H0: Vector of means is equal to a vector of zeros
              F(1,11) =     5.0374
       Prob > F(1,11) =     0.0463
```

❑

▷ Example 2

Now consider a variation on the experiment: rather than using 12 cars and running each car with and without the fuel additive, you run 24 cars, 12 with the additive and 12 without. You have the following dataset:

```
. use https://www.stata-press.com/data/r17/gasexp2, clear
. describe
Contains data from https://www.stata-press.com/data/r17/gasexp2.dta
 Observations:            24
    Variables:             4                  21 Nov 2020 13:01
--------------------------------------------------------------------------------
Variable      Storage   Display    Value
    name         type    format    label      Variable label
--------------------------------------------------------------------------------
id              byte    %9.0g                 Car ID
mpg1            byte    %9.0g                 Track 1
mpg2            float   %9.0g                 Track 2
additive        byte    %9.0g      yesno      Additive?
--------------------------------------------------------------------------------
Sorted by:
. tabulate additive
  Additive? |      Freq.     Percent        Cum.
------------+-----------------------------------
         No |         12       50.00       50.00
        Yes |         12       50.00      100.00
------------+-----------------------------------
      Total |         24      100.00
```

This is an unpaired experiment because there is no natural pairing of the cars; you want to test that the means of `mpg1` are equal for the two groups specified by `additive`, as are the means of `mpg2`:

```
. hotelling mpg1 mpg2, by(additive)

------------------------------------------------------------------------------
-> additive = No

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        mpg1 |         12          21    2.730301         17         25
        mpg2 |         12    19.91667    2.644319         16         24

------------------------------------------------------------------------------
-> additive = Yes

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        mpg1 |         12       22.75    3.250874         17         28
        mpg2 |         12          22    3.316625       16.5       27.5

2-group Hotelling's T-squared = 7.1347584
F test statistic: ((24-2-1)/(24-2)(2)) x 7.1347584 = 3.4052256

H0: Vectors of means are equal for the two groups
              F(2,21) =     3.4052
       Prob > F(2,21) =     0.0524
```

❑ Technical note

As in the paired experiment, had there been only one test track, the t test would have yielded the same results as Hotelling's test:

```
. hotelling mpg1, by(additive)

------------------------------------------------------------------------------
-> additive = No

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        mpg1 |         12          21    2.730301         17         25

------------------------------------------------------------------------------
-> additive = Yes

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
        mpg1 |         12       22.75    3.250874         17         28

2-group Hotelling's T-squared = 2.0390921
F test statistic: ((24-1-1)/(24-2)(1)) x 2.0390921 = 2.0390921

H0: Vectors of means are equal for the two groups
              F(1,22) =     2.0391
       Prob > F(1,22) =     0.1673

. ttest mpg1, by(additive)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
      No |      12          21    .7881701    2.730301    19.26525    22.73475
     Yes |      12       22.75    .9384465    3.250874    20.68449    24.81551
---------+--------------------------------------------------------------------
Combined |      24      21.875    .6264476    3.068954    20.57909    23.17091
---------+--------------------------------------------------------------------
    diff |               -1.75    1.225518               -4.291568    .7915684
------------------------------------------------------------------------------
    diff = mean(No) - mean(Yes)                                   t =  -1.4280
H0: diff = 0                                     Degrees of freedom =       22

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0837         Pr(|T| > |t|) = 0.1673          Pr(T > t) = 0.9163
```

With more than one pair of means, however, there is no t test equivalent to Hotelling's test, although there are other logically (but not practically) equivalent solutions. One is the discriminant function: if the means of mpg1 and mpg2 are different, the discriminant function should separate the groups along that dimension.

```
. regress additive mpg1 mpg2

      Source |       SS           df       MS      Number of obs   =        24
-------------+----------------------------------   F(2, 21)        =      3.41
       Model |  1.46932917         2  .734664585   Prob > F        =    0.0524
    Residual |  4.53067083        21   .21574623   R-squared       =    0.2449
-------------+----------------------------------   Adj R-squared   =    0.1730
       Total |           6        23  .260869565   Root MSE        =    .46448

------------------------------------------------------------------------------
    additive | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        mpg1 |  -.4570407   .2416657    -1.89   0.072     -.959612    .0455306
        mpg2 |   .5014605   .2376762     2.11   0.047     .0071859    .9957352
       _cons |  -.0120115   .7437049    -0.02   0.987     -1.55863    1.534607
------------------------------------------------------------------------------
```

This test would declare the means different at the 5.24% level. You could also have fit this model by using logistic regression:

```
. logit additive mpg1 mpg2

Iteration 0:   log likelihood = -16.635532
Iteration 1:   log likelihood = -13.395178
Iteration 2:   log likelihood = -13.371201
Iteration 3:   log likelihood = -13.371143
Iteration 4:   log likelihood = -13.371143

Logistic regression                                     Number of obs =     24
                                                        LR chi2(2)    =   6.53
                                                        Prob > chi2   = 0.0382
Log likelihood = -13.371143                             Pseudo R2     = 0.1962

------------------------------------------------------------------------------
    additive | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        mpg1 |  -2.306844    1.36139    -1.69   0.090    -4.975119    .3614307
        mpg2 |   2.524477   1.367373     1.85   0.065    -.1555257     5.20448
       _cons |  -2.446527   3.689821    -0.66   0.507    -9.678443     4.78539
------------------------------------------------------------------------------
```

This test would have declared the means different at the 3.82% level.

Are the means different? Hotelling's T^2 and the discriminant function reject equality at the 5.24% level. The logistic regression rejects equality at the 3.82% level.

❑

Stored results

hotelling stores the following in r():

Scalars

r(N)	number of observations	r(T2)	Hotelling's T^2
r(k)	number of variables	r(df)	degrees of freedom

Methods and formulas

See Wilks (1962, 556–561) for a general discussion. The original formulation was by Hotelling (1931) and Mahalanobis (1930, 1936).

For the test that the means of k variables are 0, let $\overline{\mathbf{x}}$ be a $1 \times k$ matrix of the means and $\mathbf{S}$ be the estimated covariance matrix. Then $T^2 = \overline{\mathbf{x}}\mathbf{S}^{-1}\overline{\mathbf{x}}'$.

For two groups, the test of equality is $T^2 = (\overline{\mathbf{x}}_1 - \overline{\mathbf{x}}_2)\mathbf{S}^{-1}(\overline{\mathbf{x}}_1 - \overline{\mathbf{x}}_2)'$.

Harold Hotelling (1895–1973) was an American economist and statistician who made many important contributions to mathematical economics, multivariate analysis, and statistical inference. After obtaining degrees in journalism and mathematics, he taught and researched at Stanford, Columbia, and the University of North Carolina. His work generalizing Student's t ratio and on principal components, canonical correlation, multivariate analysis of variance, and correlation continues to be widely used.

Prasanta Chandra Mahalanobis (1893–1972) studied physics and mathematics at Calcutta and Cambridge. He became interested in statistics and on his return to India worked on applications in anthropology, meteorology, hydrology, and agriculture. Mahalanobis became the leader in Indian statistics, specializing in multivariate problems (including what is now called the Mahalanobis distance), the design of large-scale sample surveys, and the contribution of statistics to national planning.

References

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Mahalanobis, P. C. 1930. On tests and measures of group divergence. *Journal of the Asiatic Society of Bengal* 26: 541–588.

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Olkin, I., and A. R. Sampson. 2001. Harold Hotelling. In *Statisticians of the Centuries*, ed. C. C. Heyde and E. Seneta, 454–458. New York: Springer.

Rao, C. R. 1973. Prasantha Chandra Mahalanobis, 1893–1972. *Biographical Memoirs of Fellows of The Royal Society* 19: 455–492. https://doi.org/10.1098/rsbm.1973.0017.

Wilks, S. S. 1962. *Mathematical Statistics*. New York: Wiley.

Also see

[MV] **manova** — Multivariate analysis of variance and covariance

[MV] **mvtest means** — Multivariate tests of means

[R] **regress** — Linear regression

[R] **ttest** — t tests (mean-comparison tests)