

**gschurd()** — Generalized Schur decomposition

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## Description

`gschurd(A, B, T, R, U, V, w, b)` computes the generalized Schur decomposition of two square, numeric matrices,  $A$  and  $B$ , and the [generalized eigenvalues](#). The decomposition is returned in the [Schur-form](#) matrix,  $T$ ; the upper-triangular matrix,  $R$ ; and the orthogonal (unitary) matrices,  $U$  and  $V$ . The generalized eigenvalues are returned in the complex vectors  $w$  and  $b$ .

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m)` computes the generalized Schur decomposition of two square, numeric matrices,  $A$  and  $B$ , and the [generalized eigenvalues](#), and groups the results according to whether a condition on each generalized eigenvalue is satisfied.  $f$  is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed [below](#). The number of generalized eigenvalues for which the condition is true is returned in  $m$ .

`_gschurd()` mirrors `gschurd()`, the difference being that it returns  $T$  in  $A$  and  $R$  in  $B$ .

`_gschurdgroupby()` mirrors `gschurdgroupby()`, the difference being that it returns  $T$  in  $A$  and  $R$  in  $B$ .

`_gschurd_la()` and `_gschurdgroupby_la()` are the interfaces into the LAPACK routines used to implement the above functions; see [\[M-1\] LAPACK](#). Their direct use is not recommended.

## Syntax

```
void          gschurd(A, B, T, R, U, V, w, b)
```

```
void          _gschurd(A, B, U, V, w, b)
```

```
void gschurdgroupby(A, B, f, T, R, U, V, w, b, m)
```

```
void _gschurdgroupby(A, B, f, U, V, w, b, m)
```

## Remarks and examples

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Remarks are presented under the following headings:

*Generalized Schur decomposition*  
*Grouping the results*

### Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices,  $A$  and  $B$ , can be written as

$$U' \times A \times V = T$$

$$U' \times B \times V = R$$

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where  $\mathbf{T}$  is in Schur form,  $\mathbf{R}$  is upper triangular, and  $\mathbf{U}$  and  $\mathbf{V}$  are orthogonal if  $\mathbf{A}$  and  $\mathbf{B}$  are real and are unitary if  $\mathbf{A}$  or  $\mathbf{B}$  is complex. The complex vectors  $\mathbf{w}$  and  $\mathbf{b}$  contain the generalized eigenvalues.

If  $\mathbf{A}$  and  $\mathbf{B}$  are real,  $\mathbf{T}$  is in real Schur form and  $\mathbf{R}$  is a real upper-triangular matrix. If  $\mathbf{A}$  or  $\mathbf{B}$  is complex,  $\mathbf{T}$  is in complex Schur form and  $\mathbf{R}$  is a complex upper-triangular matrix.

In the example below, we define  $\mathbf{A}$  and  $\mathbf{B}$ , obtain the generalized Schur decomposition, and list  $\mathbf{T}$  and  $\mathbf{R}$ .

```
: A = (6, 2, 8, -1\ -3, -4, -6, 4\ 0, 8, 4, 1\ -8, -7, -3, 5)
: B = (8, 0, -8, -1\ -6, -2, -6, -1\ -7, -6, 2, -6\ 1, -7, 9, 2)
: gschurd(A, B, T=., R=., U=., V=., w=., b=.)
```

```
: T
```

	1	2	3	4
1	12.99313938	1.746927947	3.931212285	-10.91622337
2	0	.014016016	6.153566902	1.908835695
3	0	-4.362999645	1.849905717	-2.998194791
4	0	0	0	-5.527285433

```
: R
```

	1	2	3	4
1	4.406836593	6.869534063	-1.840892081	1.740906311
2	0	13.88730687	0	-.6995556735
3	0	0	9.409495218	-4.659386723
4	0	0	0	9.453808732

```
: w
```

	1	2	3	4
1	12.9931394	.409611804+1.83488354i	.024799819-.111092453i	-5.52728543

```
: b
```

	1	2	3	4
1	4.406836593	4.145676341	.2509986829	9.453808732

Generalized eigenvalues can be obtained by typing

```
: w:/b
```

	1	2	3	4
1	2.94840508	.098804579+.442601735i	.098804579-.442601735i	-.584662287

## Grouping the results

`gschurdgroupby()` reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair  $\mathbf{w}$  and  $\mathbf{b}$ . It also reorders the generalized Schur form  $\mathbf{T}$ ,  $\mathbf{R}$ , and orthogonal (unitary) matrices,  $\mathbf{U}$  and  $\mathbf{V}$ , correspondingly.

We must pass `gschurdgroupby()` a [pointer](#) to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the following example, we use `gschurdgroupby()` to put the finite, real, generalized eigenvalues first. One of the arguments to `gschurdgroupby()` is a pointer to the function `onlyreal()` which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. `onlyreal()` returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```

: real scalar onlyreal(complex scalar w, real scalar b)
> {
>     if(b==0) return(0)
>     if(Im(w/b)==0) return(1)
>     return(0)
> }
: gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)

```

We obtain

```

: T
      1      2      3      4
1  12.99313938   8.19798168   6.285710813   5.563547054
2      0  -5.952366071  -1.473533834   2.750066482
3      0      0  -0.2015830885   3.882051743
4      0      0   6.337230739   1.752690714

: R
      1      2      3      4
1  4.406836593   2.267479575  -6.745927817   1.720793701
2      0  10.18086202  -2.253089622   5.74882307
3      0      0  -12.5704981      0
4      0      0      0   9.652818299

: w
      1      2      3      4
1  12.9931394  -5.95236607  .36499234+1.63500766i  .36499234-1.63500766i

: b
      1      2      3      4
1  4.406836593  10.18086202  3.694083258  3.694083258

: w:/b
      1      2      3      4
1  2.94840508  -.584662287  .098804579+.442601735i  .098804579-.442601735i

```

`m` contains the number of real, generalized eigenvalues

```

: m
2

```

## Conformability

`gschurd(A, B, T, R, U, V, w, b):`

*input:*

*A:*  $n \times n$

*B:*  $n \times n$

*output:*

*T:*  $n \times n$

*R:*  $n \times n$

*U:*  $n \times n$

*V:*  $n \times n$

*w:*  $1 \times n$

*b:*  $1 \times n$

`_gschurd(A, B, U, V, w, b):`

*input:*

*A:*  $n \times n$

*B:*  $n \times n$

*output:*

*A:*  $n \times n$

*B:*  $n \times n$

*U:*  $n \times n$

*V:*  $n \times n$

*w:*  $1 \times n$

*b:*  $1 \times n$

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m):`

*input:*

*A:*  $n \times n$

*B:*  $n \times n$

*f:*  $1 \times 1$

*output:*

*T:*  $n \times n$

*R:*  $n \times n$

*U:*  $n \times n$

*V:*  $n \times n$

*w:*  $1 \times n$

*b:*  $1 \times n$

*m:*  $1 \times 1$

`_gschurdgroupby(A, B, f, U, V, w, b, m)`:

*input:*

*A:*     $n \times n$   
*B:*     $n \times n$   
*f:*     $1 \times 1$

*output:*

*A:*     $n \times n$   
*B:*     $n \times n$   
*U:*     $n \times n$   
*V:*     $n \times n$   
*w:*     $1 \times n$   
*b:*     $1 \times n$   
*m:*     $1 \times 1$

## Diagnostics

`_gschurd()` and `_gschurdgroupby()` abort with error if *A* or *B* is a view.

`gschurd()`, `_gschurd()`, `gschurdgroupby()`, and `_gschurdgroupby()` return missing results if *A* or *B* contains missing values.

## Also see

[M-1] [LAPACK](#) — The LAPACK linear-algebra routines

[M-5] [ghessenbergd\(\)](#) — Generalized Hessenberg decomposition

[M-5] [geigensystem\(\)](#) — Generalized eigenvectors and eigenvalues

[M-4] [matrix](#) — Matrix functions