

Description

`gschurd(A, B, T, R, U, V, w, b)` computes the generalized Schur decomposition of two square, numeric matrices, *A* and *B*, and the [generalized eigenvalues](#). The decomposition is returned in the [Schur-form](#) matrix, *T*; the upper-triangular matrix, *R*; and the orthogonal (unitary) matrices, *U* and *V*. The generalized eigenvalues are returned in the complex vectors *w* and *b*.

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m)` computes the generalized Schur decomposition of two square, numeric matrices, *A* and *B*, and the [generalized eigenvalues](#), and groups the results according to whether a condition on each generalized eigenvalue is satisfied. *f* is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed [below](#). The number of generalized eigenvalues for which the condition is true is returned in *m*.

`_gschurd()` mirrors `gschurd()`, the difference being that it returns *T* in *A* and *R* in *B*.

`_gschurdgroupby()` mirrors `gschurdgroupby()`, the difference being that it returns *T* in *A* and *R* in *B*.

`_gschurd_la()` and `_gschurdgroupby_la()` are the interfaces into the LAPACK routines used to implement the above functions; see [\[M-1\] LAPACK](#). Their direct use is not recommended.

Syntax

void `gschurd(A, B, T, R, U, V, w, b)`

void `_gschurd(A, B, U, V, w, b)`

void `gschurdgroupby(A, B, f, T, R, U, V, w, b, m)`

void `_gschurdgroupby(A, B, f, U, V, w, b, m)`

Remarks and examples

Remarks are presented under the following headings:

Generalized Schur decomposition

Grouping the results

Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices, **A** and **B**, can be written as

$$\mathbf{U}' \times \mathbf{A} \times \mathbf{V} = \mathbf{T}$$

$$\mathbf{U}' \times \mathbf{B} \times \mathbf{V} = \mathbf{R}$$

where **T** is in Schur form, **R** is upper triangular, and **U** and **V** are orthogonal if **A** and **B** are real and are unitary if **A** or **B** is complex. The complex vectors **w** and **b** contain the generalized eigenvalues.

If **A** and **B** are real, **T** is in real Schur form and **R** is a real upper-triangular matrix. If **A** or **B** is complex, **T** is in complex Schur form and **R** is a complex upper-triangular matrix.

In the example below, we define A and B, obtain the generalized Schur decomposition, and list T and R.

```

: A = (6, 2, 8, -1\ -3, -4, -6, 4\ 0, 8, 4, 1\ -8, -7, -3, 5)
: B = (8, 0, -8, -1\ -6, -2, -6, -1\ -7, -6, 2, -6\ 1, -7, 9, 2)
: gschurd(A, B, T=., R=., U=., V=., w=., b=.)
: T

```

	1	2	3	4
1	12.99313938	1.746927947	3.931212285	-10.91622337
2	0	.014016016	6.153566902	1.908835695
3	0	-4.362999645	1.849905717	-2.998194791
4	0	0	0	-5.527285433

```

: R

```

	1	2	3	4
1	4.406836593	6.869534063	-1.840892081	1.740906311
2	0	13.88730687	0	-6.699556735
3	0	0	9.409495218	-4.659386723
4	0	0	0	9.453808732

```

: w

```

	1	2	3	4
1	12.9931394	.409611804+1.83488354i	.024799819- .111092453i	-5.52728543

```

: b

```

	1	2	3	4
1	4.406836593	4.145676341	.2509986829	9.453808732

Generalized eigenvalues can be obtained by typing

```

: w:/b

```

	1	2	3	4
1	2.94840508	.098804579+ .442601735i	.098804579- .442601735i	-5.84662287

Grouping the results

gschurdgroupby() reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair w and b. It also reorders the generalized Schur form T, R, and orthogonal (unitary) matrices, U and V, correspondingly.

We must pass gschurdgroupby() a pointer to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the following example, we use `gschurdgroupby()` to put the finite, real, generalized eigenvalues first. One of the arguments to `schurdgroupby()` is a pointer to the function `onlyreal()` which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. `onlyreal()` returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```

: real scalar onlyreal(complex scalar w, real scalar b)
> {
>     if(b==0) return(0)
>     if(Im(w/b)==0) return(1)
>     return(0)
> }
: gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)

```

We obtain

```

: T
      1          2          3          4
1  12.99313938   8.19798168   6.285710813   5.563547054
2           0  -5.952366071  -1.473533834   2.750066482
3           0           0   -2.2015830885   3.882051743
4           0           0   6.337230739   1.752690714

: R
      1          2          3          4
1  4.406836593   2.267479575  -6.745927817   1.720793701
2           0   10.18086202  -2.253089622   5.74882307
3           0           0  -12.5704981           0
4           0           0           0   9.652818299

: w
      1          2          3          4
1  12.9931394  -5.95236607  .36499234+1.63500766i  .36499234-1.63500766i

: b
      1          2          3          4
1  4.406836593  10.18086202  3.694083258  3.694083258

: w:/b
      1          2          3          4
1  2.94840508  -.584662287  .098804579+.442601735i  .098804579-.442601735i

```

`m` contains the number of real, generalized eigenvalues

```

: m
2

```

Conformability

`gschurd(A, B, T, R, U, V, w, b)`:

input:

A: $n \times n$

B: $n \times n$

output:

T: $n \times n$

R: $n \times n$

U: $n \times n$

V: $n \times n$

w: $1 \times n$

b: $1 \times n$

`_gschurd(A, B, U, V, w, b)`:

input:

A: $n \times n$

B: $n \times n$

output:

A: $n \times n$

B: $n \times n$

U: $n \times n$

V: $n \times n$

w: $1 \times n$

b: $1 \times n$

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m)`:

input:

A: $n \times n$

B: $n \times n$

f: 1×1

output:

T: $n \times n$

R: $n \times n$

U: $n \times n$

V: $n \times n$

w: $1 \times n$

b: $1 \times n$

m: 1×1

`_gschurdgroupby(A, B, f, U, V, w, b, m)`:

input:

A: $n \times n$

B: $n \times n$

f: 1×1

output:

A: $n \times n$

B: $n \times n$

U: $n \times n$

V: $n \times n$

w: $1 \times n$

b: $1 \times n$

m: 1×1

Diagnostics

`_gschurd()` and `_gschurdgroupby()` abort with error if *A* or *B* is a view.

`gschurd()`, `_gschurd()`, `gschurdgroupby()`, and `_gschurdgroupby()` return missing results if *A* or *B* contains missing values.

Also see

[M-1] **LAPACK** — Linear algebra package (LAPACK) routines

[M-5] **geigensystem()** — Generalized eigenvectors and eigenvalues

[M-5] **ghessenberg()** — Generalized Hessenberg decomposition

[M-4] **Matrix** — Matrix functions

Stata, Stata Press, Mata, NetCourse, and NetCourseNow are registered trademarks of StataCorp LLC. Stata and Stata Press are registered trademarks with the World Intellectual Property Organization of the United Nations. StataNow is a trademark of StataCorp LLC. Other brand and product names are registered trademarks or trademarks of their respective companies. Copyright © 1985–2025 StataCorp LLC, College Station, TX, USA. All rights reserved.



For suggested citations, see the FAQ on [citing Stata documentation](#).