polyeval() — Manipulate and evaluate polynomials

Description

polyeval(c, x) evaluates polynomial c at each value recorded in x, returning the results in a p-conformable-with-x vector. For instance, polyeval((4,2,1), (3\,5)) returns (4+2*3+3^2 \ 4+2*5+5^2) = (19\,39).

polysolve(y, x) returns the minimal-degree polynomial c fitting y = polyeval(c, x). Solution is via Lagrange’s interpolation formula.

polytrim(c) returns polynomial c with trailing zeros removed. For instance, polytrim((1,2,3,0)) returns (1,2,3). polytrim((0,0,0,0)) returns (0). Thus if n = cols(polytrim(c)), then c records an (n − 1)th degree polynomial.

polyderiv(c, i) returns the polynomial that is the i th derivative of polynomial c. For instance, polyderiv((4,2,1), 1) returns (2,2) (the derivative of 4 + 2x + x^2 is 2 + 2x). The value of the first derivative of polynomial c at x is polyeval(polyderiv(c,1), x).

polyinteg(c, i) returns the polynomial that is the i th integral of polynomial c. For instance, polyinteg((4,2,1), 1) returns (0,4,1,.3333) (the integral of 4 + 2x + x^2 is 0 + 4x + x^2 + .3333x^3). The value of the integral of polynomial c at x is polyeval(polyinteg(c,1), x).

polyadd(c_1, c_2) returns the polynomial that is the sum of the polynomials c_1 and c_2. For instance, polyadd((2,1), (3,5,1)) is (5,6,1) (the sum of 2 + x and 3 + 5x + x^2 is 5 + 6x + x^2).

polymult(c_1, c_2) returns the polynomial that is the product of the polynomials c_1 and c_2. For instance, polymult((2,1), (3,5,1)) is (6,13,7,1) (the product of 2 + x and 3 + 5x + x^2 is 6 + 13x + 7x^2 + x^3).

polydiv(c_1, c_2, c_q, c_r) calculates polynomial c_1/c_2, storing the quotient polynomial in c_q and the remainder polynomial in c_r. For instance, polydiv((3,5,1), (2,1), c_q, c_r) returns c_q=(3,1) and c_r=(-3); that is, \[ \frac{3 + 5x + x^2}{2 + x} = 3 + x \text{ with a remainder of } -3 \]
or \[ 3 + 5x + x^2 = (3 + x)(2 + x) - 3 \]

polyroots(c) find the roots of polynomial c and returns them in complex row vector (complex even if c is real). For instance, polyroots((3,5,1)) returns (-4.303+0i, -.697+0i) (the roots of 3 + 5x + x^2 are −4.303 and −.697).
Syntax

numeric vector  polyeval(numeric rowvector c, numeric vector x)
numeric rowvector  polysolve(numeric vector y, numeric vector x)
numeric rowvector  polytrim(numeric vector c)
numeric rowvector  polyderiv(numeric rowvector c, real scalar i)
numeric rowvector  polyinteg(numeric rowvector c, real scalar i)
numeric rowvector  polyadd(numeric rowvector c1, numeric rowvector c2)
numeric rowvector  polymult(numeric rowvector c1, numeric rowvector c2)
void  polydiv(numeric rowvector c1, numeric rowvector c2, c q, c r)
complex rowvector  polyroots(numeric rowvector c)

In the above, row vector c contains the coefficients for a \( \text{cols}(c) - 1 \) degree polynomial. For instance,

\[ c = (4, 2, 1) \]

records the polynomial

\[ 4 + 2x + x^2 \]

Remarks and examples

Given the real or complex coefficients c that define an \( n - 1 \) degree polynomial in x, polyroots(c) returns the \( n - 1 \) roots for which

\[ 0 = c_1 + c_2x^1 + c_3x^2 + \cdots + c_nx^{n-1} \]

polyroots(c) obtains the roots by calculating the eigenvalues of the companion matrix. The \( (n-1) \times (n-1) \) companion matrix for the polynomial defined by c is

\[
C = \begin{bmatrix}
-c_{n-1}s & -c_{n-2}s & \cdots & -c_2s & -c_1s \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0 \\
0 & 0 & \cdots & 0 & 1 \\
\end{bmatrix}
\]

where \( s = 1/c_n \) if c is real and

\[
s = C \left( \frac{\text{Re}(c_n)}{\text{Re}(c_n)^2 + \text{Im}(c_n)^2}, \frac{-\text{Im}(c_n)}{\text{Re}(c_n)^2 + \text{Im}(c_n)^2} \right)
\]

otherwise.
As in all nonsymmetric eigenvalue problems, the returned roots are complex and sorted from largest to smallest, see [M-5] eigensystem().

Conformability

polyeval(c, x):
\[
c: \quad 1 \times n, \ n > 0 \\
x: \quad r \times l \text{ or } 1 \times c \\
result: \quad r \times l \text{ or } 1 \times c
\]

polysolve(y, x):
\[
y: \quad n \times l \text{ or } 1 \times n, \ n \geq 1 \\
x: \quad n \times l \text{ or } 1 \times n \\
result: \quad 1 \times k, \ 1 \leq k \leq n
\]

polytrim(c):
\[
c: \quad 1 \times n \\
result: \quad 1 \times k, \ 1 \leq k \leq n
\]

polyderiv(c, i):
\[
c: \quad 1 \times n, \ n > 0 \\
i: \quad 1 \times l, \ i \text{ may be negative} \\
result: \quad 1 \times \max(1, n - i)
\]

polyinteg(c, i):
\[
c: \quad 1 \times n, \ n > 0 \\
i: \quad 1 \times l, \ i \text{ may be negative} \\
result: \quad 1 \times \max(1, n + i)
\]

polyadd(c_1, c_2):
\[
c_1: \quad 1 \times n_1, \ n_1 > 0 \\
c_2: \quad 1 \times n_2, \ n_2 > 0 \\
result: \quad 1 \times \max(n_1, n_2)
\]

polymult(c_1, c_2):
\[
c_1: \quad 1 \times n_1, \ n_1 > 0 \\
c_2: \quad 1 \times n_2, \ n_2 > 0 \\
result: \quad 1 \times n_1 + n_2 - 1
\]

polydiv(c_1, c_2, c_q, c_r):
\[
input: \\
c_1: \quad 1 \times n_1, \ n_1 > 0 \\
c_2: \quad 1 \times n_2, \ n_2 > 0 \\
output: \\
c_q: \quad 1 \times k_1, \ 1 \leq k_1 \leq \max(n_1 - n_2 + 1, 1) \\
c_r: \quad 1 \times k_2, \ 1 \leq k_2 \leq \max(n_1 - n_2, 1)
\]

polyroots(c):
\[
c: \quad 1 \times n, \ n > 0 \\
result: \quad 1 \times k - 1, \ k = \text{cols(polytrim}(c))
\]
Diagnostics

All functions abort with an error if a polynomial coefficient row vector is void, but they do not necessarily give indicative error messages as to the problem. Polynomial coefficient row vectors may contain missing values.

\[ \text{polyderiv}(c, i) \] returns \( c \) when \( i = 0 \). It returns \( \text{polyinteg}(c, -i) \) when \( i < 0 \). It returns \( 0 \) when \( i \) is missing (think of missing as positive infinity).

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\[ \text{polyroots}(c) \] returns a vector of missing values if any element of \( c \) equals missing.

Also see

[M-4] mathematical — Important mathematical functions