hessenbergd() — Hessenberg decomposition

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# Description

hessenbergd (A, H, Q) calculates the Hessenberg decomposition of a square, numeric matrix, A, returning the upper Hessenberg form matrix in H and the orthogonal (unitary) matrix in Q. Q is orthogonal if A is real and unitary if A is complex.

\_hessenbergd(A, Q) does the same as hessenbergd() except that it returns H in A.

\_hessenbergd\_la() is the interface to the LAPACK routines used to implement the above function; see [M-1] LAPACK. Its direct use is not recommended.

### **Syntax**

void hessenbergd(numeric matrix A, H, Q)
void \_hessenbergd(numeric matrix A, Q)

### **Remarks and examples**

The Hessenberg decomposition of a matrix, A, can be written as

$$\mathbf{Q}' \times \mathbf{A} \times \mathbf{Q} = \mathbf{H}$$

where **H** is upper Hessenberg; **Q** is orthogonal if **A** is real or unitary if **A** is complex.

A matrix **H** is in upper Hessenberg form if all entries below its first subdiagonal are zero. For example, a  $5 \times 5$  upper Hessenberg matrix looks like

	1	2	3	4	5	
1	x	x	x	x	x	
2	x	х	х	х	х	
3	0	х	х	х	х	
4	0	0	х	х	х	
5	0	0	0	х	х	

#### For instance,

: A												
	1	2	3	4	5							
1	3	2	1	-2	-5							
2	4	2	1	0	3							
3	4	4	0	1	-1							
4	5	6	7	-2	4							
5	6	7	1	2	-1							
hessenbergd(A, H=., Q=.)												
: H												
			1			2	3		4		5	
1			3	2 (	034647	745	- 552977683	-4 787	64119	-1 530	555451	
2	-9	643650	1761	7 9	3064516	313	2 878001755	5 10	85876	5 5804	122694	
3	0.1	0 1000	0	-3 4	1540238	379	-6 119229633	- 23472	00215	1 4679	32097	
4			Ő	0.	10 10 200	0	1,404136249	-1.7158	23624	98706	301994	
5			0			õ	0 0	-2 6681	28952	- 9713	398356	
Ŭ			•			•		2.0001				
· n												
. ų			1			2	3		4		5	
						-					-	
1			1			0	0		0		0	
2			0	4	1478067	779	0368006164	40477	68558	81409	997488	
3			0	4	1478067	779	4871239484	56923	09155	.5163	752637	
4			0	5	1847584	174	.8096135604	07484	49196	.2647	771074	
5			0	62	2217101	168	3253949238	.71170	92805	02216	645995	

Many algorithms use a Hessenberg decomposition in the process of finding another decomposition with more structure.

## Conformability

hessenbergd(A, H, Q): input: A:  $n \times n$ output: H:  $n \times n$ Q:  $n \times n$  $\_$ hessenbergd(A, Q): input: A:  $n \times n$ output: A:  $n \times n$ Q:  $n \times n$ 

## **Diagnostics**

\_hessenbergd() aborts with error if A is a view.

hessenbergd() and \_hessenbergd() return missing results if A contains missing values.

Karl Adolf Hessenberg (1904–1959) was born in Frankfurt am Main, Germany. He was an electrical engineer and gained degrees from the Technische Hochschule Darmstadt. His doctoral dissertation, approved in 1942, was on computation of the eigenvalues and eigensolutions of linear systems of equations. In concurrent work, he introduced what are now called Hessenberg matrices. The mathematician Gerhard Hessenberg was a near relative.

### Also see

- [M-1] LAPACK Linear algebra package (LAPACK) routines
- [M-5] schurd() Schur decomposition
- [M-4] Matrix Matrix functions

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