**mi estimate — Estimation using multiple imputations**

**Description**

`mi estimate`: `estimation_command` runs `estimation_command` on the imputed `mi` data, and adjusts coefficients and standard errors for the variability between imputations according to the combination rules by Rubin (1987).

**Menu**

Statistics > Multiple imputation

**Syntax**

Compute MI estimates of coefficients by fitting `estimation_command` to `mi` data

```
mi estimate [, options] : estimation_command ...
```

Compute MI estimates of transformed coefficients by fitting `estimation_command` to `mi` data

```
mi estimate [spec] [, options] : estimation_command ...
```

where `spec` may be one or more terms of the form `[name:] exp`. `exp` is any function of the parameter estimates allowed by `nlcom`; see [R] `nlcom`.
## mi estimate — Estimation using multiple imputations

### Options

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<tr>
<td>imputations to use</td>
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<td>variables and base and empty cells, and factor-variable labeling</td>
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### Reporting

| set confidence level; default is `level(95)`                              | `level(#)`                                  |
| display dots as estimations are performed                                 | `dots`                                      |
| display any output from `estimation_command` (and from `nlcom` if        | `noisily`                                   |
| transformations specified)                                                |                                              |
| trace `estimation_command` (and `nlcom` if transformations specified)     | `trace`                                     |
| suppress summary about groups displayed for `xt` commands                 | `nogroup`                                   |
| control output from mixed-effects commands                                | `me_options`                                |

### Advanced

| store estimation sample in variable `newvar`; available only in the       | `esample(newvar)`                           |
| flong and flongsep styles                                                 |                                              |
| allow estimation even when `estimation_command` (or `nlcom`) errors out   | `errorok`                                   |
| such imputations are discarded from the analysis                          |                                              |
| allow estimation when estimation sample varies across imputations         | `esampvaryok`                               |
| allow estimation when `estimation_command` is not one of the supported    | `cmdok`                                     |
| estimation commands                                                      |                                              |
| display legend instead of statistics                                       | `coeflegend`                               |
| suppress the warning about varying estimation samples                     | `nowarning`                                 |
| display coefficient table in exponentiated form                           | `eform_option`                              |
| post estimated coefficients and VCE to `e(b)` and `e(V)`                  | `post`                                      |
| do not perform `mi update`; see `[MI] noupdate option`                    | `noupdate`                                  |

You must `mi set` your data before using `mi estimate`; see `[MI] mi set.`

`coeflegend`, `nowarning`, `eform_option`, `post`, and `noupdate` do not appear in the dialog box.
Table options

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<td>suppress table header(s)</td>
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<tr>
<td>notable</td>
<td>suppress table(s)</td>
</tr>
<tr>
<td>nocoef</td>
<td>suppress table output related to coefficients</td>
</tr>
<tr>
<td>nocmdlegend</td>
<td>suppress command legend that appears in the presence of transformed</td>
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<td></td>
<td>coefficients when nocoef is used</td>
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<tr>
<td>notrcoef</td>
<td>suppress table output related to transformed coefficients</td>
</tr>
<tr>
<td>nolegend</td>
<td>suppress table legend(s)</td>
</tr>
<tr>
<td>nocnsr</td>
<td>do not display constraints</td>
</tr>
</tbody>
</table>

See [MI] `mi estimate postestimation` for features available after estimation. `mi estimate` is its own estimation command. The postestimation features for `mi estimate` do not include by default the postestimation features for `estimation_command`. To replay results, type `mi estimate` without arguments.

Options

The options are as follows:

- `nimputations(#)` specifies that the first # imputations be used; # must be \( M_{\text{min}} \leq # \leq M \), where \( M_{\text{min}} = 3 \) if `merror` is specified and \( M_{\text{min}} = 2 \), otherwise. The default is to use all imputations, \( M \). Only one of `nimputations()` or `imputations()` may be specified.

- `imputations(numlist)` specifies which imputations to use. The default is to use all of them. `numlist` must contain at least two numbers. If `merror` is specified, `numlist` must contain at least three numbers. Only one of `nimputations()` or `imputations()` may be specified.

- `merror` specifies to compute Monte Carlo error (MCE) estimates for the results displayed in the estimation, degrees-of-freedom, and variance-information tables. MCE estimates reflect variability of MI results across repeated uses of the same imputation procedure and are useful for determining an adequate number of imputations to obtain stable MI results; see White, Royston, and Wood (2011) for details and guidelines.

  MCE estimates are obtained by applying the jackknife procedure to multiple-imputation results. That is, the jackknife pseudovalues of MI results are obtained by omitting one imputation at a time; see [R] `jackknife` for details about the jackknife procedure. As such, the MCE computation requires at least three imputations.

  If `level()` is specified during estimation, MCE estimates are obtained for confidence intervals using the specified confidence level instead of using the default 95% confidence level. If any of the options described in [R] `eform_option` is specified during estimation, MCE estimates for the coefficients, standard errors, and confidence intervals in the exponentiated form are also computed. `merror` can also be used upon replay to display MCE estimates. Otherwise, MCE estimates are not reported upon replay even if they were previously computed.

- `ufmitest` specifies that the unrestricted fraction missing information (FMI) model test be used. The default test performed assumes equal fractions of information missing due to nonresponse for all coefficients. This is equivalent to the assumption that the between-imputation and within-imputation variances are proportional. The unrestricted test may be preferable when this assumption is suspect provided the number of imputations is large relative to the number of estimated coefficients.
nosmall specifies that no small-sample correction be made to the degrees of freedom. The small-sample correction is made by default to estimation commands that account for small samples. If the command stores residual degrees of freedom in \( e(\text{df}_r) \), individual tests of coefficients (and transformed coefficients) use the small-sample correction of Barnard and Rubin (1999) and the overall model test uses the small-sample correction of Reiter (2007). If the command does not store residual degrees of freedom, the large-sample test is used and the nosmall option has no effect.

\[
\text{saving}(\text{miestfile}, \text{replace})
\]
saves estimation results from each model fit in \text{miestfile}.\text{ster}. The replace suboption specifies to overwrite \text{miestfile}.\text{ster} if it exists. \text{miestfile}.\text{ster} can later be used by \text{mi estimate using} (see \text{MI mi estimate using}) to obtain MI estimates of coefficients or of transformed coefficients without refitting the completed-data models. This file is written in the format used by estimates use; see \text{[R] estimates save}.

---

**Tables**

All table options below may be specified at estimation time or when redisplaying previously estimated results. Table options must be specified as options to \text{mi estimate}, not to \text{estimation_command}.

\text{citable} and \text{nocitable} specify whether the standard estimation table containing parameter-specific confidence intervals is displayed. The default is \text{citable}. \text{nocitable} can be used with \text{vartable} to suppress the confidence interval table.

\text{dftable} displays a table containing parameter-specific degrees of freedom and percentages of increase in standard errors due to nonresponse. \text{dftable} implies \text{nocitable}.

\text{vartable} displays a table reporting variance information about MI estimates. The table contains estimates of within-imputation variances, between-imputation variances, total variances, relative increases in variance due to nonresponse, fractions of information about parameter estimates missing due to nonresponse, and relative efficiencies for using finite \( M \) rather than a hypothetically infinite number of imputations. \text{vartable} implies \text{citable}.

\text{table_options} control the appearance of all displayed table output:

- \text{noheader} suppresses all header information from the output. The table output is still displayed.
- \text{notable} suppresses all tables from the output. The header information is still displayed.
- \text{nocoeff} suppresses the display of tables containing coefficient estimates. This option affects the table output produced by \text{citable}, \text{dftable}, and \text{vartable}.
- \text{nocmdlegend} suppresses the table legend showing the specified command line, \text{estimation_command}, from the output. This legend appears above the tables containing transformed coefficients (or above the variance-information table if \text{vartable} is used) when \text{nocoeff} is specified.
- \text{notrcoef} suppresses the display of tables containing estimates of transformed coefficients (if specified). This option affects the table output produced by \text{citable}, \text{dftable}, and \text{vartable}.
- \text{nolegend} suppresses all table legends from the output.

\text{nocnsreport}; see \text{[R] Estimation options}.

\text{display_options}: \text{noci}, \text{nopvalues}, \text{noomitted}, \text{vsquish}, \text{noemptycells}, \text{baselevels}, \text{allbaselevels}, \text{nofvlabel}, \text{fwrap(#)}, \text{fwrapon(style)}, \text{cformat(\%fmt)}, \text{pformat(\%fmt)}, and \text{sformat(\%fmt)}; see \text{[R] Estimation options}. 


Reporting options must be specified as options to `mi estimate` and not as options to `estimation_command`.

`level(#)`; see [R] Estimation options.

dots specifies that dots be displayed as estimations are successfully completed. An x is displayed if the `estimation_command` returns an error, if the model fails to converge, or if `nlcom` fails to estimate one of the transformed coefficients specified in `spec`.

noisily specifies that any output from `estimation_command` and `nlcom`, used to obtain the estimates of transformed coefficients if transformations are specified, be displayed.

trace traces the execution of `estimation_command` and traces `nlcom` if transformations are specified.

trace implies noisily.

nogroup suppresses the display of group summary information (number of groups, average group size, minimum, and maximum) as well as other command-specific information displayed for xt commands; see the list of commands under Panel-data models in [MI] Estimation.

`me_options`: stddeviations, variance, noretable, nofetable, and estmetric. These options are relevant only with the mixed-effects command `mixed` (see [ME] mixed). See the corresponding mixed-effects commands for more information. The stddeviations option is the default with `mi estimate`. The estmetric option is implied when vartable or dftable is used.

esample( newvar ) creates newvar containing e(sample). This option is useful to identify which observations were used in the estimation, especially when the estimation sample varies across imputations (see Potential problems that can arise when using mi estimate for details). newvar is zero in the original data ($m = 0$) and in any imputations ($m > 0$) in which the estimation failed or that were not used in the computation. esample() may be specified only if the data are flong or flongsep; see [MI] mi convert to convert to one of those styles. The variable created will be super varying and therefore must not be registered; see [MI] mi varying for more explanation.

The saved estimation sample newvar may be used later with `mi extract` (see [MI] mi extract) to set the estimation sample.

errorok specifies that estimations that fail be skipped and the combined results be based on the successful individual estimation results. The default is that `mi estimate` stops if an individual estimation fails. If errorok is specified with saving(), all estimation results, including failed, are saved to a file.

esampvaryok allows estimation to continue even if the estimation sample varies across imputations. `mi estimate` stops if the estimation sample varies. If esampvaryok is specified, results from all imputations are used to compute MI estimates and a warning message is displayed at the bottom of the table. Also see the esample() option. See Potential problems that can arise when using mi estimate for more information.

cmdok allows unsupported estimation commands to be used with `mi estimate`; see [MI] Estimation for a list of supported estimation commands. Alternatively, if you want `mi estimate` to work with your estimation command, add the property `mi` to the program properties; see [P] program properties.

The following options are available with `mi estimate` but are not shown in the dialog box:

coeflegend; see [R] Estimation options. coeflegend implies nocitable and cannot be combined with citable or dftable.
novarning suppresses the warning message at the bottom of table output that occurs if the estimation sample varies and esampvaryok is specified. See Potential problems that can arise when using mi estimate for details.

eform_option; see [R] eform_option. Regardless of the estimation_command specified, mi estimate reports results in the coefficient metric under which the combination rules are applied. You may use the appropriate eform_option to redisplay results in exponentiated form, if desired. If dfetable is also specified, the reported degrees of freedom and percentage increases in standard errors are not adjusted and correspond to the original coefficient metric.

post requests that MI estimates of coefficients and their respective VCEs be posted in the usual way. This allows the use of estimation_command-specific postestimation tools with MI estimates. There are issues; see Using the command-specific postestimation tools in [MI] mi estimate postestimation. post may be specified at estimation time or when redisplaying previously estimated results.

noupdate in some cases suppresses the automatic mi update this command might perform; see [MI] noupdate option. This option is seldom used.

Remarks and examples

mi estimate requires that imputations be already formed; see [MI] mi impute. To import existing multiply imputed data, see [MI] mi import.

Remarks are presented under the following headings:

Using mi estimate
Example 1: Completed-data logistic analysis
Example 2: Completed-data linear regression analysis
Example 3: Completed-data survival analysis
Example 4: Panel data and multilevel models
Example 5: Estimating transformations
Example 6: Monte Carlo error estimates
Potential problems that can arise when using mi estimate

Using mi estimate

mi estimate estimates model parameters from multiply imputed data and adjusts coefficients and standard errors for the variability between imputations. It runs the specified estimation_command on each of the M imputed datasets to obtain the M completed-data estimates of coefficients and their VCEs. It then computes MI estimates of coefficients and standard errors by applying combination rules (Rubin 1987, 77) to the M completed-data estimates. See [MI] Intro substantive for a discussion of MI analysis and see Methods and formulas for computational details.

To use mi estimate, your data must contain at least two imputations. The basic syntax of mi estimate is

    . mi estimate: estimation_command ...

estimation_command is any estimation command from the list of supported estimation commands; see [MI] Estimation.

If you wish to estimate on survey data, type

    . mi estimate: svy: estimation_command ...

If you want to vary the number of imputations or select which imputations to use in the computations, use the nimputations() or the imputations() option, respectively.

    . mi estimate, nimputations(9): estimation_command ...
Doing so is useful to evaluate the stability of MI results. MCE estimates of the parameters are also useful for determining the stability of MI results. You can use the merror option to obtain these estimates. Your data must contain at least three imputations to use merror.

You can obtain more-detailed information about imputation results by specifying the dftable and vartable options.

You can additionally obtain estimates of transformed coefficients by specifying expressions with mi estimate; see Example 5: Estimating transformations for details.

When using mi estimate, keep in mind that

1. mi estimate is its own estimation command.
2. mi estimate uses different degrees of freedom for each estimated parameter when computing its significance level and confidence interval.
3. mi estimate reports results in the coefficient metric under which combination rules are applied regardless of the default reporting metric of the specified estimation_command. Use eform_option with mi estimate to report results in the exponentiated metric, if you wish. For example, mi estimate: logistic reports coefficients and not odds ratios as logistic. To obtain odds ratios, you must specify the or option with mi estimate:

   . mi estimate, or: logistic ...

4. mi estimate has its own reporting options and does not respect command-specific reporting options. The reporting options specified with estimation_command affect only the output of the command that is displayed when mi estimate’s noisily option is specified. Specify mi estimate’s options immediately after the mi estimate command:

   . mi estimate, options: estimation_command ...

Example 1: Completed-data logistic analysis

Recall the logistic analysis of the heart attack data from [MI] Intro substantive. The goal of the analysis was to explore the relationship between heart attacks and smoking adjusted for other factors such as age, body mass index (BMI), gender, and educational status. The original data contain missing values of BMI. The listwise-deletion analysis on the original data determined that smoking and BMI have significant impact on a heart attack. After imputing missing values of BMI, age was determined to be a significant factor as well. See A brief introduction to MI using Stata in [MI] Intro substantive for details. The data we used are stored in mheart1s20.dta.

Below we refit the logistic model using the imputed data. We also specify the dots option so that dots will be displayed as estimations are completed.
. use https://www.stata-press.com/data/r16/mheart1s20
(Fictional heart attack data; bmi missing)
. mi estimate, dots: logit attack smokes age bmi hsgrad female
Imputations (20):
          ..........10 ..........20 done

Multiple-imputation estimates Imputations = 20
Logistic regression
          Number of obs = 154
          Average RVI = 0.0312
          Largest FMI = 0.1355

DF adjustment: Large sample
          DF:        min  =  1,060.38
                      avg = 223,362.56
                      max = 493,335.88
Model F test:  Equal FMI
          F(  5,71379.3) =  3.59
Within VCE type: OIM
          Prob > F =  0.0030

|       | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|-------|--------|-----------|-------|------|----------------------|
| smoke | 1.198595 | .3578195 | 3.35  | 0.001 | .4972789 1.899911   |
| age   | .0360159 | .0154399 | 2.33  | 0.020 | .0057541 .0662776  |
| bmi   | .1039416 | .0476136 | 2.18  | 0.029 | .010514 .1973692   |
| hsgrad| .1578992 | .4049257 | 0.39  | 0.697 | -.6357464 .9515449 |
| female| -.1067433| .4164735 | -0.26 | 0.798 | -.9230191 .7095326 |
| _cons | -5.478143| 1.685075 | -3.25 | 0.001 | -8.782394 -2.173892|

The left header column reports information about the fitted MI model. The right header column reports the number of imputations and the number of observations used, the average relative variance increase (RVI) due to nonresponse, the largest fraction of missing information (FMI), a summary about parameter-specific degrees of freedom (DF), and the overall model test that all coefficients, excluding the constant, are equal to zero.

Notice first that *mi estimate* reports Student’s *t* and *F* statistics for inference although *logit* would usually report *Z* and *χ²* statistics.

*mi estimate*: *logit* is not *logit*. *mi estimate* uses Rubin’s combination rules to obtain the estimates from multiply imputed data. The variability of the MI estimates consists of two components: variability within imputations and variability between imputations. Therefore, the precision of the MI estimates is governed not only by the number of observations in the sample but also by the number of imputations. As such, even if the number of observations is large, if the number of imputations is small and the FMI are not low, the reference distribution used for inference will deviate from the normal distribution. Because in practice the number of imputations tends to be small, *mi estimate* uses a reference *t* distribution.

Returning to the output, average RVI reports the average relative increase (averaged over all coefficients) in variance of the estimates because of the missing bmi values. A relative variance increase is an increase in the variance of the estimate because of the loss of information about the parameter due to nonresponse relative to the variance of the estimate with no information lost. The closer this number is to zero, the less effect missing data have on the variance of the estimate. Note that the reported RVI will be zero if you use *mi estimate* with the complete data or with missing data that have not been imputed. In our case, average RVI is small: 0.0312.

Largest FMI reports the largest of all the FMI about coefficient estimates due to nonresponse. This number can be used to get an idea of whether the specified number of imputations is sufficient for the analysis. A rule of thumb is that $M \geq 100 \times \text{FMI}$ provides an adequate level of reproducibility of MI analysis. In our example, the largest FMI is 0.14 and the number of imputations, 20, exceeds the required number of imputations: 14 ($= 100 \times 0.14$) according to this rule.
The coefficient-specific degrees of freedom (DF) averaging 223,363 are large. They are large because the MI degrees of freedom depends not only on the number of imputations but also on the RVI due to nonresponse. Specifically, the degrees of freedom is inversely related to RVI. The closer RVI is to zero, the larger the degrees of freedom regardless of the number of imputations.

To the left of the DF, we see that the degrees of freedom is obtained under a large-sample assumption. The alternative is to use a small-sample adjustment. Whether the small-sample adjustment is applied is determined by the type of the reference distribution used for inference by the specified estimation command. For the commands that use a large-sample (normal) approximation for inference, a large-sample approximation is used when computing the MI degrees of freedom. For the commands that use a small-sample (Student’s t) approximation for inference, a small-sample approximation is used when computing the MI degrees of freedom. See Methods and formulas for details. As we already mentioned, logit assumes large samples for inference, and thus the MI degrees of freedom is computed assuming a large sample.

The model $F$ test rejects the hypothesis that all coefficients are equal to zero and thus rules out a constant-only model for heart attacks. By default, the model test uses the assumption that the fractions of missing information of all coefficients are equal (as noted by Equal FMI to the left). Although this assumption may not be supported by the data, it is used to circumvent the difficulties arising with the estimation of the between-imputation variance matrix based on a small number of imputations. See Methods and formulas and [MI] mi test for details.

mi estimate also reports the type of variance estimation used by the estimation command to compute variance estimates in the individual completed-data analysis. These completed-data variance estimates are then used to compute the within-imputation variance. In our example, the observed-information-matrix (OIM) method, the default variance-estimation method used by maximum likelihood estimation, is used to compute completed-data VCEs. This is labeled as Within VCE type: OIM in the output.

Finally, mi estimate reports a coefficient table containing the combined estimates. Unlike all other Stata estimation commands, the reported significance levels and confidence intervals in this table are based on degrees of freedom that is specific to each coefficient. Remember that the degrees of freedom depends on the relative variance increases and thus on how much information is lost about the estimated parameter because of missing data. How much information is lost is specific to each parameter and so is the degrees of freedom.

As we already saw, a summary of the coefficient-specific degrees of freedom (minimum, average, and maximum) was reported in the header. We can obtain a table containing coefficient-specific degrees of freedom by replaying the results with the dftable option:
Notice that we type `mi estimate` to replay the results, not `logit`.

The header information remains the same. In particular, degrees of freedom ranges from 1,060 to 493,336 and averages 223,363. In the table output, the columns for the confidence intervals are replaced with the DF and % Increase Std. Err. columns. We now see that the smallest degrees of freedom corresponds to the coefficient for `bmi`. We should have anticipated this because `bmi` is the only variable containing missing values in this example. The largest degrees of freedom is observed for the coefficient for `age`, which suggests that the loss of information due to nonresponse is the smallest for the estimation of this coefficient.

The last column displays as a percentage the increase in standard errors of the parameters due to nonresponse. We observe a 7% increase in the standard error for the coefficient of `bmi` and a 5% increase in the standard error for the constant. Increases in standard errors of other coefficients are negligible.

In this example, we displayed a degrees-of-freedom table on replay by specifying the `dftable` option. We could also obtain this table if we specified this option at estimation time. Alternatively, if desired, we could display both tables by specifying the `citable` and `dftable` options together.

We can obtain more detail about imputation results by specifying the `vartable` option. We specify this option on replay and also use the `nocitable` option to suppress the default confidence interval table:

```
10  mi estimate — Estimation using multiple imputations

. mi estimate, dftable
Multiple-imputation estimates Imputations  =  20
Logistic regression Number of obs  =  154
  Average RVI  =  .0312
  Largest FMI  =  .1355
DF adjustment: Large sample
  DF:  min  =  1,060.38
       avg =  223,362.56
       max =  493,335.88
Model F test: Equal FMI
  F(  5,71379.3)  =  3.59
Within VCE type: OIM
  Prob > F  =  .0030

                     | Coef.  | Std. Err. |    t  |  P>|t|  | DF  | Std. Err. | % Increase
  +-------------------+---------+------------+-------+-------+-----+------------+-------------
  attack             |         |            |       |       |     |            |             
  smokes             | 1.198595| .3578195   |  3.35 |  .001 | 320019.4 | 0.39
  age                | .0360159| .0154399   |  2.33 |  .020 | 493335.9 | 0.31
  bmi                | .1039416| .0476136   |  2.18 |  .029 | 1060.4  | 7.45
  hsgrad             | .1578992| .4049257   |  0.39 |  .697 | 165126.7 | 0.54
  female             | -.1067433| .4164735  | -0.26 |  .798 | 358078.3 | 0.37
  _cons              | -5.478143| 1.685075   | -3.25 |  .001 | 2554.8  | 4.61

Notice that we type `mi estimate` to replay the results, not `logit`.

The header information remains the same. In particular, degrees of freedom ranges from 1,060 to 493,336 and averages 223,363. In the table output, the columns for the confidence intervals are replaced with the DF and % Increase Std. Err. columns. We now see that the smallest degrees of freedom corresponds to the coefficient for `bmi`. We should have anticipated this because `bmi` is the only variable containing missing values in this example. The largest degrees of freedom is observed for the coefficient for `age`, which suggests that the loss of information due to nonresponse is the smallest for the estimation of this coefficient.

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We can obtain more detail about imputation results by specifying the `vartable` option. We specify this option on replay and also use the `nocitable` option to suppress the default confidence interval table:

```
10  mi estimate — Estimation using multiple imputations

. mi estimate, vartable nocitable
Multiple-imputation estimates Imputations  =  20
Logistic regression
Variance information

 Imputation variance
Within Between Total RVI FMI Relative
  efficiency

<table>
<thead>
<tr>
<th></th>
<th>Within</th>
<th>Between</th>
<th>Total</th>
<th>RVI</th>
<th>FMI</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>smokes</td>
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<td>.128035</td>
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<td>.999615</td>
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<td>.000238</td>
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<td>.00621</td>
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<td>.001203</td>
<td>.17345</td>
<td>.007338</td>
<td>.00729</td>
<td>.999636</td>
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<tr>
<td>_cons</td>
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<td>.233211</td>
<td>2.83948</td>
<td>.094377</td>
<td>.086953</td>
<td>.995671</td>
<td></td>
</tr>
</tbody>
</table>
```
The first three columns of the table provide the variance information specific to each parameter. As we already discussed, MI variance contains two sources of variation: within imputation and between imputation. The first two columns provide estimates for the within-imputation and between-imputation variances. The third column is a total variance that is the sum of the two variances plus an adjustment for using a finite number of imputations. The next two columns are individual RVIs and fractions of missing information (FMIs) due to nonresponse. The last column records relative efficiencies for using a finite number of imputations (20 in our example) versus the theoretically optimal infinite number of imputations.

We notice that the coefficient for age has the smallest within-imputation and between-imputation variances. The between-imputation variability is very small relative to the within-imputation variability, which is why age had such a large estimate of the degrees of freedom we observed earlier. Correspondingly, this coefficient has the smallest values for RVI and FMI. As expected, the coefficient for bmi has the highest RVI and FMI.

The reported relative efficiencies are high for all coefficient estimates, with the smallest relative efficiency, again, corresponding to bmi. These estimates, however, are only approximations and thus should not be used exclusively to determine the required number of imputations. See Royston, Carlin, and White (2009) and White, Royston, and Wood (2011) for other ways of determining a suitable number of imputations.

Example 2: Completed-data linear regression analysis

Recall the data on house resale prices from example 3 of [MI] mi impute mvn. We use the imputed data stored in mhouses1993s30.dta to examine the relationship of various predictors on price via linear regression:

```
. use https://www.stata-press.com/data/r16/mhouses1993s30
   (Albuquerque Home Prices Feb15-Apr30, 1993)
. mi estimate, ni(5): regress price tax sqft age nfeatures ne custom corner

Multiple-imputation estimates Imputations = 5
Linear regression
   Number of obs = 117
   Average RVI = 0.0685
   Largest FMI = 0.2075
   Complete DF = 109
DF adjustment: Small sample
   DF: min = 48.59
       avg = 85.22
       max = 104.79
Model F test: Equal FMI F( 7, 103.9) = 67.50
Within VCE type: OLS
       Prob > F = 0.0000

        price        Coef.     Std. Err.     t     P>|t|    [95% Conf. Interval]
-------------    --------    --------    ------    ------    ------------------------
tax          .6631356    .122443     5.42    0.000       .4195447    .9067265
sqft          .2185884    .0670182     3.26    0.002       .0856051    .3515718
age          -.0395402    1.613185    -0.02    0.981      -3.28205    3.202969
nfeatures     8.735622    13.42251     0.65    0.517      -18.01198   35.48323
ne           4.069381    36.94491     0.11    0.913      -69.4355    77.57426
custom       130.4925    42.93286     3.04    0.003        45.36257   215.6225
corner       -71.25406   40.06697    -1.78    0.078      -150.7152    8.207084
    _cons     130.2002    70.38012     1.85    0.068      -9.624642   270.025
```

By default, mi estimate uses all available imputations in the analysis. For the purpose of illustration, we use only the first 5 imputations out of the available 30 by specifying the nimputations(5) option, which we abbreviated as ni(5).
Compared with the output from the previous example, an additional result, Complete DF, is reported. Also notice that the adjustment for the degrees of freedom is now labeled as Small sample. Remember that mi estimate determines what adjustment to use based on the reference distribution used for inference by the specified estimation command.

regress uses a reference $t$ distribution with $117 - 8 = 109$ residual degrees of freedom. Thus a small-sample adjustment is used by mi estimate for the MI degrees of freedom.

Complete DF contains the degrees of freedom used for inference with complete data. It corresponds to the completed-data residual degrees of freedom stored by the command in $e(df_r)$. In most applications, the completed-data residual degrees of freedom will be the same, and so Complete DF will correspond to the complete degrees of freedom, the degrees of freedom that would have been used for inference if the data were complete. In the case when the completed-data residual degrees of freedom varies across imputations (as may happen when the estimation sample varies; see Potential problems that can arise when using mi estimate), Complete DF reports the smallest of them.

In our example, all completed-data residual degrees of freedom are equal, and Complete DF is equal to 109, the completed-data residual degrees of freedom obtained from regress. mi estimate uses the complete degrees of freedom to adjust the MI degrees of freedom for a small sample (Barnard and Rubin 1999).

Example 3: Completed-data survival analysis

Consider survival data on 48 participants in a cancer drug trial. The dataset contains information about participants’ ages, treatments received (drug or placebo), times to death measured in months, and a censoring indicator. The data are described in more detail in Cox regression with censored data of [ST] stcox. We consider a version of these data containing missing values for age. The imputed data are saved in mdrugtrs25.dta:

```stata
. use https://www.stata-press.com/data/r16/mdrugtrs25
(Patient Survival in Drug Trial)
. mi describe

Style: mlong
     last mi update 19apr2019 14:00:11, 8 days ago
   Obs.: complete 40
         incomplete  8 (M = 25 imputations)
         total   48
   Vars.: imputed:  1; age(8)
         passive:  0
         regular:  3; studytime died drug
         system:  3; _mi_m _mi_id _mi_miss
           (there are no unregistered variables)
```

The dataset contains 25 imputations for 8 missing values of age. Missing values were imputed following guidelines in White and Royston (2009).

We analyze these data using stcox with mi estimate. These data have not yet been stset, so we use mi stset (see [MI] mi XXXset) to set them and then perform the analysis using mi estimate: stcox:
. mi stset studytime, failure(died)
  
failure event:  died != 0 & died <
obs. time interval: (0, studytime]
exit on or before:  failure

48  total observations
   0  exclusions

48  observations remaining, representing
31  failures in single-record/single-failure data
744  total analysis time at risk and under observation
at risk from t = 0
earliest observed entry t = 0
last observed exit t = 39

. mi estimate, dots: stcox drug age

Imputations (25):

.........10.........20..... done

Multiple-imputation estimates
Cox regression: Breslow method for ties
  Number of obs = 48
  Average RVI = 0.1059
  Largest FMI = 0.1567
  DF adjustment: Large sample
    DF:  
      min = 998.63
      avg = 11,621.53
      max = 22,244.42
  Model F test: Equal FMI
    F( 2, 4448.6) = 13.43
  Within VCE type: OIM
    Prob > F = 0.0000

|    | Coef.  | Std. Err. |     t  |  P>|t|  |      [95% Conf. Interval]   |
|----|--------|-----------|--------|--------|----------------------------|
| _t |        |           |        |        |                            |
| drug | -2.204572 | .4589047 | -4.80  | 0.000  | -3.104057, -1.305086       |
| age  | .1242711  | .040261  | 3.09   | 0.002  | .0452652, .2032771         |

Notice that mi estimate displays the results in the coefficient metric and not in the hazard-ratio metric. By default, mi estimate reports results in the metric under which the combination rules were applied. To obtain the results as hazard ratios, we can use the hr option with mi estimate:

. mi estimate, hr

Multiple-imputation estimates
Cox regression: Breslow method for ties
  Number of obs = 48
  Average RVI = 0.1059
  Largest FMI = 0.1567
  DF adjustment: Large sample
    DF:  
      min = 998.63
      avg = 11,621.53
      max = 22,244.42
  Model F test: Equal FMI
    F( 2, 4448.6) = 13.43
  Within VCE type: OIM
    Prob > F = 0.0000

|    | Haz. Ratio | Std. Err. |     t  |  P>|t|  |      [95% Conf. Interval]   |
|----|------------|-----------|--------|--------|----------------------------|
| _t |            |           |        |        |                            |
| drug | .1102977  | .0506161 | -4.80  | 0.000  | .0448686, .2711491         |
| age  | 1.132323  | .0455885 | 3.09   | 0.002  | 1.046305, 1.225412         |

We obtain results similar to those from the corresponding example in [ST] stcox.

We specified the hr option above on replay. We can also specify it at estimation time:

. mi estimate, hr: stcox drug age
(output omitted)
Notice that the hr option must be specified with `mi estimate` to obtain hazard ratios. Specifying it with the command itself,

```
. mi estimate: stcox drug age, hr
(output omitted)
```

will not affect the output from `mi estimate` but only that of the command, `stcox`. You see `stcox`’s output only if you specify `mi estimate`’s noisily option.

See Cleves, Gould, and Marchenko (2016, sec. 9.6) for more information on Cox regression with multiply imputed data.

### Example 4: Panel data and multilevel models

We have data on the math scores of students in their third and fifth years of education. There are 887 students from 48 schools in inner London; see Mortimore et al. (1988) for more information on the study. We would like to fit a random-effects model to the fifth-year score, `math5`, on the third-year score, `math3`, using a random effect at the school level.

We created a version of the data that contains missing values for `math3` and then performed imputation following guidelines from the Stata FAQ “How can I account for clustering when creating imputations with `mi impute`?”; see https://www.stata.com/support/faqs/stat/impute_cluster.html. The resulting imputed data are saved in `mjsps5.dta`:

```
. use https://www.stata-press.com/data/r16/mjsps5, clear
    (LEA Junior School Project data (Mortimore et al., 1988) with missing values)
. mi describe
    Style:  mlong
    last mi update 19apr2019 14:00:11, 8 days ago
    Obs.:  complete 705  incomplete 182  (M = 5 imputations)
    total 887
    Vars.:  imputed: 1; math3(182)
             passive: 0
             regular: 2; school math5
             system: 3; _mi_m _mi_id _mi_miss
             (there are no unregistered variables)
```

There are five imputations for 182 missing values of the third-year score, `math3`. Variable `math3` is an imputed variable, whereas variable `math5` and variable `school`, recording school identifiers, are complete and are registered as regular.

Our random-effects model includes only a random intercept, the school effect, so we can use the `xtreg` command, or more specifically `mi estimate: xtreg`, for our primary analysis.

Without imputed data, to use `xtreg` or any other panel-data command, we must first declare data to be panel (`xt`) data by using `xtset`. With imputed data, we should use the `mi xtset` command instead. We declare `school` as our panel variable:

```
. mi xtset school
    panel variable:  school (unbalanced)
```
Next we use `mi estimate: xtreg` to regress the fifth-year math score on the third-year score.

```
. mi estimate: xtreg math5 math3
```

```
Multiple-imputation estimates Imputations = 5
Random-effects GLS regression Number of obs = 887
Group variable: school Number of groups = 48
Obs per group:
    min = 5
    avg = 18.5
    max = 62
DF adjustment: Large sample
Largest FMI = 0.1071
Average RVI = 0.0595
Model F test: Equal FMI F( 1, 381.4) = 305.71
Within VCE type: Conventional Prob > F = 0.0000

                    Coeff. Std. Err.      t    P>|t|     [95% Conf. Interval]
math3          .6101277   .0348951   17.48   0.000     .5415168    .6787385
_cons           30.48295   .3576417   85.23   0.000     29.78198    31.18392
sigma_u       2.0684286
sigma_e        5.3206673
rho            .13128791 (fraction of variance due to u_i)
```

Note: `sigma_u` and `sigma_e` are combined in the original metric.

Third-year math scores are positively associated with fifth-year math scores. Because we use a random-effects model, the coefficient on `math3` is for comparison of students from the same school or from different schools.

In the above results, multiple-imputation estimates of variance components `sigma_u` and `sigma_e` are obtained by applying Rubin's combination rules to the completed-data estimates in the original, standard deviation metric.

Alternatively, we can use the `mixed` command to fit our two-level random-effects model and to obtain variance-component estimates of the school effect. `mixed` can be used to fit more complicated multilevel models; see `[ME] mixed` for details.

We fit a two-level linear model with `mi estimate: mixed` and specify `school` as our second-level variable. `mixed` does not require prior declaration of the data, so we do not need to use `mi xtset` with `mi estimate: mixed`:
. mi estimate: mixed math5 math3 || school: , reml

Multiple-imputation estimates
Mixed-effects REML regression
Group variable: school

Imputations = 5
Number of obs = 887
Number of groups = 48

Obs per group:
min = 5
avg = 18.5
max = 62

Average RVI = 0.0574
Largest FMI = 0.1079

DF adjustment: Large sample
DF: min = 376.05
avg = 44,112.02
max = 167,428.86

Model F test: Equal FMI
F( 1, 376.0) = 305.41
Prob > F = 0.0000

|        | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|--------|--------|-----------|-------|------|---------------------|
| math5  |        |           |       |      |                     |
| math3  | .6100335 | .0349069 | 17.48 | 0.000 | .5413963           | .6786708       |
| _cons  | 30.48217 | .3536049 | 86.20 | 0.000 | 29.78911           | 31.17522       |

Random-effects Parameters

<table>
<thead>
<tr>
<th>school: Identity</th>
<th>Estimate</th>
<th>Std. Err.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd(_cons)</td>
<td>2.033826</td>
<td>.3069989</td>
<td>1.512894</td>
</tr>
<tr>
<td>sd(Residual)</td>
<td>5.321503</td>
<td>.1355669</td>
<td>5.061821</td>
</tr>
</tbody>
</table>

The estimated coefficients, random-effects standard deviations, and other statistics are similar to those from `mi estimate: xtreg`. Unlike `mi estimate: xtreg`, the `mi estimate: mixed` command combines variance components in the estimation metric described in `[ME] mixed` and then back-transforms the estimates to display results in the original metric. In our example, the reported standard deviations are exponentiated multiple-imputation estimates of the log standard-deviations.
The random-effects parameters are displayed as standard deviations. We can display variances instead by replaying the `mi estimate` command with the `variance` option:

```
  . mi estimate, variance
  Multiple-imputation estimates
  Mixed-effects REML regression
  Group variable: school
  Obs per group:  min = 5  avg = 18.5  max = 62
  Average RVI = 0.0574  Largest FMI = 0.1079
  DF adjustment: Large sample
  DF:  min = 376.05  avg = 44,112.02  max = 167,428.86
  DF: Equal FMI  F( 1, 376.0) = 305.41  Prob > F = 0.0000
```

|       | Coef. | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|-------|-------|-----------|-------|-----|---------------------|
| math5 |       |           |       |     |                     |
| math3 | .6100335 | .0349069 | 17.48 | 0.000 | .5413963 .6786708 |
| _cons | 30.48217 | .3536049 | 86.20 | 0.000 | 29.78911 31.17522 |

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std. Err.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>school: Identity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>var(_cons)</td>
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<td>1.248765</td>
<td>2.288848 7.475462</td>
</tr>
<tr>
<td>var(Residual)</td>
<td>28.3184</td>
<td>1.442839</td>
<td>25.62204 31.29852</td>
</tr>
</tbody>
</table>

Although the random-effects parameters are now displayed as variances, they are still combined and stored in the log–standard-deviation metric.

**Example 5: Estimating transformations**

Stata estimation commands usually support `lincom` and `nlcom` (see [R] `lincom` and [R] `nlcom`) to obtain estimates of the transformed coefficients after estimation by using the delta method. Because MI estimates based on a small number of imputations may not yield a valid VCE, this approach is not generally viable. Also, transformations applied to the combined coefficients are only asymptotically equivalent to the combined transformed coefficients. With a small number of imputations, these two ways of obtaining transformed coefficients can differ significantly.

Thus `mi estimate` provides its own way of combining transformed coefficients. You need to use `mi estimate`’s method for both linear and nonlinear combinations of coefficients. We are about to demonstrate how to use the method using the ratio of coefficients as an example, but what we are about to do would be equally necessary if we wanted to obtain the difference in two coefficients.

For the purpose of illustration, suppose that we want to estimate the ratio of the coefficients, say, `age` and `sqft` from example 2. We can do this by typing
. use https://www.stata-press.com/data/r16/mhouses1993s30
(Albuquerque Home Prices Feb15-Apr30, 1993)
. mi estimate (ratio: _b[age]/_b[sqft]):
> regress price tax sqft age nfeatures ne custom corner
Multiple-imputation estimates Imputations = 30
Linear regression Number of obs = 117
Average RVI = 0.0648
Largest FMI = 0.2533
Complete DF = 109
DF adjustment: Small sample
DF: min = 69.12
max = 105.51
Model F test: Equal FMI F( 7, 106.5) = 67.18
Within VCE type: OLS Prob > F = 0.0000

|      | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|------|--------|-----------|-------|------|----------------------|
| price |        |           |       |      |                      |
| tax   | .6768015 | .1241568 | 5.45  | 0.000| .4301777 - .9234253  |
| sqft  | .2118129 | .069177  | 3.06  | 0.003| .0745091 - .3491168  |
| age   | .2471445 | 1.653669 | 0.15  | 0.882| -3.051732  3.546021  |
| nfeatures | 9.288033 | 13.30469 | 0.70  | 0.487| -17.12017  35.69623 |
| ne    | 2.518996 | 36.99365 | 0.07  | 0.946| -70.90416  75.94215 |
| custom| 134.2193 | 43.29755 | 3.10  | 0.002| 48.35674  220.0818 |
| corner| -68.58686| 39.4988  | -1.72 | 0.089| -147.7934  10.61972|
| _cons | 123.9118 | 71.05816 | 1.74  | 0.085| -17.19932 265.0229 |

Transformations Average RVI = 0.2899
Largest FMI = 0.2316
Complete DF = 109
DF adjustment: Small sample
DF: min = 72.51
max = 72.51
Within VCE type: OLS prob > F = 0.0000

|      | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|------|--------|-----------|-------|------|----------------------|
| price |        |           |       |      |                      |
| ratio | 1.44401| 8.217266  | 0.18  | 0.861| -14.93485  17.82287 |

We use the `nlcom` syntax to specify the transformation: (ratio: _b[age]/_b[sqft]) defines the transformation and its name is ratio. All transformations must be specified following `mi estimate` and before the colon, and must be bound in parentheses.

A separate table containing the estimate of the ratio is displayed following the estimates of coefficients. If desired, we can suppress the table containing the estimates of coefficients by specifying the `nocoef` option. The header reports the average RVI due to nonresponse, the largest FMI, and the summaries of the degrees of freedom specific to the estimated transformations. Because we specified only one transformation, the minimum, average, and maximum degrees of freedom are the same. They correspond to the individual degrees of freedom for ratio.

See `[MI] mi test` for an example of linear transformation.
Example 6: Monte Carlo error estimates

Multiple imputation is a stochastic procedure. Each time we reimpute our data, we get different sets of imputations because of the randomness of the imputation step, and therefore we get different multiple-imputation estimates. However, we want to be able to reproduce MI results. Of course, we can always set the random-number seed to ensure reproducibility by obtaining the same imputed values. However, what if we use a different seed? Would we not want our results to be similar regardless of what seed we use? This leads us to a notion we call statistical reproducibility—we want results to be similar across repeated uses of the same imputation procedure; that is, we want to minimize the simulation error associated with our results.

To assess the level of simulation error, White, Royston, and Wood (2011) propose to use a Monte Carlo error of the MI results, defined as the standard deviation of the results across repeated runs of the same imputation procedure using the same data. The authors suggest evaluating Monte Carlo error estimates not only for parameter estimates but also for other statistics, including $p$-values and confidence intervals, as well as MI statistics including $\text{RVI}$ and $\text{FMI}$.

Clearly, as the number of imputations increases, the simulation error decreases. Consider the total MI variance $T = \bar{U} + B + B/M$ of a single parameter, where $\bar{U}$ is the within-imputation variance and $B$ is the between-imputation variance; see Methods and formulas for details. The term $B/M$ reflects the increase in variance due to using a finite number of imputations, and its square root defines the Monte Carlo error associated with a single parameter. In general, Monte Carlo error estimates are obtained by applying a jackknife procedure to MI results. That is, an MCE estimate of an MI statistic is the standard error of the mean of the pseudovalues for that statistic, computed by omitting one imputation at a time; see [R] jackknife for technical details.

Consider our heart attack data analysis from example 1. Let’s compute Monte Carlo error estimates of MI results. To obtain MCE estimates, we specify the `merror` option during estimation:
Multiple-imputation estimates

| attack  | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|---------|--------|-----------|-------|------|----------------------|
| smokes  | 1.198595 | .3578195 | 3.35  | 0.001 | .4972789 -1.899911  |
|         | .0068541 | .0008565 | 0.01  | 0.000 | .0056572 .0082212   |
| age     | .0360159 | .0154399 | 2.33  | 0.020 | .0057541 .0662776   |
|         | .0002654 | .0000351 | 0.01  | 0.001 | .0002319 .0003108   |
| bmi     | .1039416 | .0476136 | 2.18  | 0.029 | .010514 .1973692    |
|         | .0038014 | .0008904 | 0.09  | 0.006 | .0039928 .0044049   |
| hsggrad | .1578992 | .4049257 | 0.39  | 0.697 | -.6357464 .9515449  |
|         | .0091517 | .0010209 | 0.02  | 0.016 | .0086215 .0100602   |
| female  | -.1067433 | .4164735 | -0.26 | 0.798 | -.9230191 .7095326  |
|         | .0077566 | .0009279 | 0.02  | 0.015 | .006985 .0088408    |
| _cons   | -5.478143 | 1.685075 | -3.25 | 0.001 | -8.782394 -2.173892 |
|         | .1079841 | .0248274 | 0.07  | 0.000 | .1310618 .1050817   |

Note: Values displayed beneath estimates are Monte Carlo error estimates.

As the note describes, MCE estimates are displayed beneath parameter estimates. Following practical guidelines from White, Royston, and Wood (2011), MCE estimates of coefficients should be less than 10% of the standard errors of the coefficients; MCE estimates of test statistics should be approximately 0.1; and MCE estimates of p-values should be approximately 0.01 when the true p-value is 0.05 and 0.02 when the true p-value is 0.1. Our results based on 20 imputations satisfy these conditions, so we can be reasonably sure about the statistical reproducibility of our results.

We can also see Monte Carlo error estimates for other MI statistics reported by the vartable option. To redisplay Monte Carlo error estimates, we use the merror option upon replay. We also suppress the coefficient table by using the nocitable option.
Multiple-imputation estimates
Logistic regression

<table>
<thead>
<tr>
<th>Imputation variance</th>
<th>Within</th>
<th>Between</th>
<th>Total</th>
<th>RVI</th>
<th>FMI</th>
<th>Relative efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>smokes</td>
<td>.127048</td>
<td>.00094</td>
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<td>.007711</td>
<td>.999615</td>
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</table>

Note: Values displayed beneath estimates are Monte Carlo error estimates.

MCE estimates of all statistics are small.

What if we want to see MCE estimates of odds ratios? We know that we can use the or option on replay to redisplay results as odds ratios. However, using this option in combination with merror upon replay will not display MCE estimates of odds ratios:

```
. mi estimate, or merror
Multiple-imputation estimates
Logistic regression
```

```
Number of obs = 154
Average RVI = 0.0312
Largest FMI = 0.1355
DF adjustment: Large sample
      DF:     min = 1,060.38
             avg = 223,362.56
             max = 493,335.88
Model F test: Equal FMI
      F(  5,71379.3) = 3.59
Within VCE type: OIM
     Prob > F = 0.0030
```

| attack | Odds Ratio | Std. Err. | t  | P>|t| | [95% Conf. Interval] |
|--------|------------|-----------|----|-----|----------------------|
| smokes | 3.315455   | 1.186334  | 3.35| 0.001| 1.644241 6.685298   |
| age    | 1.036672   | .0160061  | 2.33| 0.020| 1.005771 1.068523   |
| bmi    | 1.109536   | .052829   | 2.18| 0.029| 1.010569 1.218194   |
| hsggrad| 1.171048   | .4741875  | 0.39| 0.697| .5295401 2.589707   |
| female | .8987564   | .3743082  | -0.26| 0.798| .3973177 2.033041   |
| _cons  | .0041771   | .0070387  | -3.25| 0.001| .0001534 1.137342   |

Note: _cons estimates baseline odds.
Note: Monte Carlo error estimates are not available for exponentiated coefficients.

The same applies to a combination of the level() and merror options specified on replay to try to display MCE estimates of confidence intervals for a confidence level other than the one used during estimation.
To compute MCE estimates for odds ratios in addition to coefficients, you need to specify the `or` option in combination with `mcerror` during estimation. Similarly, to compute MCE estimates for confidence intervals with a specific confidence level, you need to specify the `level()` option in combination with `mcerror` during estimation. Otherwise, MCE estimates of 95% confidence intervals are computed.

```
.mi estimate, merror or level(90): logit attack smokes age bmi hsggrad female
```

Multiple-imputation estimates

Logistic regression

Number of obs = 154
Average RVI = 0.0312
Largest FMI = 0.1355

DF adjustment: Large sample

DF: min = 1,060.38
avg = 223,362.56
max = 493,335.88

Model F test: Equal FMI

F(5,71379.3) = 3.59
Within VCE type: OIM

| Variable | Odds Ratio | Std. Err. | t | P>|t| | [90% Conf. Interval] |
|----------|------------|-----------|---|------|----------------------|
| attack   | 3.315455   | 1.186334  | 3.35 | 0.001 | 1.840491 5.97245 |
|          | 0.227267   | 0.104806  | 2.01 | 0.000 | 0.1017398 0.477351 |
| smokes   | 1.036672   | 0.0160061 | 2.33 | 0.020 | 1.010676 1.063337 |
|          | 0.002752   | 0.000039  | 0.01 | 0.001 | 0.002388 0.003221 |
| age      | 1.109536   | 0.052829  | 2.18 | 0.029 | 1.025885 1.200007 |
|          | 0.042178   | 0.001033  | 0.09 | 0.006 | 0.040064 0.0051089 |
| bmi      | 1.171048   | 0.4741875 | 0.39 | 0.697 | 0.6016087 2.279478 |
|          | 0.0107188  | 0.0049031 | 0.02 | 0.16 | 0.0052248 0.02254 |
| hsggrad  | 0.8987564  | 0.3743082 | -0.26 | 0.798 | 0.4530363 1.782998 |
|          | 0.0069686  | 0.00341   | 0.02 | 0.15 | 0.0032087 0.0154128 |
| female   | 0.9041771  | 0.0070387 | -3.25 | 0.001 | 0.00261 0.0668412 |
|          | 0.0004519  | 0.0007338 | 0.07 | 0.000 | 0.0000336 0.0068716 |

Note: _cons estimates baseline odds.
Note: Values displayed beneath estimates are Monte Carlo error estimates.

Similarly to the MCE estimates for coefficients, the MCE estimates for odds ratios are within acceptable limits.

If you wish to obtain Monte Carlo error estimates of confidence intervals for a number of different confidence levels, a more computationally efficient way of doing so is to use `mi estimate using` (see `[MI] mi estimate using`).

First, use `mi estimate` to save individual estimation results from a model to an estimation file:

```
.mi estimate, saving(miest): ...
```

Then use `mi estimate using` to obtain MCE estimates for different confidence intervals,

```
.mi estimate using miest, merror level(90) ...
```

or for odds ratios,

```
.mi estimate using miest, merror or ...
```

without refitting the model.
Potential problems that can arise when using mi estimate

There are two problems that can arise when using mi estimate:

1. The estimation sample varies across imputations.
2. Different covariates are omitted across the imputations.

mi estimate watches for and issues an error message if either of these problems occur. Below we explain how each can arise and what to do about it. If you see one of these messages, be glad that mi estimate mentioned the problem, because otherwise, it might have gone undetected. A varying-estimation sample may result in biased or inefficient estimates. Different covariates being omitted always results in the combined results being biased.

If the first problem arises, mi estimate issues the error message “estimation sample varies between \( m = \# \) and \( m = \# \).” mi estimate expects that when it runs the estimation command on the first imputation, on the second, and so on, the estimation command will use the same observations in each imputation. mi estimate does not just count, it watches which observations are used.

Perhaps the difference is due to a past mistake, such as not having imputed all the missing values. Perhaps you even corrupted your mi data so that the imputed variable is missing in some imputations and not in others.

Another reason the error can arise is because you specified an if condition based on imputed or passive variables. mi estimate considers this a mistake but, if this is your intent, you can reissue the mi estimate command and include the esampvaryok option.

Finally, it is possible that the varying observations are merely a characteristic of the estimator when combined with the two different imputed datasets. In this case, just as in the previous one, you can reissue mi estimate with the esampvaryok option.

The easy way to diagnose why you got this error is to use mi xeq (see [MI] mi xeq) to run the estimation command separately on the two imputations mentioned in the error message. Alternatively, you can rerun the mi estimate command immediately with the esampvaryok option and with the esample(varname) option, which will create in new variable varname the e(sample) from each of the individual estimations. If you use the second approach, you must first mi convert your data to flong or flongsep if they are not recorded in that style already; see [MI] mi convert for details.

The second problem we mentioned concerns omitted variables as opposed to omitted observations. mi estimate reports that “omitted variables vary” and goes on to mention the two imputations between which the variation was detected.

This can be caused when you include factor variables but did not specify base categories. It was the base categories that differed in the two imputations. That could happen if you specified i.group. By default, Stata chooses to omit the most frequent category. If group were imputed or passive, then the most frequent category could vary between two imputations. The solution is to specify the base category for yourself by typing, for instance, b2.group; see [U] 11.4.3 Factor variables.

There are other possible causes. Varying omitted variables 1) includes different variables being omitted in the two imputations and 2) includes no variables being omitted in one imputation and, in the other, one or more variables being omitted.

When different variables are being omitted, it is usually caused by collinearity, and one of the variables needs to be dropped from the model. Variables x1 and x2 are collinear; sometimes the estimation command is choosing to omit x1 and other times, x2. The solution is that you choose which to omit by removing it from your model.

If no variables were omitted in one of the imputations, the problem is more difficult to explain. Say that you included i.group in your model, the base category remained the same for the two
imputations, but in one of the imputations, no one is observed in group 3, and thus no coefficient for

group 3 could be estimated. You choices are to accept that you cannot estimate a group 3 coefficient

and combine group 3 with, say, group 4, or to drop all imputations in which there is no one in group

3. If you want to drop imputations 3, 9, and 12, you type \texttt{mi set m - (3,9,12)}; see \cite{MI} \texttt{mi set}.

\section*{Technical note}

As we already mentioned, \texttt{mi estimate} obtains MI estimates by using the combination rules to

pool results from the specified command executed separately on each imputation. As such, certain

concepts (for example, likelihood function) and most postestimation tools specific to the command

may not be applicable to the MI estimates; see \textit{Analysis of multiply imputed data} in \cite{MI} \textit{Intro substantive}. MI estimates may not even have a valid variance–covariance matrix associated

with them when the number of imputations is smaller than the number of estimated parameters. For these

reasons, the system matrices \( e(b) \) and \( e(V) \) are not set by \texttt{mi estimate}. If desired, you can save

the MI estimates and their variance–covariance estimates in \( e(b) \) and \( e(V) \) by specifying the post

option. See \cite{MI} \texttt{mi estimate postestimation} for postestimation tools available after \texttt{mi estimate}.

\section*{Stored results}

\texttt{mi estimate} stores the following in \( e() \):

\begin{verbatim}
Scalars
  e(df_avg[Q]_mi)      average degrees of freedom
  e(df_c_mi)           complete degrees of freedom (if originally stored by estimation_command in e(df_r))
  e(df_max[Q]_mi)      maximum degrees of freedom
  e(df_min[Q]_mi)      minimum degrees of freedom
  e(df_m_mi)           MI model test denominator (residual) degrees of freedom
  e(df_r_mi)           MI model test numerator (model) degrees of freedom
  e(esampvary_mi)      varying-estimation sample flag (0 or 1)
  e(F_mi)              model test \( F \) statistic
  e(k_exp_mi)          number of expressions (transformed coefficients)
  e(M_mi)              number of imputations
  e(N_mi)              number of observations (minimum, if varies)
  e(N_min_mi)          minimum number of observations
  e(N_max_mi)          maximum number of observations
  e(N_g_mi)            number of groups
  e(g_min_mi)          smallest group size
  e(g_avg_mi)          average group size
  e(g_max_mi)          largest group size
  e(p_mi)              MI model test \( p \)-value
  e(cilevel_mi)        confidence level used to compute Monte Carlo error estimates of confidence intervals
  e(fmi_max[Q]_mi)     largest FMI
  e(rvi_avg[Q]_mi)     average RVI
  e(rvi_avg_F_mi)      average RVI associated with the residual degrees of freedom for model test
  e(ufmi_mi)           1 if unrestricted FMI model test is performed, 0 if equal FMI model test is performed
\end{verbatim}
Macros
- `e(mi)`
- `e(cmdline_mi)` command as typed
- `e(prefix_mi)` name of estimation_command
- `e(cmd_mi)` mi estimate (equals `e(cmd_mi)` when `post` is used)
- `e(title_mi)` “Multiple-imputation estimates”
- `e(wvce_mi)` title used to label within-imputation variance in the table header
- `e(modeltest_mi)` title used to label the model test in the table header
- `e(ddf_adjust_mi)` title used to label the degrees-of-freedom adjustment in the table header
- `e(exp_names_mi)` names of expressions specified in spec
- `e(rc_mi)` return codes for each imputation
- `e(m Mi)` specified imputation numbers
- `e(m_est Mi)` imputation numbers used in the computation
- `e(names_vvl Mi)` command-specific e() macro names that contents varied across imputations
- `e(names_vvm Mi)` command-specific e() matrix names that values varied across imputations
- `e(names_vvs Mi)` command-specific e() scalar names that values varied across imputations
- `e(b Mi)` MI estimates of coefficients (equals `e(b Mi)`; stored only if `post` is used)
- `e(V Mi)` variance–covariance matrix (equals `e(V Mi)`; stored only if `post` is used)
- `e(Cns Mi)` constraint matrix, for constrained estimation only (equals `e(Cns Mi)`; stored only if `post` is used)
- `e(N_g Mi)` group counts
- `e(g_min Mi)` group-size minimums
- `e(g_avg Mi)` group-size averages
- `e(g_max Mi)` group-size maximums
- `e(b_Q Mi)` MI estimates of coefficients (or transformed coefficients)
- `e(V_Q Mi)` variance–covariance matrix (total variance)
- `e(Cns Mi)` constraint matrix (for constrained estimation only)
- `e(W_Q Mi)` within-imputation variance matrix
- `e(B_Q Mi)` between-imputation variance matrix
- `e(rel_Q Mi)` parameter-specific relative efficiencies
- `e(rvi_Q Mi)` parameter-specific RVIs
- `e(df_Q Mi)` parameter-specific degrees of freedom 
- `e(pive_Q Mi)` parameter-specific percentages increase in standard errors
- `e(vs_names_vs Mi)` values of command-specific e() scalar vs_names that varied across imputations

vs_names include (but are not restricted to) `df_r`, `N`, `N_strata`, `N_psu`, `N_pop`, `N_sub`, `N_postrata`, `N_stdize`, `N_subpop`, `N_over`, and converged.

Results `N_g Mi`, `g_min Mi`, `g_avg Mi`, and `g_max Mi` are stored for panel-data models only. The results are stored as matrices for mixed-effects models and as scalars for other panel-data models.

If transformations are specified, the corresponding estimation results are stored with the _Q Mi suffix, as described above.

Command-specific e() results that remain constant across imputations are also stored. Command-specific results that vary from imputation to imputation are posted as missing, and their names are stored in the corresponding macros `e(names_vvl Mi)`, `e(names_vvm Mi)`, and `e(names_vvs Mi)`. For some command-specific e() scalars (see vs_names above), their values from each imputation are stored in a corresponding matrix with the _vs Mi suffix.

### Methods and formulas
Let \( \mathbf{q} \) define a column vector of parameters of interest. For example, \( \mathbf{q} \) may be a vector of coefficients (or functions of coefficients) from a regression model. Let \( \{ (\bar{\mathbf{q}}_i, \hat{U}_i) : i = 1, 2, \ldots, M \} \)
be the completed-data estimates of \( q \) and the respective variance–covariance estimates from \( M \) imputed datasets.

The MI estimate of \( q \) is

\[
\bar{q}_M = \frac{1}{M} \sum_{i=1}^{M} \hat{q}_i
\]

The variance–covariance estimate (VCE) of \( \bar{q}_M \) (total variance) is

\[
T = \bar{U} + (1 + \frac{1}{M})B
\]

where \( \bar{U} = \sum_{i=1}^{M} \hat{U}_i/M \) is the within-imputation variance–covariance matrix and \( B = \sum_{i=1}^{M} (q_i - \bar{q}_M)(q_i - \bar{q}_M)/(M - 1) \) is the between-imputation variance–covariance matrix.

Methods and formulas are presented under the following headings:

*Univariate case

*Multivariate case

**Univariate case**

Let \( Q, \bar{Q}_M, B, \bar{U}, \) and \( T \) correspond to the scalar analogues of the above formulas. Univariate inferences are based on the approximation

\[
T^{-1/2}(Q - \bar{Q}_M) \sim t_\nu
\]

where \( t_\nu \) denotes a Student’s \( t \) distribution with \( \nu \) degrees of freedom, which depends on the number of imputations, \( M \), and the increase in variance of estimates due to missing data. Under the large-sample assumption with respect to complete data, the degrees of freedom is

\[
\nu_{\text{large}} = (M - 1) \left(1 + \frac{1}{r}\right)^2
\]

where

\[
r = \frac{(1 + M^{-1})B}{\bar{U}}
\]

is an RVI due to missing data. Under the small-sample assumption, the degrees of freedom is

\[
\nu_{\text{small}} = \left(\frac{1}{\nu_{\text{large}}} + \frac{1}{\hat{\nu}_{\text{obs}}}\right)^{-1}
\]

where \( \hat{\nu}_{\text{obs}} = \nu_c(\nu_c + 1)(1 - \gamma)/(\nu_c + 3), \gamma = (1 + 1/M)B/T \), and \( \nu_c \) are the complete degrees of freedom, the degrees of freedom used for inference when data are complete (Barnard and Rubin 1999).

The small-sample adjustment (4) is applied to the degrees of freedom \( \nu \) when the specified command stores the residual degrees of freedom in \( e(\text{df x}) \). This number of degrees of freedom is used as the complete degrees of freedom, \( \nu_{\text{c}} \), in the computation. (If \( e(\text{df x}) \) varies across imputations, the smallest is used in the computation, resulting in conservative inference.) If \( e(\text{df x}) \) is not set by the specified command or if the \text{nosmall} option is specified, then (2) is used to compute the degrees of freedom, \( \nu \).
Parameter-specific significance levels, confidence intervals, and degrees of freedom as reported by `mi estimate` are computed using the formulas above.

The percentage of standard-error increase due to missing data, as reported by `mi estimate`, `dftable`, is computed as \{(T/U)^{1/2} - 1\} \times 100\%.

The FMI\s due to missing data and relative efficiencies reported by `mi estimate`, `vartable` are computed as follows.

In the large-sample case, the fraction of information about \(Q\) missing due to nonresponse (Rubin 1987, 77) is

\[
\lambda = \frac{r + 2/(\nu_{\text{large}} + 3)}{r + 1}
\]

where the RVI, \(r\), is defined in (3). In the small-sample case, the fraction of information about \(Q\) missing due to nonresponse (Barnard and Rubin 1999, 953) is

\[
\lambda = 1 - \frac{\lambda(\nu_{\text{small}}) \bar{U}}{\lambda(\nu_c) T}
\]

where \(\lambda(u) = (u + 1)/(u + 3)\).

The relative (variance) efficiency of using \(M\) imputations versus the infinite number of imputations is \(RE = (1 + \lambda/M)^{-1}\) (Rubin 1987, 114).

Also see Rubin (1987, 76–77) and Schafer (1997, 109–111) for details.

**Multivariate case**

The approximation (1) can be generalized to the multivariate case:

\[
(q - \bar{q}_M)T^{-1}(q - \bar{q}_M)'/k \sim F_{k,\nu}
\]  

where \(F_{k,\nu}\) denotes an \(F\) distribution with \(k = \text{rank}(T)\) numerator degrees of freedom and \(\nu\) denominator degrees of freedom defined as in (2), where the RVI, \(r\), is replaced with the average RVI, \(r_{\text{ave}}:\)

\[
r_{\text{ave}} = (1 + 1/M)\text{tr}(B\bar{U}^{-1})/k
\]

The approximation (5) is inadequate with a small number of imputations because the between-imputation variance, \(B\), cannot be estimated reliably based on small \(M\). Moreover, when \(M\) is smaller than the number of estimated parameters, \(B\) does not have a full rank. As such, the total variance, \(T\), may not be a valid variance–covariance matrix for \(q\).

One solution is to assume that the between-imputation and within-imputation matrices are proportional, that is \(B = \bar{\lambda} \times \bar{U}\) (Rubin 1987, 78). This assumption implies that FMI\s of all estimated parameters are equal. Under this assumption, approximation (5) becomes

\[
(1 + r_{\text{ave}})^{-1}(q - \bar{q}_M)\bar{U}^{-1}(q - \bar{q}_M)'/k \sim F_{k,\nu_{\text{ave}}}
\]

where \(k = \text{rank}(U)\) and \(\nu_{\text{ave}}\) is computed as described in Li et al. (1991, 1067).

Also see Rubin (1987, 77–78) and Schafer (1997, 112–114) for details.

We refer to (6) as an equal FMI test and to (5) as the unrestricted FMI test. By default, `mi estimate` uses the approximation (6) for the model test. If the `ufmitest` option is specified, it uses the approximation (5) for the model test.
Similar to the univariate case, the degrees of freedom $\nu_\star$ and $\nu$ are adjusted for small samples when the command stores the completed-data residual degrees of freedom in \texttt{e(df\_r)}.

In the small-sample case, the degrees of freedom $\nu_\star$ is computed as described in Reiter (2007) (in the rare case, when $k(M - 1) \leq 4$, $\nu_\star = (k + 1)\nu_1 / 2$, where $\nu_1$ is the degrees of freedom from Barnard and Rubin [1999]). In the small-sample case, the degrees of freedom $\nu$ is computed as described in Barnard and Rubin (1999) and Marchenko and Reiter (2009).

Acknowledgments

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References


Also see

[MI] mi estimate postestimation — Postestimation tools for mi estimate
[MI] mi estimate using — Estimation using previously saved estimation results
[MI] Intro — Introduction to mi
[MI] Intro substantive — Introduction to multiple-imputation analysis
[MI] Glossary