**Description**

`polyeval(c, x)` evaluates polynomial `c` at each value recorded in `x`, returning the results in a p-conformable-with-`x` vector. For instance, `polyeval((4,2,1), (3\5))` returns \(4+2*3+3^2 \ 4+2*5+5^2) = (19\39)\).

`polysolve(y, x)` returns the minimal-degree polynomial `c` fitting \(y = \text{polyeval}(c, x)\). Solution is via Lagrange’s interpolation formula.

`polytrim(c)` returns polynomial `c` with trailing zeros removed. For instance, `polytrim((1,2,3,0))` returns \((1,2,3)\). Thus if \(n = \text{cols(polytrim}(c))\), then \(c\) records an \((n-1)\)th degree polynomial.

`polyderiv(c, i)` returns the polynomial that is the \(i\)th derivative of polynomial `c`. For instance, `polyderiv((4,2,1), 1)` returns \((2,2)\) (the derivative of \(4+2x+x^2\) is \(2+2x\)). The value of the first derivative of polynomial `c` at `x` is `polyeval(polyderiv(c,1), x)`.

`polyinteg(c, i)` returns the polynomial that is the \(i\)th integral of polynomial `c`. For instance, `polyinteg((4,2,1), 1)` returns \((0,4,1,.3333)\) (the integral of \(4+2x+x^2\) is \(0+4x+x^2+.3333x^3\)). The value of the integral of polynomial `c` at `x` is `polyeval(polyinteg(c,1), x)`.

`polyadd(c1, c2)` returns the polynomial that is the sum of the polynomials `c1` and `c2`. For instance, `polyadd((2,1), (3,5,1))` is \((5,6,1)\) (the sum of \(2+x\) and \(3+5x+x^2\) is \(5+6x+x^2\)).

`polymult(c1, c2)` returns the polynomial that is the product of the polynomials `c1` and `c2`. For instance, `polymult((2,1), (3,5,1))` is \((6,13,7,1)\) (the product of \(2+x\) and \(3+5x+x^2\) is \(6+13x+7x^2+x^3\)).

`polydiv(c1, c2, c_q, c_r)` calculates polynomial \(c_1/c_2\), storing the quotient polynomial in `c_q` and the remainder polynomial in `c_r`. For instance, `polydiv((3,5,1), (2,1), c_q, c_r)` returns \(c_q=(3,1)\) and \(c_r=(-3)\); that is, \[
\frac{3 + 5x + x^2}{2 + x} = 3 + x \text{ with a remainder of } -3
\]
or \[
3 + 5x + x^2 = (3 + x)(2 + x) - 3
\]

`polyroots(c)` find the roots of polynomial `c` and returns them in complex row vector (complex even if `c` is real). For instance, `polyroots((3,5,1))` returns \((-4.303+0i, -.697+0i)\) (the roots of \(3+5x+x^2\) are \(-4.303\) and \(-.697\)).
Syntax

\[ \text{numeric vector} \quad \text{polyeval(numeric rowvector } c, \text{ numeric vector } x) \]
\[ \text{numeric rowvector} \quad \text{polysolve(numeric vector } y, \text{ numeric vector } x) \]
\[ \text{numeric rowvector} \quad \text{polytrim(numeric vector } c) \]
\[ \text{numeric rowvector} \quad \text{polyderiv(numeric rowvector } c, \text{ real scalar } i) \]
\[ \text{numeric rowvector} \quad \text{polyinteg(numeric rowvector } c, \text{ real scalar } i) \]
\[ \text{numeric rowvector} \quad \text{polyadd(numeric rowvector } c_1, \text{ numeric rowvector } c_2) \]
\[ \text{numeric rowvector} \quad \text{polymult(numeric rowvector } c_1, \text{ numeric rowvector } c_2) \]
\[ \text{void} \quad \text{polydiv(numeric rowvector } c_1, \text{ numeric rowvector } c_2, c_q, c_r) \]
\[ \text{complex rowvector} \quad \text{polyroots(numeric rowvector } c) \]

In the above, row vector \( c \) contains the coefficients for a \( \text{cols}(c) - 1 \) degree polynomial. For instance,

\[ c = (4, 2, 1) \]

records the polynomial

\[ 4 + 2x + x^2 \]

Remarks and examples

Given the real or complex coefficients \( c \) that define an \( n - 1 \) degree polynomial in \( x \), \text{polyroots}(c) returns the \( n - 1 \) roots for which

\[ 0 = c_1 + c_2x^1 + c_3x^2 + \cdots + c_nx^{n-1} \]

\text{polyroots}(c) obtains the roots by calculating the eigenvalues of the companion matrix. The \( (n-1) \times (n-1) \) companion matrix for the polynomial defined by \( c \) is

\[
C = \begin{bmatrix}
-s & -c_2 & \cdots & -c_{n-1} & -c_1 \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0 \\
0 & 0 & \cdots & 0 & 1 \\
\end{bmatrix}
\]

where \( s = 1/c_n \) if \( c \) is real and

\[ s = C \left( \frac{\text{Re}(c_n)}{\text{Re}(c_n)^2 + \text{Im}(c_n)^2}, \frac{-\text{Im}(c_n)}{\text{Re}(c_n)^2 + \text{Im}(c_n)^2} \right) \]

otherwise.
As in all nonsymmetric eigenvalue problems, the returned roots are complex and sorted from largest to smallest, see [M-5] \texttt{eigensystem()}.

### Conformability

\texttt{polyeval}(c, x):
\[
c: \quad 1 \times n, \ n > 0
\]
\[
x: \quad r \times 1 \ \text{or} \ 1 \times c
\]
\[
result: \quad r \times 1 \ \text{or} \ 1 \times c
\]

\texttt{polysolve}(y, x):
\[
y: \quad n \times 1 \ \text{or} \ 1 \times n, \ n \geq 1
\]
\[
x: \quad n \times 1 \ \text{or} \ 1 \times n
\]
\[
result: \quad 1 \times k, \ 1 \leq k \leq n
\]

\texttt{polytrim}(c):
\[
c: \quad 1 \times n
\]
\[
result: \quad 1 \times k, \ 1 \leq k \leq n
\]

\texttt{polyderiv}(c, i):
\[
c: \quad 1 \times n, \ \ n > 0
\]
\[
i: \quad 1 \times 1, \ \ i \ \text{may be negative}
\]
\[
result: \quad 1 \times \max(1, n - i)
\]

\texttt{polyinteg}(c, i):
\[
c: \quad 1 \times n, \ n > 0
\]
\[
i: \quad 1 \times 1, \ \ i \ \text{may be negative}
\]
\[
result: \quad 1 \times \max(1, n + i)
\]

\texttt{polyadd}(c_1, c_2):
\[
c_1: \quad 1 \times n_1, \ n_1 > 0
\]
\[
c_2: \quad 1 \times n_2, \ n_2 > 0
\]
\[
result: \quad 1 \times \max(n_1, n_2)
\]

\texttt{polymult}(c_1, c_2):
\[
c_1: \quad 1 \times n_1, \ n_1 > 0
\]
\[
c_2: \quad 1 \times n_2, \ n_2 > 0
\]
\[
result: \quad 1 \times n_1 + n_2 - 1
\]

\texttt{polydiv}(c_1, c_2, c_q, c_r):
\[
input:
\]
\[
c_1: \quad 1 \times n_1, \ n_1 > 0
\]
\[
c_2: \quad 1 \times n_2, \ n_2 > 0
\]
\[
output:
\]
\[
c_q: \quad 1 \times k_1, \ 1 \leq k_1 \leq \max(n_1 - n_2 + 1, 1)
\]
\[
c_r: \quad 1 \times k_2, \ 1 \leq k_2 \leq \max(n_1 - n_2, 1)
\]

\texttt{polyroots}(c):
\[
c: \quad 1 \times n, \ n > 0
\]
\[
result: \quad 1 \times k - 1, \ k = \text{cols}(\text{polytrim}(c))
Diagnostics

All functions abort with an error if a polynomial coefficient row vector is void, but they do not necessarily give indicative error messages as to the problem. Polynomial coefficient row vectors may contain missing values.

\[ \text{polyderiv}(c, i) \] returns \( c \) when \( i = 0 \). It returns \( \text{polyinteg}(c, -i) \) when \( i < 0 \). It returns \( 0 \) when \( i \) is missing (think of missing as positive infinity).

\[ \text{polyinteg}(c, i) \] returns \( c \) when \( i = 0 \). It returns \( \text{polyderiv}(c, -i) \) when \( i < 0 \). It aborts with error if \( i \) is missing (think of missing as positive infinity).

\[ \text{polyroots}(c) \] returns a vector of missing values if any element of \( c \) equals missing.

Also see

[M-4] Mathematical — Important mathematical functions