

**qrsolve()** — Solve  $AX=B$  for  $X$  using QR decomposition

Description  
Diagnostics

Syntax  
Also see

Remarks and examples

Conformability

## Description

`qrsolve(A, B, ...)` uses QR decomposition to solve  $AX = B$  and returns  $X$ . When  $A$  is singular or nonsquare, `qrsolve()` computes a least-squares generalized solution. When *rank* is specified, in it is placed the rank of  $A$ .

`_qrsolve(A, B, ...)`, does the same thing, except that it destroys the contents of  $A$  and it overwrites  $B$  with the solution. Returned is the rank of  $A$ .

In both cases, *tol* specifies the tolerance for determining whether  $A$  is of full rank. *tol* is interpreted in the standard way—as a multiplier for the default if  $tol > 0$  is specified and as an absolute quantity to use in place of the default if  $tol \leq 0$  is specified; see [\[M-1\] tolerance](#).

## Syntax

*numeric matrix*    `qrsolve(A, B)`  
*numeric matrix*    `qrsolve(A, B, rank)`  
*numeric matrix*    `qrsolve(A, B, rank, tol)`  
  
*real scalar*        `_qrsolve(A, B)`  
*real scalar*        `_qrsolve(A, B, tol)`

where

*A*:    *numeric matrix*  
*B*:    *numeric matrix*  
*rank*: irrelevant; *real scalar* returned  
*tol*:    *real scalar*

## Remarks and examples

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`qrsolve(A, B, ...)` is suitable for use with square and possibly rank-deficient matrix  $A$ , or when  $A$  has more rows than columns. When  $A$  is square and full rank, `qrsolve()` returns the same solution as `lusolve()` (see [\[M-5\] lusolve\(\)](#)), up to roundoff error. When  $A$  is singular, `qrsolve()` returns a generalized (least-squares) solution.

Remarks are presented under the following headings:

[Derivation](#)  
[Relationship to inversion](#)  
[Tolerance](#)

## Derivation

We wish to solve for  $X$

$$AX = B \tag{1}$$

Perform QR decomposition on  $A$  so that we have  $A = QRP'$ . Then (1) can be rewritten as

$$QRP'X = B$$

Premultiplying by  $Q'$  and remembering that  $Q'Q = QQ' = I$ , we have

$$RP'X = Q'B \tag{2}$$

Define

$$Z = P'X \tag{3}$$

Then (2) can be rewritten as

$$RZ = Q'B \tag{4}$$

It is easy to solve (4) for  $Z$  because  $R$  is upper triangular. Having  $Z$ , we can obtain  $X$  via (3), because  $Z = P'X$ , premultiplied by  $P$  (and if we remember that  $PP' = I$ ), yields

$$X = PZ$$

For more information on QR decomposition, see [M-5] `qrd()`.

## Relationship to inversion

For a general discussion, see *Relationship to inversion* in [M-5] `lusolve()`.

For an inverse based on QR decomposition, see [M-5] `qrinv()`. `qrinv(A)` amounts to `qrsolve(A, I(rows(A)))`, although it is not actually implemented that way.

## Tolerance

The default tolerance used is

$$eta = 1e-13 * trace(abs(R))/rows(R)$$

where  $R$  is the upper-triangular matrix of the QR decomposition; see *Derivation* above. When  $A$  is less than full rank, by, say,  $d$  degrees of freedom, then  $R$  is also rank deficient by  $d$  degrees of freedom and the bottom  $d$  rows of  $R$  are essentially zero. If the  $i$ th diagonal element of  $R$  is less than or equal to  $eta$ , then the  $i$ th row of  $Z$  is set to zero. Thus if the matrix is singular, `qrsolve()` provides a generalized solution.

If you specify  $tol > 0$ , the value you specify is used to multiply  $eta$ . You may instead specify  $tol \leq 0$ , and then the negative of the value you specify is used in place of  $eta$ ; see [M-1] `tolerance`.

## Conformability

`qrsolve(A, B, rank, tol)`:

*input:*

*A:*  $m \times n$ ,  $m \geq n$   
*B:*  $m \times k$   
*tol:*  $1 \times 1$  (optional)

*output:*

*rank:*  $1 \times 1$  (optional)  
*result:*  $n \times k$

`_qrsolve(A, B, tol)`:

*input:*

*A:*  $m \times n$ ,  $m \geq n$   
*B:*  $m \times k$   
*tol:*  $1 \times 1$  (optional)

*output:*

*A:*  $0 \times 0$   
*B:*  $n \times k$   
*result:*  $1 \times 1$

## Diagnostics

`qrsolve(A, B, ...)` and `_qrsolve(A, B, ...)` return a result containing missing if  $A$  or  $B$  contain missing values.

`_qrsolve(A, B, ...)` aborts with error if  $A$  or  $B$  are views.

## Also see

[M-5] [qrinv\(\)](#) — Generalized inverse of matrix via QR decomposition

[M-5] [qrd\(\)](#) — QR decomposition

[M-5] [solvelower\(\)](#) — Solve  $AX=B$  for  $X$ ,  $A$  triangular

[M-5] [cholsolve\(\)](#) — Solve  $AX=B$  for  $X$  using Cholesky decomposition

[M-5] [lusolve\(\)](#) — Solve  $AX=B$  for  $X$  using LU decomposition

[M-5] [svsolve\(\)](#) — Solve  $AX=B$  for  $X$  using singular value decomposition

[M-5] [solve\\_tol\(\)](#) — Tolerance used by solvers and inverters

[M-4] [matrix](#) — Matrix functions

[M-4] [solvers](#) — Functions to solve  $AX=B$  and to obtain  $A$  inverse