

**estat heterogeneity** — Compute multivariate heterogeneity statistics

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## Description

`estat heterogeneity` computes multivariate heterogeneity statistics after a random-effects multivariate meta-regression fit by `meta mvregress`. By default, the multivariate Cochran and Jackson–White–Riley heterogeneity statistics are computed, but the White statistic is also available.

## Menu for estat

Statistics > Postestimation

## Syntax

```
estat heterogeneity [ , statistics ]
```

*statistics*

Description

Main

`cochran`

Cochran statistics; the default

`jwriley` [ (*depnames*) ]

Jackson–White–Riley statistics; the default for all dependent variables

`white`

White statistics

`all`

all heterogeneity statistics

You may specify one or multiple statistics simultaneously.

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

## Options

Main

`cochran`, the default, specifies that the multivariate Cochran  $I_Q^2$  and  $H_Q^2$  heterogeneity statistics be computed. These statistics are computed jointly for all dependent variables and thus are not available for each dependent variable separately. These statistics are displayed by default.

`jwriley`, the default, and `jwriley(depnames)` specify that Jackson–White–Riley heterogeneity statistics  $I_{JWR}^2$  and  $R_{JWR}$  be computed. These statistics are available jointly for all dependent variables, separately for each dependent variable, or jointly for any subset of dependent variables. The statistics for all dependent variables, separately and jointly, are displayed by default.

`jwriley` computes the Jackson–White–Riley heterogeneity statistics jointly for all dependent variables and separately for each dependent variable. For example, if you have three dependent variables `y1`, `y2`, and `y3`, then the Jackson–White–Riley heterogeneity statistics are computed separately for each variable and jointly for all variables, resulting in four  $I^2_{JWR}$  statistics and four  $R_{JWR}$  statistics.

`jwriley(depnames)` computes the Jackson–White–Riley statistics jointly for the specified dependent variables `depnames`. For example, if you have three dependent variables `y1`, `y2`, and `y3`, you may specify `jwriley(y1 y2)` to compute heterogeneity statistics based on dependent variables `y1` and `y2` jointly.

`white` specifies that White  $I^2$  statistics be computed. These statistics are available only separately for each dependent variable. With one dependent variable, the White  $I^2$  statistic reduces to the univariate  $I^2$  statistic as reported by [META] [meta summarize](#) or [META] [meta regress](#).

`all` specifies that all heterogeneity statistics be reported. This option implies `cochran`, `jwriley`, and `white`.

## Remarks and examples

[stata.com](https://www.stata.com)

### ► Example 1: Jackson–White–Riley heterogeneity statistics

Consider the dataset from [example 8](#) of [META] [meta mvregress](#). We fit a random-effects multivariate meta-regression using maximum likelihood estimation and an `exchangeable` covariance structure for the between-study covariance matrix:

```
. use https://www.stata-press.com/data/r17/smokecess
(Smoking cessation interventions)
. quietly meta mvregress y*, wcovvariables(v*)
> random(mle, covariance(exchangeable))
```

We can then compute the multivariate Jackson–White–Riley heterogeneity statistics by typing

```
. estat heterogeneity, jwriley
Method: Jackson-White-Riley
yb:
  I2 (%) = 81.83
  R = 2.35
yc:
  I2 (%) = 90.60
  R = 3.26
yd:
  I2 (%) = 62.84
  R = 1.64
Joint:
  I2 (%) = 78.61
  R = 2.16
```

The `jwriley` option displays the Jackson–White–Riley heterogeneity statistics separately for each dependent variable and jointly for all dependent variables. See [example 4](#) of [META] [meta mvregress](#) for the interpretation of these statistics.

One feature that is unique to the Jackson–White–Riley heterogeneity statistics is the possibility of assessing heterogeneity jointly for a subset of dependent variables. Below, we specify the `jwriley(yb yc)` option to assess heterogeneity jointly for these two dependent variables.

```
. estat heterogeneity, jwiley(yb yc)
Method: Jackson-White-Riley
yb yc:
    I2 (%) = 86.02
        R = 2.67
```

The value of  $I^2$  is 86.02%, which suggests that there is considerable heterogeneity among the effect sizes corresponding to these variables; see [example 4](#) of [\[META\] meta mvregress](#) for details about the interpretation of  $I^2_{JWR}$  and  $R_{JWR}$ .

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## Stored results

estat heterogeneity stores the following in `r()`:

|                        |  |
|------------------------|--|
| Scalars                |  |
| <code>r(I2_Q)</code>   | Cochran $I^2$ heterogeneity statistic (option <code>cochran</code> )             |
| <code>r(H2)</code>     | $H^2$ heterogeneity statistic (option <code>cochran</code> )                     |
| Matrices               |  |
| <code>r(R)</code>      | Jackson-White-Riley $R$ heterogeneity statistics (option <code>jwiley</code> )   |
| <code>r(I2_JWR)</code> | Jackson-White-Riley $I^2$ heterogeneity statistics (option <code>jwiley</code> ) |
| <code>r(I2_W)</code>   | White $I^2$ heterogeneity statistics (option <code>white</code> )                |

## Methods and formulas

Methods and formulas are presented under the following headings:

- [Brief overview of heterogeneity statistics](#)
- [Cochran heterogeneity statistics](#)
- [Jackson-White-Riley heterogeneity statistics](#)
- [White heterogeneity statistics](#)

See [Methods and formulas](#) of [\[META\] meta mvregress](#).

Consider a random-effects multivariate meta-regression

$$\hat{\theta}_j = \mathbf{X}_j \boldsymbol{\beta} + \mathbf{u}_j + \boldsymbol{\epsilon}_j, \quad \boldsymbol{\epsilon}_j \sim N_d(\mathbf{0}, \boldsymbol{\Lambda}_j)$$

where  $\mathbf{u}_j = (u_{1j}, u_{2j}, \dots, u_{dj})' \sim N_d(\mathbf{0}, \boldsymbol{\Sigma})$  ( $j = 1, \dots, K$ ) define random effects,  $\mathbf{x}_j = (1, x_{1j}, \dots, x_{p-1,j})$  is a  $1 \times p$  vector of moderators (covariates),  $\mathbf{X}_j = \mathbf{x}_j \otimes I_d$  is a  $d \times dp$  design matrix, and  $\boldsymbol{\beta} = (\beta'_1, \beta'_2, \dots, \beta'_d)'$  is a  $dp \times 1$  vector of unknown regression coefficients.

The random-effects estimator for regression coefficients is

$$\hat{\boldsymbol{\beta}}^* = \left( \sum_{j=1}^K \mathbf{X}'_j \mathbf{W}_j^* \mathbf{X}_j \right)^{-1} \sum_{j=1}^K \mathbf{X}'_j \mathbf{W}_j^* \hat{\theta}_j$$

where  $\mathbf{W}_j^* = (\hat{\boldsymbol{\Sigma}} + \boldsymbol{\Lambda}_j)^{-1}$ . The corresponding covariance matrix is given by

$$\text{Var}(\hat{\boldsymbol{\beta}}^*) = \left( \sum_{j=1}^K \mathbf{X}'_j \mathbf{W}_j^* \mathbf{X}_j \right)^{-1} \tag{1}$$

Let  $\mathbf{W}_j = \mathbf{\Lambda}_j^{-1}$ . The fixed-effects estimator for regression coefficients is

$$\hat{\boldsymbol{\beta}} = \left( \sum_{j=1}^K \mathbf{X}'_j \mathbf{W}_j \mathbf{X}_j \right)^{-1} \sum_{j=1}^K \mathbf{X}'_j \mathbf{W}_j \hat{\boldsymbol{\theta}}_j$$

and the corresponding covariance matrix is

$$\text{Var}(\hat{\boldsymbol{\beta}}) = \left( \sum_{j=1}^K \mathbf{X}'_j \mathbf{W}_j \mathbf{X}_j \right)^{-1} \quad (2)$$

## Brief overview of heterogeneity statistics

`estat heterogeneity` supports the following heterogeneity statistics: Cochran  $I^2_Q$  and  $H^2_Q$ , Jackson–White–Riley  $R_{JWR}$  and  $I^2_{JWR}$ , and White  $I^2_W$ . Some statistics can be computed separately for each outcome, some can be computed jointly for all outcomes, and some can be computed both jointly and separately. We summarize how each statistic can quantify heterogeneity in [table 1](#), which is a modified version of [Jackson, White, and Riley \(2012, table V\)](#).

Table 1. Quantifying heterogeneity for various heterogeneity statistics

| Statistic            | Quantifies heterogeneity for |                      |                     |
|----------------------|------------------------------|----------------------|---------------------|
|                      | each outcome                 | all outcomes jointly | subsets of outcomes |
| $I^2_Q, H^2_Q$       | No                           | Yes                  | No                  |
| $I^2_{JWR}, R_{JWR}$ | Yes                          | Yes                  | Yes                 |
| $I^2_W$              | Yes                          | No                   | No                  |

## Cochran heterogeneity statistics

The Cochran  $I^2_Q$  and  $H^2_Q$  are based on  $Q_M$ , a multivariate extension of the Cochran statistic  $Q_{\text{res}}$  defined in (3) in [Methods and formulas](#) of [\[META\] meta regress](#). These statistics reduce to the  $I^2_{\text{res}}$  and  $H^2_{\text{res}}$  [see (4) and (5) in [Methods and formulas](#) of [\[META\] meta regress](#)] based on the DerSimonian–Laird random-effects method in the univariate case.

$$I^2_Q = 100 \times \max \left( 0, \frac{Q_M - N + dp}{Q_M} \right)$$

$$H^2_Q = \max \left( 1, \frac{Q_M}{N - dp} \right)$$

where  $Q_M = \sum_{j=1}^K (\hat{\boldsymbol{\theta}}_j - \mathbf{X}_j \hat{\boldsymbol{\beta}})' \mathbf{W}_j (\hat{\boldsymbol{\theta}}_j - \mathbf{X}_j \hat{\boldsymbol{\beta}})$ .

One drawback of the Cochran statistics is that they assess only the impact of heterogeneity jointly for all outcomes. It may be of interest to investigate separately the extent of each outcome’s contribution to the total variability. This will be possible with all heterogeneity statistics discussed below.

## Jackson–White–Riley heterogeneity statistics

Jackson, White, and Riley (2012) proposed to extend the heterogeneity statistic  $R$  of Higgins and Thompson (2002) (not to be confused with the  $R^2$  statistic reported after [META] meta regress) to the multivariate setting. They then used the  $R$  statistic to construct an  $I^2$  statistic that can quantify heterogeneity more conveniently. These  $I^2$  and  $R$  statistics have an advantage over other versions of heterogeneity statistics because they may be computed for any subset of outcome variables.

For all outcomes jointly, the statistic  $R$  is defined as the ratio of the volumes of the confidence regions for the regression coefficients under the random-effects and fixed-effects models raised to the power  $1/dp$ ,

$$R_{\text{JWR}} = \left( \frac{\text{Vol}_{\text{RE}}}{\text{Vol}_{\text{FE}}} \right)^{1/dp}$$

where  $\text{Vol}_{\text{RE}}$  and  $\text{Vol}_{\text{FE}}$  are the volumes of the confidence regions under the random-effects and fixed-effects models, respectively. It can be shown (see Jackson, White, and Riley [2012, appendix]) that the volumes of these regions are proportional to the square root of the determinants of the covariance matrices defined in (1) and (2),

$$\text{Vol}_{\text{RE}} \propto \left| \text{Var}(\widehat{\beta}^*) \right|^{1/2} \quad \text{and} \quad \text{Vol}_{\text{FE}} \propto \left| \text{Var}(\widehat{\beta}) \right|^{1/2}$$

In what follows, we will define the JWR  $R$  and  $I^2$  statistics in terms of these determinants.

For all outcome jointly, the JWR  $R$  and  $I^2$  statistics are given by

$$R_{\text{JWR}} = \left\{ \frac{\left| \text{Var}(\widehat{\beta}^*) \right|}{\left| \text{Var}(\widehat{\beta}) \right|} \right\}^{1/2dp} = \left\{ \left| \text{Var}(\widehat{\beta}^*) \right| \left| \text{Var}(\widehat{\beta}) \right|^{-1} \right\}^{1/2dp}$$

$$I_{\text{JWR}}^2 = 100 \times \max \left( 0, \frac{\left| \text{Var}(\widehat{\beta}^*) \right|^{1/dp} - \left| \text{Var}(\widehat{\beta}) \right|^{1/dp}}{\left| \text{Var}(\widehat{\beta}^*) \right|^{1/dp}} \right) = 100 \times \max \left( 0, \frac{R_{\text{JWR}}^2 - 1}{R_{\text{JWR}}^2} \right)$$

where  $\text{Var}(\widehat{\beta}^*)$  and  $\text{Var}(\widehat{\beta})$  are the estimated covariance matrices of the regression coefficients under the random-effects and fixed-effects models, defined in (1) and (2), respectively.

For each outcome  $i$ , the expressions for the JWR statistics are

$$R_{\text{JWR}, i} = \left\{ \left| \text{Var}(\widehat{\beta}_i^*) \right| \left| \text{Var}(\widehat{\beta}_i) \right|^{-1} \right\}^{1/2dp}$$

$$I_{\text{JWR}, i}^2 = 100 \times \max \left( 0, \frac{R_{\text{JWR}, i}^2 - 1}{R_{\text{JWR}, i}^2} \right)$$

where  $\text{Var}(\widehat{\beta}_i^*)$  and  $\text{Var}(\widehat{\beta}_i)$  are the  $p \times p$  submatrices of  $\text{Var}(\widehat{\beta}^*)$  and  $\text{Var}(\widehat{\beta})$  corresponding to the regression coefficient parameters of the  $i$ th outcome.

When the `jriley(depnames)` option is specified, it is possible to compute the JWR statistics for any subset of outcomes defined by `depnames`. For a subset  $\{s\}$  of outcome variables,

$$R_{\text{JWR}, \{s\}} = \left\{ \left| \text{Var}(\widehat{\beta}_{\{s\}}^*) \right| \left| \text{Var}(\widehat{\beta}_{\{s\}}) \right|^{-1} \right\}^{1/2\nu}$$

$$I_{\text{JWR}, \{s\}}^2 = 100 \times \max \left( 0, \frac{R_{\text{JWR}, \{s\}}^2 - 1}{R_{\text{JWR}, \{s\}}^2} \right)$$

where  $\widehat{\beta}_{\{s\}}^*$  and  $\widehat{\beta}_{\{s\}}$  represent the set of regression coefficients corresponding to the subset  $\{s\}$  and  $\nu$  is the number of the corresponding estimated regression coefficients.

## White heterogeneity statistics

White (2011) (option `white`) suggested to compute the  $I^2$  statistic separately for each outcome but did not provide a method to quantify heterogeneity jointly based on all outcomes. For each outcome  $i$ , the White  $I^2$  statistic is defined as

$$I_{\text{W}, i}^2 = \frac{\widehat{\Sigma}_{ii}}{\widehat{\Sigma}_{ii} + s_{\text{W}, i}^2}$$

where  $\widehat{\Sigma}_{ii}$  is the  $i$ th diagonal element of the estimated between-study covariance matrix  $\widehat{\Sigma}$  described in *Iterative methods for computing  $\Sigma$*  and *Noniterative method for computing  $\Sigma$*  of [META] **meta mvregress**.

The typical within-study variance  $s_{\text{W}, i}^2$  is computed from a univariate meta-regression of the  $i$ th outcome on the moderators. Therefore,  $s_{\text{W}, i}^2$  is computed in the same way as  $s^2$  in *Residual heterogeneity measures* in [META] **meta regress** and is given by

$$s_{\text{W}, i}^2 = \frac{n_i - p}{\text{tr}(\mathbf{P})}$$

where  $n_i$  is the number of observations on outcome  $i$ , which equals  $K$  if there are no missing values, and  $\mathbf{P}$  is given by (1) in *Methods and formulas* of [META] **meta regress**, with  $\mathbf{A} = \mathbf{W}$  being the  $n_i \times n_i$  diagonal matrix containing inverse-variance weights for outcome  $i$ .

The  $I_{\text{W}, i}^2$  statistic reduces to  $I_{\text{res}}^2$  reported by (4) in *Methods and formulas* of [META] **meta regress** when  $d = 1$ .

## References

- Higgins, J. P. T., and S. G. Thompson. 2002. Quantifying heterogeneity in a meta-analysis. *Statistics in Medicine* 21: 1539–1558. <https://doi.org/10.1002/sim.1186>.
- Jackson, D., I. R. White, and R. D. Riley. 2012. Quantifying the impact of between-study heterogeneity in multivariate meta-analyses. *Statistics in Medicine* 31: 3805–3820. <https://doi.org/10.1002/sim.5453>.
- White, I. R. 2011. Multivariate random-effects meta-regression: Updates to `mvmeta`. *Stata Journal* 11: 240–254.

## Also see

- [META] **meta mvregress** — Multivariate meta-regression
- [META] **meta mvregress postestimation** — Postestimation tools for meta mvregress
- [U] **20 Estimation and postestimation commands**