### Description

\( \sin(Z) \), \( \cos(Z) \), and \( \tan(Z) \) return the appropriate trigonometric functions. Angles are measured in radians. All return real if the argument is real and complex if the argument is complex.

\[ \sin(x) \text{, } x \text{ real, returns the sine of } x. \text{ } \sin() \text{ returns a value between } -1 \text{ and } 1. \]

\[ \sin(z) \text{, } z \text{ complex, returns the complex sine of } z, \text{ mathematically defined as } \{ \exp(i \times z) - \exp(-i \times z) \}/2i. \]

\[ \cos(x) \text{, } x \text{ real, returns the cosine of } x. \cos() \text{ returns a value between } -1 \text{ and } 1. \]

\[ \cos(z) \text{, } z \text{ complex, returns the complex cosine of } z, \text{ mathematically defined as } \{ \exp(i \times z) + \exp(-i \times z) \}/2. \]

\[ \tan(x) \text{, } x \text{ real, returns the tangent of } x. \]

\[ \tan(z) \text{, } z \text{ complex, returns the complex tangent of } z, \text{ mathematically defined as } \sin(z)/\cos(z). \]

\( \text{asin}(Z) \), \( \text{acos}(Z) \), and \( \text{atan}(Z) \) return the appropriate inverse trigonometric functions. Returned results are in radians. All return real if the argument is real and complex if the argument is complex.

\[ \text{asin}(x) \text{, } x \text{ real, returns arcsine in the range } [-\pi/2, \pi/2]. \text{ If } x < -1 \text{ or } x > 1, \text{ missing (.) is returned.} \]

\[ \text{asin}(z) \text{, } z \text{ complex, returns the complex arcsine, mathematically defined as } -i \times \ln\{i \times z + \sqrt{1 - z \times z}\}. \text{ Re(} \text{asin}(\) \text{)} \text{ is chosen to be in the interval } [-\pi/2, \pi/2]. \]

\[ \text{acos}(x) \text{, } x \text{ real, returns arccosine in the range } [0, \pi]. \text{ If } x < -1 \text{ or } x > 1, \text{ missing (.) is returned.} \]

\[ \text{acos}(z) \text{, } z \text{ complex, returns the complex arccosine, mathematically defined as } -i \times \ln\{z + \sqrt{z \times z - 1}\}. \text{ Re(} \text{acos}(\) \text{)} \text{ is chosen to be in the interval } [0, \pi]. \]

\[ \text{atan}(x) \text{, } x \text{ real, returns arctangent in the range } (-\pi/2, \pi/2). \]

\[ \text{atan}(z) \text{, } z \text{ complex, returns the complex arctangent, mathematically defined as } \ln\{(1 + iz)/(1 - iz)\}/(2i). \text{ Re(} \text{atan}(\) \text{)} \text{ is chosen to be in the interval } [0, \pi]. \]

\( \text{atan2}(X, Y) \) returns the radian value in the range \((-\pi, \pi]\) of the angle of the vector determined by \((X,Y)\), the result being in the range \([0, \pi]\) for quadrants 1 and 2 and \([0, -\pi]\) for quadrants 4 and 3. \( X \) and \( Y \) must be real. \( \text{atan2}(X, Y) \) is equivalent to \( \text{arg}(C(X, Y)) \).

\( \text{arg}(Z) \) returns the arctangent of \( \text{Im}(Z)/\text{Re}(Z) \) in the correct quadrant, the result being in the range \((-\pi, \pi]; [0, \pi]\) in quadrants 1 and 2 and \([0, -\pi]\) in quadrants 4 and 3. \( \text{arg}(Z) \) is equivalent to \( \text{atan2}(\text{Re}(Z), \text{Im}(Z)) \).

\( \sinh(Z) \), \( \cosh(Z) \), and \( \tanh(Z) \) return the hyperbolic sine, cosine, and tangent, respectively. The returned value is real if the argument is real and complex if the argument is complex.
\[ \sinh(x), \ x \text{ real}, \] returns the inverse hyperbolic sine of \( x \), mathematically defined as \( \frac{\exp(x) - \exp(-x)}{2} \).

\[ \sinh(z), \ z \text{ complex}, \] returns the complex hyperbolic sine of \( z \), mathematically defined as \( \frac{\exp(z) - \exp(-z)}{2} \).

\[ \cosh(x), \ x \text{ real}, \] returns the inverse hyperbolic cosine of \( x \), mathematically defined as \( \frac{\exp(x) + \exp(-x)}{2} \).

\[ \cosh(z), \ z \text{ complex}, \] returns the complex hyperbolic cosine of \( z \), mathematically defined as \( \frac{\exp(z) + \exp(-z)}{2} \).

\[ \tanh(x), \ x \text{ real}, \] returns the inverse hyperbolic tangent of \( x \), mathematically defined as \( \frac{\sinh(x)}{\cosh(x)} \).

\[ \tanh(z), \ z \text{ complex}, \] returns the complex hyperbolic tangent of \( z \), mathematically defined as \( \frac{\sinh(z)}{\cosh(z)} \).

\[ \text{asinh}(Z), \ \text{acosh}(Z), \ \text{and} \ \text{atanh}(Z) \] return the inverse hyperbolic sine, cosine, and tangent, respectively. The returned value is real if the argument is real and complex if the argument is complex.

\[ \text{asinh}(x), \ x \text{ real}, \] returns the inverse hyperbolic sine.

\[ \text{asinh}(z), \ z \text{ complex}, \] returns the complex inverse hyperbolic sine, mathematically defined as \( \ln\{z + \sqrt{z \cdot z + 1}\} \). \( \text{Im(asinh())} \) is chosen to be in the interval \( [-\pi/2, \pi/2] \).

\[ \text{acosh}(x), \ x \text{ real}, \] returns the inverse hyperbolic cosine. If \( x < 1 \), missing (.) is returned.

\[ \text{acosh}(z), \ z \text{ complex}, \] returns the complex inverse hyperbolic cosine, mathematically defined as \( \ln\{z + \sqrt{z \cdot z - 1}\} \). \( \text{Im(acosh())} \) is chosen to be in the interval \( [-\pi, \pi] \); \( \text{Re(acosh())} \) is chosen to be nonnegative.

\[ \text{atanh}(x), \ x \text{ real}, \] returns the inverse hyperbolic tangent. If \( |x| > 1 \), missing (.) is returned.

\[ \text{atanh}(z), \ z \text{ complex}, \] returns the complex inverse hyperbolic tangent, mathematically defined as \( \ln\{(1 + z)/(1 - z)\}/2 \). \( \text{Im(atanh())} \) is chosen to be in the interval \( [-\pi/2, \pi/2] \).

\( \pi() \) returns the value of \( \pi \).
Syntax

\[
\begin{align*}
\text{numeric matrix} & \quad \sin(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \cos(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \tan(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \text{asin}(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \text{acos}(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \text{atan}(\text{numeric matrix } Z) \\
\text{real matrix} & \quad \text{atan2}(\text{real matrix } X, \text{real matrix } Y) \\
\text{real matrix} & \quad \text{arg}(\text{complex matrix } Z) \\
\text{numeric matrix} & \quad \sinh(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \cosh(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \tanh(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \text{asinh}(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \text{acosh}(\text{numeric matrix } Z) \\
\text{numeric matrix} & \quad \text{atanh}(\text{numeric matrix } Z) \\
\text{real scalar} & \quad \text{pi}() \\
\end{align*}
\]

Conformability

\[\text{atan2}(X, Y):
\begin{align*}
X: & \quad r_1 \times c_1 \\
Y: & \quad r_2 \times c_2, \quad X \text{ and } Y \text{ r-conformable} \\
result: & \quad \max(r_1, r_2) \times \max(c_1, c_2)
\end{align*}\]

\text{pi}() \text{ returns a } 1 \times 1 \text{ scalar.}

All other functions return a matrix of the same dimension as input containing element-by-element calculated results.

Diagnostics

All functions return missing for real arguments when the result would be complex. For instance, \(\text{acos}(2) = .,\) whereas \(\text{acos}(2+0i) = -1.317i.\)

Also see

[M-4] scalar — Scalar mathematical functions