trace() — Trace of square matrix

Description

trace(A) returns the sum of the diagonal elements of A. Returned result is real if A is real, complex if A is complex.

trace(A, B) returns trace(AB), the calculation being made without calculating or storing the off-diagonal elements of AB. Returned result is real if A and B are real and is complex otherwise.

trace(A, B, t) returns trace(AB) if t = 0 and returns trace(A'B) otherwise, where, if either A or B is complex, transpose is understood to mean conjugate transpose. Returned result is real if A and B are real and is complex otherwise.

Syntax

numeric scalar  trace(numeric matrix A)
numeric scalar  trace(numeric matrix A, numeric matrix B)
numeric scalar  trace(numeric matrix A, numeric matrix B, real scalar t)

Remarks and examples

trace(A, B) returns the same result as trace(A*B) but is more efficient if you do not otherwise need to calculate A*B.

trace(A, B, 1) returns the same result as trace(A'B) but is more efficient.

For real matrices A and B,

\[ \text{trace}(A') = \text{trace}(A) \]
\[ \text{trace}(AB) = \text{trace}(BA) \]

and for complex matrices,

\[ \text{trace}(A') = \text{conj}(\text{trace}(A)) \]
\[ \text{trace}(AB) = \text{trace}(BA) \]

where, for complex matrices, transpose is understood to mean conjugate transpose.
Thus for real matrices,

<table>
<thead>
<tr>
<th>To calculate</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>trace(AB)</td>
<td>trace(A, B)</td>
</tr>
<tr>
<td>trace(A'B)</td>
<td>trace(A, B, 1)</td>
</tr>
<tr>
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<td>conj(trace(A, B))</td>
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Transpose in the first column means conjugate transpose.

**Conformability**

\[
\text{trace}(A):
\]

\[
A: \quad n \times n
\]

\[
\text{result:} \quad 1 \times 1
\]

\[
\text{trace}(A, B):
\]

\[
A: \quad n \times m
\]

\[
B: \quad m \times n
\]

\[
\text{result:} \quad 1 \times 1
\]

\[
\text{trace}(A, B, t)
\]

\[
A: \quad n \times m \text{ if } t = 0, \quad m \times n \text{ otherwise}
\]

\[
B: \quad m \times n
\]

\[
t: \quad 1 \times 1
\]

\[
\text{result:} \quad 1 \times 1
\]

**Diagnostics**

\[
\text{trace}(A) \text{ aborts with error if } A \text{ is not square.}
\]

\[
\text{trace}(A, B) \text{ and } \text{trace}(A, B, t) \text{ abort with error if the matrices are not conformable or their product is not square.}
\]

The trace of a \(0 \times 0\) matrix is 0.
Also see