qrinv() — Generalized inverse of matrix via QR decomposition

Description

qrinv(A, ...) returns the inverse or generalized inverse of real or complex matrix A: \( m \times n \), \( m \geq n \). If optional argument rank is specified, the rank of A is returned there.

_qrinv(A, ...) does the same thing except that, rather than returning the result, it overwrites the original matrix A with the result. _qrinv() returns the rank of A.

In both cases, optional argument tol specifies the tolerance for determining singularity; see Remarks and examples below.

Syntax

```
numeric matrix  qrinv(numeric matrix A)
numeric matrix  qrinv(numeric matrix A, rank)
numeric matrix  qrinv(numeric matrix A, rank, real scalar tol)
real scalar     _qrinv(numeric matrix A)
real scalar     _qrinv(numeric matrix A, real scalar tol)
```

where the type of rank is irrelevant; the rank of A is returned there.

Remarks and examples

qrinv() and _qrinv() are most often used on square and possibly rank-deficient matrices but may be used on nonsquare matrices that have more rows than columns. Also see [M-5] pinv() for an alternative. See [M-5] luinv() for a more efficient way to obtain the inverse of full-rank, square matrices, and see [M-5] invsym() for inversion of real, symmetric matrices.

When A is of full rank, the inverse calculated by qrinv() is essentially the same as that computed by the faster luinv(). When A is singular, qrinv() and _qrinv() compute a generalized inverse, \( A^* \), which satisfies

\[
A(A^*)A = A \\
(A^*)A(A^*) = A^*
\]

This generalized inverse is also calculated for nonsquare matrices that have more rows than columns and, then returned is a least-squares solution. If A is \( m \times n \), \( m \geq n \), and if the rank of A is equal to \( n \), then \( (A^*)A = I \), ignoring roundoff error.

qrinv(A) is implemented as qrsolve(A, I(rows(A))); see [M-5] qrsolve() for details and for use of the optional tol argument.
Conformability

qrinv(A, rank, tol):

input:
- $A$: $m \times n$, $m \geq n$
- $tol$: $1 \times 1$ (optional)

output:
- $rank$: $1 \times 1$ (optional)
- $result$: $n \times m$

_qrinv(A, tol):

input:
- $A$: $m \times n$, $m \geq n$
- $tol$: $1 \times 1$ (optional)

output:
- $A$: $n \times m$
- $result$: $1 \times 1$ (containing rank)

Diagnostics

The inverse returned by these functions is real if $A$ is real and is complex if $A$ is complex.

qrinv(A, ...) and _qrinv(A, ...) return a result containing missing values if A contains missing values.

_qrinv(A, ...) aborts with error if $A$ is a view.


Also see

[M-5] cholinv() — Symmetric, positive-definite matrix inversion
[M-5] invsym() — Symmetric real matrix inversion
[M-5] luinv() — Square matrix inversion
[M-5] pinv() — Moore–Penrose pseudoinverse
[M-5] qr.solve() — Solve AX=B for X using QR decomposition
[M-5] solve_tol() — Tolerance used by solvers and inverters
[M-4] Solvers — Functions to solve AX=B and to obtain A inverse