polyeval() — Manipulate and evaluate polynomials

Description

polyeval(c, x) evaluates polynomial c at each value recorded in x, returning the results in a p-conformable-with-x vector. For instance, polyeval((4,2,1), (3\5)) returns (4+2*3+3^2 \ 4+2*5+5^2) = (19\39).

polysolve(y, x) returns the minimal-degree polynomial c fitting y = polyeval(c, x). Solution is via Lagrange's interpolation formula.

polytrim(c) returns polynomial c with trailing zeros removed. For instance, polytrim((1,2,3,0)) returns (1,2,3). polytrim((0,0,0,0)) returns (0). Thus if n = cols(polytrim(c)), then c records an $(n-1)$th degree polynomial.

polyderiv(c, i) returns the polynomial that is the ith derivative of polynomial c. For instance, polyderiv((4,2,1), 1) returns (2,2) (the derivative of $4+2x+x^2$ is $2+2x$). The value of the first derivative of polynomial c at x is polyeval(polyderiv(c,1),x).

polyinteg(c, i) returns the polynomial that is the ith integral of polynomial c. For instance, polyinteg((4,2,1), 1) returns (0,4,1,.3333) (the integral of $4+2x+x^2$ is $0+4x+x^2+.3333x^3$). The value of the integral of polynomial c at x is polyeval(polyinteg(c,1), x).

polyadd(c_1, c_2) returns the polynomial that is the sum of the polynomials c_1 and c_2. For instance, polyadd((2,1), (3,5,1)) is (5,6,1) (the sum of $2+x$ and $3+5x+x^2$ is $5+6x+x^2$).

polymult(c_1, c_2) returns the polynomial that is the product of the polynomials c_1 and c_2. For instance, polymult((2,1), (3,5,1)) is (6,13,7,1) (the product of $2+x$ and $3+5x+x^2$ is $6+13x+7x^2+x^3$).

polydiv(c_1, c_2, c_q, c_r) calculates polynomial c_1/c_2, storing the quotient polynomial in c_q and the remainder polynomial in c_r. For instance, polydiv((3,5,1), (2,1), c_q, c_r) returns c_q=(3,1) and c_r=(-3); that is,

$$\frac{3+5x+x^2}{2+x} = 3+x \text{ with a remainder of } -3$$

or

$$3+5x+x^2 = (3+x)(2+x)-3$$

polyroots(c) find the roots of polynomial c and returns them in complex row vector (complex even if c is real). For instance, polyroots((3,5,1)) returns (-4.303+0i, -.697+0i) (the roots of $3+5x+x^2$ are -4.303 and $-.697$).

Syntax

numeric vector	`polyeval(`*numeric rowvector c*`,` *numeric vector x*`)`
numeric rowvector	`polysolve(`*numeric vector y*`,` *numeric vector x*`)`
numeric rowvector	`polytrim(`*numeric vector c*`)`
numeric rowvector	`polyderiv(`*numeric rowvector c*`,` *real scalar i*`)`
numeric rowvector	`polyinteg(`*numeric rowvector c*`,` *real scalar i*`)`
numeric rowvector	`polyadd(`*numeric rowvector* c_1`,` *numeric rowvector* c_2`)`
numeric rowvector	`polymult(`*numeric rowvector* c_1`,` *numeric rowvector* c_2`)`
void	`polydiv(`*numeric rowvector* c_1`,` *numeric rowvector* c_2`,` c_q`,` c_r`)`
complex rowvector	`polyroots(`*numeric rowvector c*`)`

In the above, row vector c contains the coefficients for a `cols(`c`)` -1 degree polynomial. For instance,

$$c = (4, 2, 1)$$

records the polynomial

$$4 + 2x + x^2$$

Remarks and examples

stata.com

Given the real or complex coefficients c that define an $n-1$ degree polynomial in x, `polyroots(`c`)` returns the $n-1$ roots for which

$$0 = c_1 + c_2x^1 + c_3x^2 + \cdots + c_nx^{n-1}$$

`polyroots(`c`)` obtains the roots by calculating the eigenvalues of the companion matrix. The $(n-1) \times (n-1)$ companion matrix for the polynomial defined by c is

$$C = \begin{bmatrix} -c_{n-1}s & -c_{n-2}s & \cdots & -c_2s & -c_1s \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

where $s = 1/c_n$ if c is real and

$$s = C\left(\frac{\mathrm{Re}(c_n)}{\mathrm{Re}(c_n)^2 + \mathrm{Im}(c_n)^2}, \frac{-\mathrm{Im}(c_n)}{\mathrm{Re}(c_n)^2 + \mathrm{Im}(c_n)^2}\right)$$

otherwise.

As in all nonsymmetric eigenvalue problems, the returned roots are complex and sorted from largest to smallest, see [M-5] **eigensystem( )**.

Conformability

`polyeval(`c`,` x`)`:

c:	$1 \times n,\ n > 0$		
x:	$r \times 1$	or	$1 \times c$
result:	$r \times 1$	or	$1 \times c$

`polysolve(`y`,` x`)`:

y:	$n \times 1$	or	$1 \times n,\ n \geq 1$
x:	$n \times 1$	or	$1 \times n$
result:	$1 \times k,$		$1 \leq k \leq n$

`polytrim(`c`)`:

c:	$1 \times n$	
result:	$1 \times k,$	$1 \leq k \leq n$

`polyderiv(`c`,` i`)`:

c:	$1 \times n,$	$n > 0$
i:	$1 \times 1,$	i may be negative
result:	$1 \times \max(1, n - i)$	

`polyinteg(`c`,` i`)`:

c:	$1 \times n,\ n > 0$	
i:	$1 \times 1,$	i may be negative
result:	$1 \times \max(1, n + i)$	

`polyadd(`c_1`,` c_2`)`:

c_1:	$1 \times n_1,$	$n_1 > 0$
c_2:	$1 \times n_2,$	$n_2 > 0$
result:	$1 \times \max(n_1, \quad n_2)$	

`polymult(`c_1`,` c_2`)`:

c_1:	$1 \times n_1,$	$n_1 > 0$
c_2:	$1 \times n_2,$	$n_2 > 0$
result:	$1 \times n_1 + n_2 - 1$	

`polydiv(`c_1`,` c_2`,` c_q`,` c_r`)`:

input:		
c_1:	$1 \times n_1,$	$n_1 > 0$
c_2:	$1 \times n_2,$	$n_2 > 0$
output:		
c_q:	$1 \times k_1,$	$1 \leq k_1 \leq \max(n_1 - n_2 + 1, 1)$
c_r:	$1 \times k_2,$	$1 \leq k_2 \leq \max(n_1 - n_2, 1)$

`polyroots(`c`)`:

c:	$1 \times n_i,$	$n > 0$
result:	$1 \times k - 1,$	$k =$ `cols(polytrim(`c`))`

Diagnostics

All functions abort with an error if a polynomial coefficient row vector is void, but they do not necessarily give indicative error messages as to the problem. Polynomial coefficient row vectors may contain missing values.

`polyderiv(`c`, `i`)` returns c when $i = 0$. It returns `polyinteg(`c`, `$-i$`)` when $i < 0$. It returns `(0)` when i is missing (think of missing as positive infinity).

`polyinteg(`c`, `i`)` returns c when $i = 0$. It returns `polyderiv(`c`, `$-i$`)` when $i < 0$. It aborts with error if i is missing (think of missing as positive infinity).

`polyroots(`c`)` returns a vector of missing values if any element of c equals missing.

Also see

[M-4] **Mathematical** — Important mathematical functions