matexpym() — Exponentiation and logarithms of symmetric matrices

Description

matexpym(A) returns the matrix exponential of the symmetric (Hermitian) matrix A.
matlogsym(A) returns the matrix natural logarithm of the symmetric (Hermitian) matrix A.

_matexpym(A) and _matlogsym(A) do the same thing as matexpym() and matlogsym(), but instead of returning the result, they store the result in A.

Syntax

numeric matrix matexpym(numeric matrix A)
numeric matrix matlogsym(numeric matrix A)
void _matexpym(numeric matrix A)
void _matlogsym(numeric matrix A)

Remarks and examples

Do not confuse matexpym(A) with exp(A), nor matlogsym(A) with log(A).

matexpym(2*matlogsym(A)) produces the same result as A*A. exp() and log() return elementwise results.

Exponentiated results and logarithms are obtained by extracting the eigenvectors and eigenvalues of A, performing the operation on the eigenvalues, and then rebuilding the matrix. That is, first X and L are found such that

$$AX = X \times \text{diag}(L)$$  \hspace{1cm} (1)

For symmetric (Hermitian) matrix A, X is orthogonal, meaning $$X'X = XX' = I$$. Thus

$$A = X \times \text{diag}(L) \times X'$$  \hspace{1cm} (2)

matexpym(A) is then defined

$$A = X \times \text{diag}(\exp(L)) \times X'$$  \hspace{1cm} (3)

and matlogsym(A) is defined

$$A = X \times \text{diag}(\log(L)) \times X'$$  \hspace{1cm} (4)

(1) is obtained via symeigensystem(); see [M-5] eigensystem().
Conformability

\text{matexp}(A), \text{matlog}(A):
\begin{align*}
A &: \quad n \times n \\
\text{result} &: \quad n \times n
\end{align*}

\_\text{matexp}(A), \_\text{matlog}(A):
\begin{align*}
\text{input} \\
A &: \quad n \times n \\
\text{output} \\
A &: \quad n \times n
\end{align*}

Diagnostics

\text{matexp}(A), \text{matlog}(A), \_\text{matexp}(A), \text{and} \_\text{matlog}(A) \text{ return missing results if} A \text{ contains missing values.}

Also:

1. These functions do not check that \( A \) is symmetric or Hermitian. If \( A \) is a real matrix, only the lower triangle, including the diagonal, is used. If \( A \) is a complex matrix, only the lower triangle and the real parts of the diagonal elements are used.

2. These functions return a matrix of the same storage type as \( A \).

For \text{symlog}(A), \text{this means that if} A \text{ is real and the result cannot be expressed as a real, a matrix of missing values is returned. If you want the generalized solution, code matlogsym(C(A)). This is the same rule as with scalars: \log(-1) \text{ evaluates to missing, but} \log(C(-1)) \text{ is 3.14159265i.}

3. These functions are guaranteed to return a matrix that is numerically symmetric, Hermitian, or symmetric only if theory states that the matrix should be symmetric, Hermitian, or symmetric only. See [M-5] \text{matpowersym()} \text{ for a discussion of this issue.}

For the functions here, real function \exp(x) \text{ is defined for all real values of} x \text{ (ignoring overflow), and thus the matrix returned by matexp()} \text{ will be symmetric (Hermitian).}

The same is not true for \text{matlogsym}. \log(x) \text{ is not defined for} x = 0, \text{ so if any of the eigenvalues of} A \text{ are 0 or very small, a matrix of missing values will result. Also,} \log(x) \text{ is complex for} x < 0, \text{ and thus if any of the eigenvalues are negative, the resulting matrix will be (1) missing if} A \text{ is real stored as real, (2) symmetric only if} A \text{ contains reals stored as complex, and (3) general if} A \text{ is complex.}

Also see

[M-5] \text{eigensystem() — Eigenvectors and eigenvalues}

[M-5] \text{matpowersym() — Powers of a symmetric matrix}

[M-4] \text{Matrix — Matrix functions}