qrinv( ) — Generalized inverse of matrix via QR decomposition

Description

qrinv(A, ...) returns the inverse or generalized inverse of real or complex matrix \( A: m \times n, m \geq n \). If optional argument \( \text{rank} \) is specified, the rank of \( A \) is returned there.

\_qrinv(A, ...) does the same thing except that, rather than returning the result, it overwrites the original matrix \( A \) with the result. \_qrinv() returns the rank of \( A \).

In both cases, optional argument \( \text{tol} \) specifies the tolerance for determining singularity; see Remarks and examples below.

Syntax

\[ \text{numeric matrix} \quad \text{qrinv(numeric matrix } A) \]

\[ \text{numeric matrix} \quad \text{qrinv(numeric matrix } A, \text{ rank}) \]

\[ \text{numeric matrix} \quad \text{qrinv(numeric matrix } A, \text{ rank, real scalar } \text{tol}) \]

\[ \text{real scalar} \quad \_\text{qrinv(numeric matrix } A) \]

\[ \text{real scalar} \quad \_\text{qrinv(numeric matrix } A, \text{ real scalar } \text{tol}) \]

where the type of \( \text{rank} \) is irrelevant; the rank of \( A \) is returned there.

Remarks and examples

qrinv() and \_qrinv() are most often used on square and possibly rank-deficient matrices but may be used on nonsquare matrices that have more rows than columns. Also see [M-5] pinv() for an alternative. See [M-5] luinv() for a more efficient way to obtain the inverse of full-rank, square matrices, and see [M-5] invsym() for inversion of real, symmetric matrices.

When \( A \) is of full rank, the inverse calculated by qrinv() is essentially the same as that computed by the faster luinv(). When \( A \) is singular, qrinv() and \_qrinv() compute a generalized inverse, \( A^* \), which satisfies

\[ A(A^*)A = A \]

\[ (A^*)A(A^*) = A^* \]

This generalized inverse is also calculated for nonsquare matrices that have more rows than columns and, then returned is a least-squares solution. If \( A \) is \( m \times n, m \geq n \), and if the rank of \( A \) is equal to \( n \), then \((A^*)A = I\), ignoring roundoff error.

qrinv(A) is implemented as qrsolve(A, I(rows(A))); see [M-5] qrsolve() for details and for use of the optional \text{tol} argument.
Conformability

\texttt{qrinv}(A, \textit{rank}, \textit{tol}): \\
input: \\
A: \ m \times n, \ m \geq n \\
\textit{tol}: \ 1 \times 1 \ (optional) \\
output: \\
\textit{rank}: \ 1 \times 1 \ (optional) \\
result: \ n \times m \\

\texttt{qrinv}(A, \textit{tol}): \\
input: \\
A: \ m \times n, \ m \geq n \\
\textit{tol}: \ 1 \times 1 \ (optional) \\
output: \\
A: \ n \times m \\
result: \ 1 \times 1 \ (containing \ rank) \\

Diagnostics

The inverse returned by these functions is real if \( A \) is real and is complex if \( A \) is complex.

\texttt{qrinv}(A, \ldots) \ and \ \texttt{qrinv}(A, \ldots) \ return \ a \ result \ containing \ missing \ values \ if \ A \ contains \ missing \ values.

\texttt{qrinv}(A, \ldots) \ aborts \ with \ error \ if \ A \ is \ a \ view.

See \[M-5\] \texttt{qrsolve()} \ and \ [M-1] \textbf{Tolerance} \ for \ information \ on \ the \ optional \ \textit{tol} \ argument.

Also see

[M-5] \texttt{cholinv()} — Symmetric, positive-definite matrix inversion

[M-5] \texttt{invsym()} — Symmetric real matrix inversion

[M-5] \texttt{luinv()} — Square matrix inversion

[M-5] \texttt{pinv()} — Moore–Penrose pseudoinverse

[M-5] \texttt{qrsolve()} — Solve AX=B for X using QR decomposition

[M-5] \texttt{solve_tol()} — Tolerance used by solvers and inverters

[M-4] \textbf{Matrix} — Matrix functions

[M-4] \textbf{Solvers} — Functions to solve AX=B and to obtain A inverse