# discrim Ida postestimation — Postestimation tools for discrim Ida

Postestimation commands	predict	estat	Remarks and examples
Stored results	Methods and formulas	References	Also see

# **Postestimation commands**

The following postestimation commands are of special interest after discrim lda:

Command	Description
estat anova	ANOVA summaries table
estat canontest	tests of the canonical discriminant functions
estat classfunctions	classification functions
estat classtable	classification table
estat correlations	correlation matrices and p-values
estat covariance	covariance matrices
estat errorrate	classification error-rate estimation
estat grdistances	Mahalanobis and generalized squared distances between the group means
estat grmeans	group means and variously standardized or transformed means
estat grsummarize	group summaries
estat list	classification listing
estat loadings	canonical discriminant-function coefficients (loadings)
estat manova	MANOVA table
estat structure	canonical structure matrix
estat summarize	estimation sample summary
loadingplot	plot standardized discriminant-function loadings
scoreplot	plot discriminant-function scores
screeplot	plot eigenvalues

The following standard postestimation commands are also available:

Command	Description
* estimates	cataloging estimation results
predict	group membership, probabilities of group membership, etc.

\*All estimates subcommands except table and stats are available; see [R] estimates.

# predict

## **Description for predict**

predict creates a new variable containing predictions such as group classifications, probabilities, Mahalanobis squared distances, discriminant function scores, leave-one-out probabilities, leave-one-out group classifications, and leave-one-out Mahalanobis squared distances.

# Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic options]
```

predict [type] { stub\* | newvarlist } [if ] [in] [, statistic options ]

statistic	Description
Main	
<u>c</u> lassification	group membership classification; the default when one variable is specified and group() is not specified
pr	probability of group membership; the default when group() is specified or when multiple variables are specified
<u>mah</u> alanobis	Mahalanobis squared distance between observations and groups
<u>dsc</u> ore	discriminant function score
<u>clsc</u> ore	group classification function score
* <u>looc</u> lass	leave-one-out group membership classification; may be used only when one new variable is specified
*loopr	leave-one-out probability of group membership
* <u>loom</u> ahal	leave-one-out Mahalanobis squared distance between observations and groups
options	Description
Main	
<pre>group(group)</pre>	the group for which the statistic is to be calculated
Options	
<u>pri</u> ors( <i>priors</i> ) <u>tie</u> s( <i>ties</i> )	group prior probabilities; defaults to e(grouppriors) how ties in classification are to be handled; defaults to e(ties)

priors	Description
equal proportional matname matrix_exp	equal prior probabilities group-size-proportional prior probabilities row or column vector containing the group prior probabilities matrix expression providing a row or column vector of the group prior probabilities
ties	Description
<u>m</u> issing <u>r</u> andom <u>f</u> irst	ties in group classification produce missing values ties in group classification are broken randomly ties in group classification are set to the first tied group

You specify one new variable with classification or looclass; either one or e(N\_groups) new variables with pr, loopr, mahalanobis, loomahal, or clscore; and one to e(f) new variables with dscore.

Unstarred statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.

group() is not allowed with classification, dscore, or looclass.

## **Options for predict**

Main

- classification, the default, calculates the group classification. Only one new variable may be specified.
- pr calculates group membership posterior probabilities. If you specify the group() option, specify one new variable. Otherwise, you must specify e(N\_groups) new variables.
- mahalanobis calculates the squared Mahalanobis distance between the observations and group means. If you specify the group() option, specify one new variable. Otherwise, you must specify e(N\_groups) new variables.
- dscore produces the discriminant function score. Specify as many variables as leading discriminant functions that you wish to score. No more than e(f) variables may be specified.
- clscore produces the group classification function score. If you specify the group() option, specify one new variable. Otherwise, you must specify e(N\_groups) new variables.
- looclass calculates the leave-one-out group classifications. Only one new variable may be specified. Leave-one-out calculations are restricted to e(sample) observations.
- loopr calculates the leave-one-out group membership posterior probabilities. If you specify the group()
  option, specify one new variable. Otherwise, you must specify e(N\_groups) new variables. Leaveone-out calculations are restricted to e(sample) observations.
- loomahal calculates the leave-one-out squared Mahalanobis distance between the observations and group means. If you specify the group() option, specify one new variable. Otherwise, you must specify e(N\_groups) new variables. Leave-one-out calculations are restricted to e(sample) observations.

group (group) specifies the group for which the statistic is to be calculated and can be specified using

#1, #2, ..., where #1 means the first category of the e(groupvar) variable, #2 the second category, etc.;

the values of the e(groupvar) variable; or

the value labels of the e(groupvar) variable if they exist.

group() is not allowed with classification, dscore, or looclass.

Options

priors(*priors*) specifies the prior probabilities for group membership. If priors() is not specified, e(grouppriors) is used. The following *priors* are allowed:

priors (equal) specifies equal prior probabilities.

priors (proportional) specifies group-size-proportional prior probabilities.

priors (matname) specifies a row or column vector containing the group prior probabilities.

- priors (*matrix\_exp*) specifies a matrix expression providing a row or column vector of the group prior probabilities.
- ties (*ties*) specifies how ties in group classification will be handled. If ties () is not specified, e(ties) is used. The following *ties* are allowed:

ties(missing) specifies that ties in group classification produce missing values.

ties (random) specifies that ties in group classification are broken randomly.

ties(first) specifies that ties in group classification are set to the first tied group.

# estat

## **Description for estat**

estat anova presents a table summarizing the one-way ANOVAs for each variable in the discriminant analysis.

estat canontest presents tests of the canonical discriminant functions. Presented are the canonical correlations, eigenvalues, proportion and cumulative proportion of variance, and likelihood-ratio tests for the number of nonzero eigenvalues.

estat classfunctions displays the classification functions.

estat correlations displays the pooled within-group correlation matrix, between-groups correlation matrix, total-sample correlation matrix, and/or the individual group correlation matrices. Two-tailed *p*-values for the correlations may also be requested.

estat covariance displays the pooled within-group covariance matrix, between-groups covariance matrix, total-sample covariance matrix, and/or the individual group covariance matrices.

estat grdistances provides Mahalanobis squared distances between the group means along with the associated F statistics and significance levels. Also available are generalized squared distances.

estat grmeans provides group means, total-sample standardized group means, pooled within-group standardized means, and canonical functions evaluated at the group means.

estat loadings present the canonical discriminant-function coefficients (loadings). Unstandardized, pooled within-class standardized, and total-sample standardized coefficients are available.

estat manova presents the MANOVA table associated with the discriminant analysis.

estat structure presents the canonical structure matrix.

## Menu for estat

Statistics > Postestimation

## Syntax for estat

ANOVA summaries table

estat <u>an</u>ova

Tests of the canonical discriminant functions

estat <u>can</u>ontest

Classification functions

estat <u>classf</u>unctions [, classfunctions\_options]

Correlation matrices and p-values

estat <u>cor</u>relations [, *correlations\_options*]

Covariance matrices

estat covariance [, covariance\_options]

Mahalanobis and generalized squared distances between the group means

estat grdistances [, grdistances\_options]

```
Group means and variously standardized or transformed means
```

estat grmeans [, grmeans\_options]

Canonical discriminant-function coefficients (loadings)

estat loadings [, loadings\_options]

MANOVA table

estat <u>man</u>ova

Canonical structure matrix

estat structure [, format(%fmt)]

classfunctions_options	Description
Main adjustequal <u>for</u> mat(% <i>fmt</i> )	adjust the constant even when priors are equal numeric display format; default is %9.0g
Options <u>priors(priors)</u> <u>nopriors</u>	group prior probabilities; defaults to e(grouppriors) suppress display of prior probabilities

correlations_options	Description					
Main						
within	display pooled within-group correlation matrix; the default					
_ between	display between-groups correlation matrix					
total	display total-sample correlation matrix					
groups	display the correlation matrix for each group					
all	display all the above					
p	display two-sided p-values for requested correlations					
<u>for</u> mat(% <i>fmt</i> )	numeric display format; default is %9.0g					
nohalf	display full matrix even if symmetric					
covariance_options	Description					
Main						
within	display pooled within-group covariance matrix; the default					
between	display between-groups covariance matrix					
<u>t</u> otal	display total-sample covariance matrix					
groups	display the covariance matrix for each group					
all	display all the above					
<u>for</u> mat(% <i>fmt</i> )	numeric display format; default is %9.0g					
nohalf	display full matrix even if symmetric					
grdistances_options	Description					
Main						
<u>mah</u> alanobis[(f p)]	display Mahalanobis squared distances between group means; the default					
generalized	display generalized Mahalanobis squared distances between group means					
all	equivalent to mahalanobis(f p) generalized					
<pre><u>format(%fmt)</u></pre>	numeric display format; default is %9.0g					
Options						
priors(priors)	group prior probabilities; defaults to e(grouppriors)					
grmeans_options	Description					
Main	-					
raw	display untransformed and unstandardized group means					
<u>i</u> aw totalstd	display total-sample standardized group means					
<u>u</u> otaista withinstd	display pooled within-group standardized group means					
canonical	display canonical functions evaluated at group means					
all	display all the mean tables					
<u></u>	alophaj un die mean alores					

loadings_options	Description
Main	
<u>st</u> andardized	display pooled within-group standardized canonical discriminant function coefficients; the default
$\underline{tot}$ alstandardized	display the total-sample standardized canonical discriminant function coefficients
<u>unst</u> andardized	display unstandardized canonical discriminant function coefficients
all	display all the above
<u>for</u> mat(% <i>fmt</i> )	numeric display format; default is %9.0g

collect is allowed with all estat commands; see [U] 11.1.10 Prefix commands.

#### **Options for estat**

Options for estat are presented under the following headings:

Options for estat classfunctions Options for estat correlations Options for estat covariance Options for estat grdistances Options for estat grmeans Options for estat loadings Option for estat structure

## **Options for estat classfunctions**

[ Main ]

adjustequal specifies that the constant term in the classification function be adjusted for prior probabilities even though the priors are equal. By default, equal prior probabilities are not used in adjusting the constant term. adjustequal has no effect with unequal prior probabilities.

format (% fmt) specifies the matrix display format. The default is format (%9.0g).

Options

- priors (*priors*) specifies the group prior probabilities. The prior probabilities affect the constant term in the classification function. By default, *priors* is determined from e(grouppriors). See *Options for predict* for the *priors* specification. By common convention, when there are equal prior probabilities the adjustment of the constant term is not performed. See adjustequal to override this convention.
- nopriors specifies that the prior probabilities not be displayed. By default, the prior probabilities used in determining the constant in the classification functions are displayed as the last row in the classification functions table.

#### Options for estat correlations

Main

within specifies that the pooled within-group correlation matrix be displayed. This is the default.

between specifies that the between-groups correlation matrix be displayed.

total specifies that the total-sample correlation matrix be displayed.

groups specifies that the correlation matrix for each group be displayed.

all is the same as specifying within, between, total, and groups.

p specifies that two-sided p-values be computed and displayed for the requested correlations.

format (% fmt) specifies the matrix display format. The default is format (%8.5f).

nohalf specifies that, even though the matrix is symmetric, the full matrix be printed. The default is to print only the lower triangle.

## Options for estat covariance

∫ Main `

within specifies that the pooled within-group covariance matrix be displayed. This is the default.

between specifies that the between-groups covariance matrix be displayed.

total specifies that the total-sample covariance matrix be displayed.

groups specifies that the covariance matrix for each group be displayed.

all is the same as specifying within, between, total, and groups.

format (% fmt) specifies the matrix display format. The default is format (%9.0g).

nohalf specifies that, even though the matrix is symmetric, the full matrix be printed. The default is to print only the lower triangle.

## **Options for estat grdistances**

\_\_\_ Main 🗋

mahalanobis[(f p)] specifies that a table of Mahalanobis squared distances between group means be presented. mahalanobis(f) adds F tests for each displayed distance and mahalanobis(p) adds the associated p-values. mahalanobis(f p) adds both. The default is mahalanobis.

generalized specifies that a table of generalized Mahalanobis squared distances between group means be presented. generalized starts with what is produced by the mahalanobis option and adds a term accounting for prior probabilities. Prior probabilities are provided with the priors() option, or if priors() is not specified, by the values in e(grouppriors). By common convention, if prior probabilities are equal across the groups, the prior probability term is omitted and the results from generalized will equal those from mahalanobis.

all is equivalent to specifying mahalanobis(f p) and generalized.

format (% fmt) specifies the matrix display format. The default is format (%9.0g).

Options

priors (*priors*) specifies the group prior probabilities and affects only the output of the generalized option. By default, *priors* is determined from e(grouppriors). See *Options for predict* for the *priors* specification.

## Options for estat grmeans

```
Main
```

raw, the default, displays a table of group means.

totalstd specifies that a table of total-sample standardized group means be presented.

withinstd specifies that a table of pooled within-group standardized group means be presented.

canonical specifies that a table of the unstandardized canonical discriminant functions evaluated at the group means be presented.

all is equivalent to specifying raw, totalstd, withinstd, and canonical.

## **Options for estat loadings**

Main

standardized specifies that the pooled within-group standardized canonical discriminant function coefficients be presented. This is the default.

- totalstandardized specifies that the total-sample standardized canonical discriminant function coefficients be presented.
- unstandardized specifies that the unstandardized canonical discriminant function coefficients be presented.

all is equivalent to specifying standardized, totalstandardized, and unstandardized.

format(% *fmt*) specifies the matrix display format. The default is format(%9.0g).

## Option for estat structure

\_\_\_ Main ]

format(% fmt) specifies the matrix display format. The default is format(%9.0g).

# **Remarks and examples**

Remarks are presented under the following headings:

Classification tables, error rates, and listings ANOVA, MANOVA, and canonical correlations Discriminant and classification functions Scree, loading, and score plots Means and distances Covariance and correlation matrices Predictions

## Classification tables, error rates, and listings

After discrim, including discrim lda, you can obtain classification tables, error-rate estimates, and listings; see [MV] discrim estat.

#### Example 1: Predictive linear discriminant analysis

Example 1 of [MV] **manova** introduces the apple tree rootstock data from Andrews and Herzberg (1985, 357–360) and used in Rencher and Christensen (2012, 184). Descriptive linear discriminant analysis is often used after a multivariate analysis of variance (MANOVA) to explore the differences between groups found to be significantly different in the MANOVA.

We first examine the predictive aspects of the linear discriminant model on these data by examining classification tables, error-rate estimate tables, and classification listings.

To illustrate the ability of discrim lda and the postestimation commands of handling unequal prior probabilities, we perform our LDA using prior probabilities of 0.2 for the first four rootstock groups and 0.1 for the last two rootstock groups.

```
. use https://www.stata-press.com/data/r19/rootstock
(Table 6.2. Rootstock data, Rencher and Christensen (2012))
. discrim lda y1 y2 y3 y4, group(rootstock) priors(.2, .2, .2, .2, .1, .1)
Linear discriminant analysis
Resubstitution classification summary
```

Кеу
Number
Percent

True	Classified							
rootstock	1	2	3	4	5	6	Total	
1	7	0	0	1	0	0	8	
	87.50	0.00	0.00	12.50	0.00	0.00	100.00	
2	0	4	2	1	1	0	8	
	0.00	50.00	25.00	12.50	12.50	0.00	100.00	
3	0	1	6	1	0	0	8	
	0.00	12.50	75.00	12.50	0.00	0.00	100.00	
4	3	0	1	4	0	0	8	
	37.50	0.00	12.50	50.00	0.00	0.00	100.00	
5	0	3	2	0	2	1	8	
	0.00	37.50	25.00	0.00	25.00	12.50	100.00	
6	3	0	0	0	2	3	8	
	37.50	0.00	0.00	0.00	25.00	37.50	100.00	
Total	13	8	11	7	5	4	48	
	27.08	16.67	22.92	14.58	10.42	8.33	100.00	
Priors	0.2000	0.2000	0.2000	0.2000	0.1000	0.1000		

The prior probabilities are reported at the bottom of the table. The classification results are based, in part, on the selection of prior probabilities.

With only 8 observations per rootstock and six rootstock groups, we have small cell counts in our table, with many zero cell counts. Because resubstitution classification tables give an overly optimistic view of classification ability, we use the estat classtable command to request a leave-one-out (LOO) classification table and request the reporting of average posterior probabilities in place of percentages.

```
. estat classtable, probabilities loo
```

```
Leave-one-out average-posterior-probabilities classification table
```

Кеу						
Number Average post	terior pr	obabilit	у			
True		ssified				
rootstock	1	2	3	4	5	6
1	5	0	0	2	0	1
	0.6055	•	•	0.6251	•	0.3857
2	0	4	2	1	1	0
		0.6095	0.7638	0.3509	0.6607	•
3	0	1	6	1	0	0
		0.5520	0.7695	0.4241	•	•
4	4	0	1	3	0	0
	0.5032	•	0.7821	0.5461	•	•
5	0	3	2	0	2	1
	•	0.7723	0.5606	•	0.4897	0.6799
6	3	0	0	0	2	3
	0.6725	•	•	•	0.4296	0.5763
Total	12	8	11	7	5	5
	0.5881	0.6634	0.7316	0.5234	0.4999	0.5589
Priors	0.2000	0.2000	0.2000	0.2000	0.1000	0.1000

Zero cell counts report a missing value for the average posterior probability. We did not specify the priors() option with estat classtable, so the prior probabilities used in our LDA model were used.

estat errorrate estimates the error rates for each group. We use the pp option to obtain estimates based on the posterior probabilities instead of the counts.

	stat errorrate	••				
Erro	or rate estimat	ed from post	terior proba	bilities		
	Error rate	rootstock 1	2	3	4	5
	Stratified	.2022195	.431596	.0868444	.4899799	.627472
	Unstratified	.2404022	.41446	.1889412	.5749832	.4953118
	Priors	.2	.2	.2	.2	.1
	Error rate	rootstock 6	Total			
	Stratified	.6416429	.3690394			
	Unstratified	.4027382	.3735623			
	Priors	.1				

We did not specify the priors () option, and estat errorrate defaulted to using the prior probabilities from the LDA model. Both stratified and unstratified estimates are shown for each rootstock group and for the overall total. See [MV] **discrim estat** for an explanation of the error-rate estimation.

We can list the classification results and posterior probabilities from our discriminant analysis model by using the estat list command. estat list allows us to specify which observations we wish to examine and what classification and probability results to report.

We request the LOO classification and LOO probabilities for all misclassified observations from the fourth rootstock group. We also suppress the resubstitution classification and probabilities from being displayed.

	Classifica	ation	LOO Pro	babiliti	es			
Obs	True LO	C1.	1	2	3	4	5	6
25 26 27 29 32	4 4 4 4 4	1 * 3 * 1 * 1 * 1 *	0.5433 0.0216 0.3506 0.6134 0.5054	0.1279 0.0199 0.1860 0.0001 0.0011	0.0997 0.7821 0.0583 0.0005 0.0017	0.0258 0.1458 0.2342 0.2655 0.4856	0.0636 0.0259 0.0702 0.0002 0.0002	0.1397 0.0048 0.1008 0.1202 0.0059

. estat list if rootstock==4, misclassified class(loo noclass) pr(loo nopr)

\* indicates misclassified observations

Four of the five misclassifications for rootstock group 4 were incorrectly classified as belonging to rootstock group 1.

## ANOVA, MANOVA, and canonical correlations

There is a mathematical relationship between Fisher's LDA and one-way MANOVA. They are both based on the eigenvalues and eigenvectors of the same matrix,  $W^{-1}B$  (though in MANOVA the matrices are labeled **E** and **H** for error and hypothesis instead of **W** and **B** for within and between). See [MV] manova and [R] anova for more information on MANOVA and ANOVA. Researchers often wish to examine the MANOVA and univariate ANOVA results corresponding to their LDA model.

Canonical correlations are also mathematically related to Fisher's LDA. The canonical correlations between the discriminating variables and indicator variables constructed from the group variable are based on the same eigenvalues and eigenvectors as MANOVA and Fisher's LDA. The information from a canonical correlation analysis gives insight into the importance of each discriminant function in the discrimination. See [MV] canon for more information on canonical correlations.

The estat manova, estat anova, and estat canontest commands display MANOVA, ANOVA, and canonical correlation information after discrim lda.

### Example 2: MANOVA, ANOVA, and canonical correlation corresponding to LDA

Continuing with the apple tree rootstock example, we examine the MANOVA, ANOVA, and canonical correlation results corresponding to our LDA.

. estat man	nova							
		Number of obs =		48				
			W = Wilks' lambda P = Pillai's trace		L = Lawley-Hotelling trace R = Roy's largest root			ace
	Source	St	atistic	df	F(df1,	df2) =	F	Prob>F
	rootstock	W P	0.1540	5	20.0 20.0	130.3 168.0	4.07	0.0000 a 0.0000 a
		L R	2.9214 1.8757		20.0 5.0	150.0 42.0		0.0000 a 0.0000 u
-	Residual			42				
	Total			47				

e = exact, a = approximate, u = upper bound on F

. estat anova

Univariate ANOVA summaries

Variable	Model MS	Resid MS	Total MS	R-sq	Adj. R-sq	F	Pr > F
y1			.29377189				
y2 y3	6.1139358	4.2908128	11.297777 4.484762	0.5876	0.5385	11.969	0.0000
y4	2.4930912	1.7225248	1.8044999	0.5914	0.5428	12.158	0.0000
Number of obs	s = 48	Model	df = 5	Resid	ual df =	42	

All four of the MANOVA tests reject the null hypothesis that the six rootstock groups have equal means. See example 1 of [MV] **manova** for an explanation of the MANOVA table.

estat anova presents a concise summary of univariate ANOVAs run on each of our four discriminating variables. Variables y3, trunk girth at 15 years, and y4, weight of tree above ground at 15 years, show the highest level of significance of the four variables.

estat canontest displays the canonical correlations and associated tests that correspond to our LDA model.

. estat	. estat canontest									
Canonio	Canonical linear discriminant analysis									
Fcn	Canon. Corr.	Eigen- value	Vari Prop.	ance Cumul.	Like- lihood Ratio	F	df1	df2	Prob>F	
1	0.8076	1.87567	0.6421	0.6421	0.1540	4.9369	20	130.3	0.0000	a
2	0.6645	.790694	0.2707	0.9127	0.4429	3.1879	12	106.1	0.0006	a
3	0.4317	.229049	0.0784	0.9911	0.7931	1.6799	6	82	0.1363	е
4	0.1591	.025954	0.0089	1.0000	0.9747	.54503	2	42	0.5839	е
	I									

HO: This and smaller canon. corr. are zero; e = exact F, a = approximate F

The number of nonzero eigenvalues in Fisher's LDA is  $\min(q-1, p)$  With q = 6 groups, and p = 4discriminating variables, there are four nonzero eigenvalues. The four eigenvalues and the corresponding canonical correlations of  $\mathbf{W}^{-1}\mathbf{B}$ , ordered from largest to smallest, are reported along with the proportion and cumulative proportion of variance accounted for by each of the discriminant functions. Using one discriminant dimension is insufficient for capturing the variability of our four-dimensional data. With two dimensions we account for 91% of the variance. Using three of the four dimensions accounts for 99% of the variance. Little is gained from the fourth discriminant dimension.

Also presented are the likelihood-ratio tests of the null hypothesis that each canonical correlation and all smaller canonical correlations from this model are zero. The letter a is placed beside the p-values of the approximate F tests, and the letter e is placed beside the *p*-values of the exact F tests. The first two tests are highly significant, indicating that the first two canonical correlations are likely not zero. The third test has a p-value of 0.1363, so that we fail to reject that the third and fourth canonical correlation are zero.

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## Discriminant and classification functions

See [MV] discrim Ida for a discussion of linear discriminant functions and linear classification functions for LDA.

Discriminant functions are produced from Fisher's LDA. The discriminant functions provide a set of transformations from the original p-dimensional (the number of discriminating variables) space to the minimum of p and q-1 (the number of groups minus 1) dimensional space. The discriminant functions are ordered in importance.

Classification functions are by-products of the Mahalanobis approach to LDA. There are always qclassification functions-one for each group. They are not ordered by importance, and you cannot use a subset of them for classification.

A table showing the discriminant function coefficients is available with estat loadings (see example 3), and a table showing the classification function coefficients is available with estat classfunctions (see example 4).

## Example 3: Canonical discriminant functions and canonical structures

We continue with the apple tree rootstock example. The canonical discriminant function coefficients (loadings) are available through the estat loadings command. Unstandardized, pooled within-group standardized, and total-sample standardized coefficients are available. The all option requests all three, and the format() option provides control over the numeric display format used in the tables.

_	-								
. estat loadings, all format(%6.2f)									
Canonical discrim:	Canonical discriminant function coefficients								
	func~1	func~2	func~3	func~4					
y1	-3.05	1.14	-1.00	23.42					
	1.70								
•	-4.23								
	0.48								
•		12.20							
_cons	10.40	12.20	-3.33	-12.47					
Standardized canor	nical dis	criminan	t functi	on coeff	icients				
	func~1	func~2	func~3	func~4					
v1	-0.27	0.10	-0.09	2.04					
	0.92								
	-1.35								
y3 y4		2.33							
·	1								
Total-sample stand	dardized	canonica	l discri	minant f	unction coefficients				
	func~1	func~2	func~3	func~4					
y1	-0.28	0.10	-0.09	2.14					
•	1.00								
	-1.99								
•									
y4	0.14	3.45	-1.05	0.93					

The unstandardized canonical discriminant function coefficients shown in the first table are the function coefficients that apply to the unstandardized discriminating variables—y1 through y4 and a constant term. See example 5 for a graph, known as a score plot, that plots the observations transformed by these unstandardized canonical discriminant function coefficients.

The standardized canonical discriminant function coefficients are the coefficients that apply to the discriminating variables after they have been standardized by the pooled within-group covariance. These coefficients are appropriate for interpreting the importance and relationship of the discriminating variables within the discriminant functions. See example 5 for a graph, known as a loading plot, that plots these standardized coefficients.

The total-sample standardized canonical discriminant function coefficients are the coefficients that apply to the discriminating variables after they have been standardized by the total-sample covariance. See *Methods and formulas* of [MV] **discrim lda** for references discussing which of within-group and total-sample standardization is most appropriate.

For both styles of standardization, variable y1 has small (in absolute value) coefficients for the first three discriminant functions. This indicates that y1 does not play an important part in these discriminant functions. Because the fourth discriminant function accounts for such a small percentage of the variance, we ignore the coefficients from the fourth function when assessing the importance of the variables.

Some sources, see Huberty (1994), advocate the interpretation of structure coefficients, which measure the correlation between the discriminating variables and the discriminant functions, instead of standardized discriminant function coefficients; see the discussion in example 1 of [MV] discrim Ida for references to this dispute. The estat structure command displays structure coefficients.

```
. estat structure, format(%9.6f)
Canonical structure
                   function1 function2 function3 function4
                   -0.089595
                               0.261416
                                          0.820783
                                                     0.499949
              y1
              y2
                   -0.086765
                               0.431180
                                          0.898063
                                                     0.006158
              yЗ
                   -0.836986
                               0.281362
                                          0.457902 -0.103031
             y4
                  -0.793621
                               0.572890
                                          0.162901 -0.124206
```

Using structure coefficients for interpretation, we conclude that y1 is important for the second and third discriminant functions.

4

## Example 4: LDA classification functions

Switching from Fisher's approach to LDA to Mahalanobis's approach to LDA, we examine what are called classification functions with the estat classfunctions command. Classification functions are applied to the unstandardized discriminating variables. The classification function that results in the largest value for an observation indicates the group to assign the observation.

Continuing with the rootstock LDA, we specify the format() option to control the display format of the classification coefficients.

```
. estat classfunctions, format(%8.3f)
```

```
Classification functions
```

assilication ful								
	rootstock							
	1	2	3	4	5	6		
y1	314,640	317.120	324.589	307.260	316.767	311.301		
y2	-59.417	-63.981	-65.152	-59.373	-65.826	-63.060		
уЗ	149.610	168.161	154.910	147.652	168.221	160.622		
y4	-161.178	-172.644	-150.356	-153.387	-172.851	-175.477		
_cons	-301.590	-354.769	-330.103	-293.427	-349.847	-318.099		
Priors	0.200	0.200	0.200	0.200	0.100	0.100		

The prior probabilities, used in constructing the coefficient for the constant term, are displayed as the last row in the table. We did not specify the priors() option, so the prior probabilities defaulted to those in our LDA model, which has rootstock group 5 and 6 with prior probabilities of 0.1, whereas the other groups have prior probabilities of 0.2.

See example 10 for applying the classification function to data by using the predict command.

4

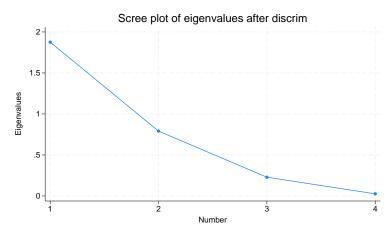
## Scree, loading, and score plots

Examples of discriminant function loading plots and score plots (see [MV] **scoreplot**) can be found in example 3 of [MV] **discrim lda** and example 1 of [MV] **candisc**. Also available after discrim lda are scree plots; see [MV] **screeplot**.

## Example 5: Scree, loading, and score plots

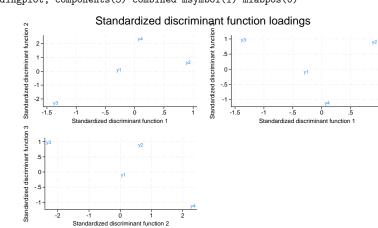
Continuing with our rootstock example, the scree plot of the four nonzero eigenvalues we previously saw in the output of estat canontest in example 2 are graphed using the screeplot command.

```
. screeplot
```



The *Remarks and examples* in [MV] screeplot concerning the use of scree plots for selecting the number of components in the context of pca apply also for selecting the number of discriminant functions after discrim lda. With these four eigenvalues, it is not obvious whether to choose the top two or three eigenvalues. From the estat canontest output of example 2, the first two discriminant functions account for 91% of the variance, and three discriminant functions account for 99% of the variance.

The loadingplot command (see [MV] scoreplot) allows us to graph the pooled within-group standardized discriminant coefficients (loadings) that we saw in tabular form from the estat loadings command of example 3. By default only the loadings from the first two functions are graphed. We override this setting with the components(3) option, obtaining graphs of the first versus second, first versus third, and second versus third function loadings. The combined option switches from a matrix graph to a combined graph. The msymbol(i) option removes the plotting points, leaving the discriminating variable names in the graph, and the option mlabpos(0) places the discriminating variable names in the positions of the plotted points.



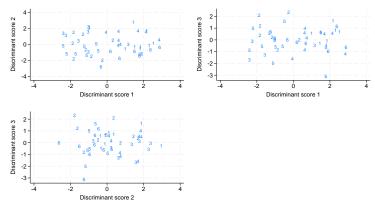
. loadingplot, components(3) combined msymbol(i) mlabpos(0)

Variable y1, trunk girth at 4 years, is near the origin in all three graphs, indicating that it does not play a strong role in discriminating among our six rootstock groups. y4, weight of tree above ground at 15 years, does not play much of a role in the first discriminant function but does in the second and third discriminant functions.

The corresponding three score plots are easily produced with the scoreplot command; see [MV] scoreplot. Score plots graph the discriminant function-transformed observations (called scores).

. scoreplot, components(3) combined msymbol(i)

#### Discriminant function scores



There is a lot of overlap, but some separation of the rootstock groups is apparent. One of the observations from group 6 seems to be sitting by itself in the bottom of the two graphs that have discriminant function 3 as the y axis. In example 11, we identify this point by using the predict command.

## Means and distances

The estat grsummarize command is available after all discrim commands and will display means, medians, minimums, maximums, standard deviations, group sizes, and more for the groups; see [MV] discrim estat. After discrim lda, the estat grmeans command will also display group means. It, however, has options for displaying the within-group standardized group means, the total-sample standardized group means, and the canonical discriminant functions evaluated at the group means.

## Example 6: Standardized group means and canonical discriminant functions at the means

We introduce the estat grmeans command with the iris data originally from Anderson (1935), introduced in example 3 of [MV] discrim Ida.

```
. use https://www.stata-press.com/data/r19/iris
(Iris data)
. discrim lda seplen sepwid petlen petwid, group(iris) notable
```

The notable option of discrim suppressed the classification table.

By default, estat grmeans displays a table of the means of the discriminating variables for each group. You could obtain the same information along with other statistics with the estat grsummarize command; see [MV] discrim estat.

```
. estat grmeans
Group means
                    iris
                       Setosa Versico~r Virginica
          seplen
                        5.006
                                    5.936
                                                6.588
          sepwid
                        3.428
                                     2.77
                                                2.974
          petlen
                        1.462
                                     4.26
                                                5.552
                         .246
                                    1.326
                                                2.026
          petwid
```

Differences in the iris species can be seen within these means. For instance, the petal lengths and widths of the *Iris setosa* are smaller than those of the other two species. See example 1 of [MV] **discrim** estat for further exploration of these differences.

The main purpose of estat grmeans is to present standardized or transformed means. The totalstd and withinstd options request the two available standardizations.

```
. estat grmeans, totalstd withinstd
Total-sample standardized group means
                   iris
                      Setosa Versico~r Virginica
          seplen
                   -1.011191
                                .1119073
                                           .8992841
          sepwid
                    .8504137
                              -.6592236
                                          -.1911901
                    -1.30063
                                .2843712
                                           1.016259
          petlen
                   -1.250704
                                .1661774
                                           1.084526
          petwid
```

Pooled within-group standardized group means

	iris Setosa	Versico~r	Virginica
seplen	-1.626555	.1800089	1.446546
sepwid	1.091198	8458749	2453234
petlen	-5.335385	1.166534	4.16885
petwid	-4.658359	.6189428	4.039416

The first table presents the total-sample standardized group means on the discriminating variables. These are the means for each group on the total-sample standardized discriminating variables.

The second table presents the pooled within-group standardized means on the discriminating variables. Instead of using the total-sample variance, the pooled within-group variance is used to standardize the variables. Of most interest in the context of an LDA is the within-group standardization.

The canonical option of estat grmeans displays the discriminant functions evaluated at the group means and gives insight into what the functions do to the groups.

. estat grmeans, canonical						
Group means on canonical variables						
iris   function1 function2						
Setosa Versicolor Virginica	-7.6076 1.825049 5.78255	215133 .7278996 5127666				

The first function places *Iris setosa* strongly negative and *Iris virginica* strongly positive with *Iris versicolor* in between. The second function places *Iris virginica* and *Iris setosa* negative and *Iris versicolor* positive.

The Mahalanobis distance between the groups in an LDA helps in assessing which groups are similar and which are different.

## Example 7: Mahalanobis distance between groups

Continuing with the iris example, we use the estat grdistances command to view the squared Mahalanobis distances between the three iris species.

. estat grdistances							
Mahalanobis squared distances between groups							
	iris						
iris	Setosa	Versicolor	Virginica				
Setosa	0						
Versicolor	89.864186	0					
Virginica	179.38471	17.201066	0				

*Iris setosa* is farthest from *Iris virginica* with a squared Mahalanobis distance of 179. *Iris versicolor* and *Iris virginica* are closest with a squared Mahalanobis distance of 17.

Are these distances significant? Requesting F statistics and p-values associated with these Mahalanobis squared distances between means will help answer that question. The mahalanobis() option requests F tests, p-values, or both.

4

. estat grdistances, mahalanobis(f p) Mahalanobis squared distances between groups

Кеу			
Mahalanobis F with 4 and p-value	ance		
	iris		
iris	Setosa	Versicolor	Virginica
Setosa	0		
	0		
	1		
Versicolor	89.864186	0	
	550.18889	0	
	3.902e-86	1	
Virginica	179.38471 1098.2738		0 0
	9.20e-107	9.515e-42	1

All three of the means are statistically significantly different from one another.

The generalized squared distance between groups starts with the Mahalanobis squared distance between groups and adjusts for prior probabilities when they are not equal. With equal prior probabilities there will be no difference between the generalized squared distance and Mahalanobis squared distance. The priors() option specifies the prior probabilities for calculating the generalized squared distances.

To illustrate, we select prior probabilities of 0.2 for *I. setosa*, 0.3 for *I. versicolor*, and 0.5 for *I. virginica*.

. estat grdistances, generalized priors(.2, .3, .5) Generalized squared distances between groups

iris	iris Setosa	Versicolor	Virginica
Setosa	3.2188758	92.272131	180.77101
Versicolor	93.083061	2.4079456	18.587361
Virginica	182.60359	19.609012	1.3862944

This matrix is not symmetric and does not have zeros on the diagonal.

4

# **Covariance and correlation matrices**

Equal group covariance matrices is an important assumption underlying LDA. The estat covariance command displays the group covariance matrices, the pooled within-group covariance matrix, the between-groups covariance matrix, and the total-sample covariance matrix. The estat correlation command provides the corresponding correlation matrices, with an option to present *p*-values with the correlations.

## Example 8: Group covariances and correlations

Continuing our examination of LDA on the iris data, we request to see the pooled within-group covariance matrix and the covariance matrices for the three iris species.

. estat covariance, within groups							
Pooled within-grou	up covariance	e matrix					
	seplen	sepwid	petlen	petwid			
seplen sepwid petlen petwid	.2650082 .0927211 .1675143 .0384014	.1153878 .0552435 .0327102	.1851878 .0426653	.0418816			
Group covariance n	natrices						
iris: Setosa							
	seplen	sepwid	petlen	petwid			
seplen sepwid petlen petwid	.124249 .0992163 .0163551 .0103306	.1436898 .011698 .009298	.0301592 .0060694	.0111061			
iris: Versicolo	r						
	seplen	sepwid	petlen	petwid			
seplen sepwid petlen petwid	.2664327 .0851837 .182898 .0557796	.0984694 .0826531 .0412041	.2208163	.0391061			
iris: Virginica							
	seplen	sepwid	petlen	petwid			
seplen sepwid petlen petwid	.4043429 .0937633 .3032898 .0490939	.1040041 .0713796 .0476286	.3045878 .0488245	.0754327			

All variables have positive covariance-not surprising for physical measurements (length and width).

We could have requested the between-groups covariance matrix and the total-sample covariance matrix. Options of estat covariance control how the covariance matrices are displayed.

Correlation matrices are also easily displayed. With estat correlations we show the pooled within-group correlation matrix, and add the p option to request display of *p*-values with the correlations. The *p*-values help us evaluate whether the correlations are statistically significant.

. estat corr, p Pooled within-group correlation matrix

Кеу				
Correlation Two-sided p-	-value			
	seplen	sepwid	petlen	petwid
seplen	1.00000			
sepwid	0.53024 0.00000	1.00000		
petlen	0.75616 0.00000	0.37792	1.00000	
petwid	0.36451 0.00001	0.47053 0.00000	0.48446 0.00000	1.00000

All correlations are statistically significant. The largest correlation is between the petal length and the sepal length.

4

## Predictions

The predict command after discrim lda has options for obtaining classifications, probabilities, Mahalanobis squared distances from observations to group means, and the leave-one-out (LOO) estimates of all of these. You can also obtain the discriminant scores and classification scores for observations. The predictions can be obtained in or out of sample.

## Example 9: Out-of-sample LDA classification and probabilities

We use the riding-mower data from Johnson and Wichern (2007) introduced in example 1 of [MV] **discrim** to illustrate out-of-sample prediction of classification and probabilities after an LDA.

```
. use https://www.stata-press.com/data/r19/lawnmower2
(Johnson and Wichern (2007) table 11.1)
. discrim lda lotsize income, group(owner) notable
```

Now we see how the LDA model classifies observations with income of \$90,000, \$110,000, and \$130,000, each with a lot size of 20,000 square feet. We add 3 observations to the bottom of our dataset containing these values and then use predict to obtain the classifications and probabilities.

.9818

```
. input
        owner
               income lots~e
25. .
       90 20
26. . 110 20
27. . 130 20
28. end
. predict grp in 25/L, class
(24 missing values generated)
. predict pr* in 25/L, pr
(24 missing values generated)
. list in 25/L
       owner
                         lotsize
                                               pr1
                                                           pr2
               income
                                   grp
25.
                 90.0
                            20.0
                                      0
                                          .5053121
                                                      .4946879
26.
                110.0
                            20.0
                                      1
                                          .1209615
                                                      .8790385
```

20.0

The observation with income of \$90,000 was classified as a nonowner, but it was a close decision with probabilities of 0.505 for nonowner and 0.495 for owner. The two other observations, with \$110,000 and \$130,000 income, were classified as owners, with higher probability of ownership for the higher income.

.0182001

1

4

The estat list, estat classtable, and estat errorrate commands (see [MV] discrim estat) obtain their information by calling predict. The LOO listings and tables from these commands are obtained by calling predict with the looclass and loopr options.

In addition to predictions and probabilities, we can obtain the classification scores for observations.

#### Example 10: Classification scores

27.

In example 4, we used the estat classfunctions command to view the classification functions for the LDA of the apple tree rootstock data. We can use predict to obtain the corresponding classification scores—the classification function applied to observations.

```
. use https://www.stata-press.com/data/r19/rootstock, clear
(Table 6.2. Rootstock data, Rencher and Christensen (2012))
```

```
. discrim lda y1 y2 y3 y4, group(rootstock) priors(.2,.2,.2,.2,.1,.1) notable
```

- . predict clscr\*, clscore
- . format clscr\* %6.1f
- . list rootstock clscr\* in 1/3, noobs

130.0

rootst~k	clscr1	clscr2	clscr3	clscr4	clscr5	clscr6
1		303.7 324.1				307.1 326.0
1				326.1 309.3		326.0

-3.11

-1.93

We did not specify the priors() option, so predict used the prior probabilities that were specified with our LDA model in determining the constant term in the classification function; see example 4 for a table of the classification functions. Observations may be classified to the group with largest score. The first 3 observations belong to rootstock group 1 and are successfully classified as belonging to group 1 because the classification score in clscr1 is larger than the classification scores for the other groups.

Scoring the discriminating variables by using Fisher's canonical discriminant functions is accomplished with the dscore option of predict.

## Example 11: Scoring the discriminant variables

Using the rootstock data in example 5, we noticed 1 observation, from group 6, near the bottom of the score plot where the third discriminant function was the y axis. The observation has a score for the third discriminant function that appears to be below -3. We will use the dscore option of predict to find the observation.

0.606

1.59

-1.44

. pre	dict ds*, ds	core							
. for	mat ds* %5.0g	B							
. lis	t rootstock	y* ds* i	f ds3 <	-3					
	rootst~k	y1	y2	уЗ	y4	ds1	ds2	ds3	ds4

3.14

Observation 42 is the one producing that third discriminant score.

0.840

# Stored results

Scalars

42.

estat anova stores the following in r():

6

0.75

r(N) r(df_m) r(df_r) Matrices	number of observations model degrees of freedom residual degrees of freedom
r(anova_stats)	ANOVA statistics for the model
estat canontest store	es the following in r():
Scalars r(N) r(N_groups) r(k) r(f) Matrices r(stat)	number of observations number of groups number of variables number of canonical discriminant functions canonical discriminant statistics

estat classfunction stores the following in r():

# Matrices r(C) classification function matrix r(priors) group prior probabilities

 $\triangleleft$ 

## estat correlations stores the following in r():

Matrices

r(Rho)	pooled within-group correlation matrix (within only)
r(P)	two-sided <i>p</i> -values for pooled within-group correlations (within and p only)
r(Rho_between)	between-groups correlation matrix (between only)
r(P_between)	two-sided $p$ -values for between-groups correlations (between and p only)
r(Rho_total)	total-sample correlation matrix (total only)
r(P_total)	two-sided <i>p</i> -values for total-sample correlations (total and p only)
r(Rho_#)	group # correlation matrix (groups only)
r(P_#)	two-sided $p$ -values for group $\#$ correlations (groups and p only)

estat covariance stores the following in r():

#### Matrices

pooled within-group covariance matrix (within only)
between-groups covariance matrix (between only)
total-sample covariance matrix (total only)
group # covariance matrix (groups only)

## estat grdistances stores the following in r():

Scalars	
r(df1)	numerator degrees of freedom (mahalanobis only)
r(df2)	denominator degrees of freedom (mahalanobis only)
Matrices	
r(sqdist)	Mahalanobis squared distances between group means (mahalanobis only)
r(F_sqdist)	F statistics for tests that the Mahalanobis squared distances between group means are zero (mahalanobis only)
r(P_sqdist)	<i>p</i> -value for tests that the Mahalanobis squared distances between group means are zero (mahalanobis only)
r(gsqdist)	generalized squared distances between group means (generalized only)

## estat grmeans stores the following in r():

#### Matrices

r(means)	group means (raw only)
r(stdmeans)	total-sample standardized group means (totalstd only)
r(wstdmeans)	pooled within-group standardized group means (withinstd only)
r(cmeans)	group means on canonical variables (canonical only)

## estat loadings stores the following in r():

#### Matrices

r(L_std)	within-group standardized canonical discriminant function coefficients
	(standardized only)
$r(L_totalstd)$	total-sample standardized canonical discriminant function coefficients
	(totalstandardized only)
r(L_unstd)	unstandardized canonical discriminant function coefficients
	(unstandardized only)

## estat manova stores the following in r():

#### Scalars

r(N) r(df_m)	number of observations model degrees of freedom
r(df_r)	residual degrees of freedom
Matrices	
r(stat_m)	multivariate statistics for the model

estat structure stores the following in r():

Matrices

r(canstruct) canonical structure matrix

# Methods and formulas

See Methods and formulas of [MV] discrim Ida for background on what is produced by predict, estat classfunctions, estat grdistances, estat grmeans, estat loadings, and estat structure. See [MV] discrim estat for more information on estat classtable, estat errorrate, estat grsummarize, and estat list. See [R] anova for background information on the ANOVAs summarized by estat anova; see [MV] manova for information on the MANOVA shown by estat manova; and see [MV] canon for background information on canonical correlations and related tests shown by estat canontest.

# References

Anderson, E. 1935. The irises of the Gaspé Peninsula. Bulletin of the American Iris Society 59: 2-5.

- Andrews, D. F., and A. M. Herzberg, eds. 1985. Data: A Collection of Problems from Many Fields for the Student and Research Worker. New York: Springer. https://doi.org/10.1007/978-1-4612-5098-2.
- Huberty, C. J. 1994. Applied Discriminant Analysis. New York: Wiley.
- Johnson, R. A., and D. W. Wichern. 2007. Applied Multivariate Statistical Analysis. 6th ed. Englewood Cliffs, NJ: Prentice Hall.
- Rencher, A. C., and W. F. Christensen. 2012. Methods of Multivariate Analysis. 3rd ed. Hoboken, NJ: Wiley. https://doi.org/10.1002/9781118391686.

# Also see

- [MV] discrim Ida Linear discriminant analysis
- [MV] candisc Canonical linear discriminant analysis
- [MV] canon Canonical correlations
- [MV] discrim Discriminant analysis
- [MV] discrim estat Postestimation tools for discrim
- [MV] manova Multivariate analysis of variance and covariance
- [MV] scoreplot Score and loading plots
- [MV] screeplot Scree plot of eigenvalues
- [U] 20 Estimation and postestimation commands

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