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halton( ) — Generate a Halton or Hammersley set

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Description

halton(n, d) returns an $n \times d$ matrix containing a Halton set of length n and dimension d.

halton(n, d, $start$) does the same thing, but the first row of the returned matrix contains the sequences starting at index $start$. The default is $start = 1$.

halton(n, d, $start$, $hammersley$), with $hammersley \neq 0$, returns a Hammersley set of length n and dimension d with the first row of the returned matrix containing the sequences starting at index $start$.

_halton(x) modifies the $n \times d$ matrix x so that it contains a Halton set of dimension d of length n.

_halton(x, $start$) does the same thing, but the first row of the returned matrix contains the sequences starting at index $start$. The default is $start = 1$.

_halton(x, $start$, $hammersley$), with $hammersley \neq 0$, returns a Hammersley set of length n and dimension d with the first row of the returned matrix containing the sequences starting at index $start$.

ghalton(n, $base$, u) returns an $n \times 1$ vector containing a (generalized) Halton sequence using base $base$ and starting from scalar $0 < u < 1$, or a nonnegative index $u = 0, 1, 2, \ldots$.

Syntax

real matrix	halton(*real scalar n*, *real scalar d*)
real matrix	halton(*real scalar n*, *real scalar d*, *real scalar start*)
real matrix	halton(*real scalar n*, *real scalar d*, *real scalar start*, *real scalar hammersley*)
void	_halton(*real matrix x*)
void	_halton(*real matrix x*, *real scalar start*)
void	_halton(*real matrix x*, *real scalar start*, *real scalar hammersley*)
real colvector	ghalton(*real scalar n*, *real scalar base*, *real scalar u*)

Remarks and examples

The Halton sequences are generated from the first d primes and generally have more uniform coverage over the unit cube of dimension d than that of sequences generated from pseudouniform random numbers. However, Halton sequences based on large primes ($d > 10$) can be highly correlated, and their coverage can be worse than that of the pseudorandom uniform sequences.

The Hammersley set contains the sequence $(2 \times i - 1)/(2 \times n)$, $i = 1, \ldots, n$, in the first dimension and Halton sequences for dimensions $2, \ldots, d$.

_halton() modifies x and can be used when repeated calls are made to generate long sequences in blocks. Here update the *start* index between calls by using $start = start + \mathtt{rows}(x)$.

ghalton() uses the base *base*, preferably a prime, and generates a Halton sequence using $0 < u < 1$ or a nonnegative index as the starting value. If u is uniform $(0, 1)$, the sequence is a randomized Halton sequence. If u is a nonnegative integer, the sequence is the standard Halton sequence starting at index $u + 1$.

Conformability

halton(*n*, *d*, *start*, *hammersley*):
   input:

n:	1×1	
d:	1×1	
start:	1×1	(optional)
hammersley:	1×1	(optional)

   output:

result:	$n \times d$

_halton(*x*, *start*, *hammersley*):
   input:

x:	$n \times d$	
start:	1×1	(optional)
hammersley:	1×1	(optional)

   output:

x:	$n \times d$

ghalton(*n*, *base*, *u*):
   input:

n:	1×1
base:	1×1
u:	1×1

   output:

result:	$n \times 1$

Diagnostics

The maximum dimension, d, is 20. The scalar index *start* must be a positive integer, and the scalar u must be such that $0 < u < 1$ or a nonnegative integer. For ghalton(), *base* must be an integer greater than or equal to 2.

John Henry Halton (1931–) was born in Brussels, Belgium. He studied mathematics and physics at Cambridge and then at Oxford, where he was a doctoral student of J. M. Hammersley. His career has included positions in government, industry, and academia in both Britain and the United States, latterly as a Professor of Computer Sciences at the University of Wisconsin in Madison and the University of North Carolina in Chapel Hill. Halton's work arises from problems in pure mathematics, theoretical physics, statistics, probability theory, and engineering, and has led to devising and rigorously analyzing algorithms of a combinatorial, probabilistic, and geometric nature. It encompasses tests for statistical relationships between random variables, the hydrodynamic theory of lubrication, probabilistic properties of the Traveling Salesman Problem, and most theoretical aspects of the Monte Carlo method, including the theory and use of both pseudorandom and quasirandom sequences and sets.

John Michael Hammersley (1920–2004) was born in Helensburgh, Dunbartonshire, Scotland. His mathematical studies at Cambridge were interspersed with military service in the Royal Artillery. Hammersley contributed to the use of radar in predicting the flight paths of enemy aircraft and was promoted to the rank of major. After graduation, he moved to Oxford where his research covered several areas of mathematics and statistics. Hammersley is best known for achievements in the study of spatial disorder, especially percolation models, and of stochastic processes generally. He was a pioneer in the use of Monte Carlo methods and was senior author of an important early monograph (Hammersley and Handscomb 1964). Hammersley contributed to reform of mathematical teaching in Britain, emphasizing the value of solving concrete problems. He was elected a Fellow of the Royal Society.

References

Grimmett, G., and D. Welsh. 2007. John Michael Hammersley, 21 March 1920–2 May 2004. *Biographical Memoirs of Fellows of the Royal Society* 53: 163–183. https://doi.org/10.1098/rsbm.2007.0001.

Hammersley, J. M., and D. C. Handscomb. 1964. *Monte Carlo Methods.* London: Methuen.

Kolenikov, S. 2012. Scrambled Halton sequences in Mata. *Stata Journal* 12: 29–44.

Also see

[M-4] **Mathematical** — Important mathematical functions