gschurd( ) — Generalized Schur decomposition

Description Syntax Remarks and examples Conformability Diagnostics Also see

Description

`gschurd(`*A*`,` *B*`,` *T*`,` *R*`,` *U*`,` *V*`,` *w*`,` *b*`)` computes the generalized Schur decomposition of two square, numeric matrices, *A* and *B*, and the generalized eigenvalues. The decomposition is returned in the Schur-form matrix, *T*; the upper-triangular matrix, *R*; and the orthogonal (unitary) matrices, *U* and *V*. The generalized eigenvalues are returned in the complex vectors *w* and *b*.

`gschurdgroupby(`*A*`,` *B*`,` *f*`,` *T*`,` *R*`,` *U*`,` *V*`,` *w*`,` *b*`,` *m*`)` computes the generalized Schur decomposition of two square, numeric matrices, *A* and *B*, and the generalized eigenvalues, and groups the results according to whether a condition on each generalized eigenvalue is satisfied. *f* is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed below. The number of generalized eigenvalues for which the condition is true is returned in *m*.

`_gschurd()` mirrors `gschurd()`, the difference being that it returns *T* in *A* and *R* in *B*.

`_gschurdgroupby()` mirrors `gschurdgroupby()`, the difference being that it returns *T* in *A* and *R* in *B*.

`_gschurd_la()` and `_gschurdgroupby_la()` are the interfaces into the LAPACK routines used to implement the above functions; see [M-1] **LAPACK**. Their direct use is not recommended.

Syntax

void `gschurd(`*A*`,` *B*`,` *T*`,` *R*`,` *U*`,` *V*`,` *w*`,` *b*`)`

void `_gschurd(`*A*`,` *B*`,` *U*`,` *V*`,` *w*`,` *b*`)`

void `gschurdgroupby(`*A*`,` *B*`,` *f*`,` *T*`,` *R*`,` *U*`,` *V*`,` *w*`,` *b*`,` *m*`)`

void `_gschurdgroupby(`*A*`,` *B*`,` *f*`,` *U*`,` *V*`,` *w*`,` *b*`,` *m*`)`

Remarks and examples

Remarks are presented under the following headings:

Generalized Schur decomposition
Grouping the results

Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices, **A** and **B**, can be written as

$$\mathbf{U}' \times \mathbf{A} \times \mathbf{V} = \mathbf{T}$$

$$\mathbf{U}' \times \mathbf{B} \times \mathbf{V} = \mathbf{R}$$

where **T** is in Schur form, **R** is upper triangular, and **U** and **V** are orthogonal if **A** and **B** are real and are unitary if **A** or **B** is complex. The complex vectors **w** and **b** contain the generalized eigenvalues.

If **A** and **B** are real, **T** is in real Schur form and **R** is a real upper-triangular matrix. If **A** or **B** is complex, **T** is in complex Schur form and **R** is a complex upper-triangular matrix.

In the example below, we define `A` and `B`, obtain the generalized Schur decomposition, and list `T` and `R`.

```
: A = (6, 2, 8, -1\-3, -4, -6, 4\0, 8, 4, 1\-8, -7, -3, 5)
: B = (8, 0, -8, -1\-6, -2, -6, -1\-7, -6, 2, -6\1, -7, 9, 2)
: gschurd(A, B, T=., R=., U=., V=., w=., b=.)
: T
                 1              2              3              4
  1   12.99313938     1.746927947     3.931212285   -10.91622337
  2             0      .014016016     6.153566902    1.908835695
  3             0   -4.362999645     1.849905717   -2.998194791
  4             0              0              0   -5.527285433

: R
                 1              2              3              4
  1   4.406836593     6.869534063   -1.840892081    1.740906311
  2             0     13.88730687              0   -.6995556735
  3             0              0    9.409495218   -4.659386723
  4             0              0              0    9.453808732

: w
                 1                        2                        3              4
  1   12.9931394   .409611804+1.83488354i   .024799819-.111092453i   -5.52728543

: b
                 1              2              3              4
  1   4.406836593    4.145676341    .2509986829    9.453808732
```

Generalized eigenvalues can be obtained by typing

```
: w:/b
                 1                        2                        3              4
  1   2.94840508   .098804579+.442601735i   .098804579-.442601735i   -.584662287
```

Grouping the results

`gschurdgroupby()` reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair `w` and `b`. It also reorders the generalized Schur form `T`, `R`, and orthogonal (unitary) matrices, `U` and `V`, correspondingly.

We must pass `gschurdgroupby()` a pointer to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the following example, we use `gschurdgroupby()` to put the finite, real, generalized eigenvalues first. One of the arguments to `schurdgroupby()` is a pointer to the function `onlyreal()` which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. `onlyreal()` returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```
: real scalar onlyreal(complex scalar w, real scalar b)
> {
>         if(b==0) return(0)
>         if(Im(w/b)==0) return(1)
>         return(0)
> }
: gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)
```

We obtain

```
: T
                 1              2              3              4
  +-------------------------------------------------------------+
1 |  12.99313938    8.19798168    6.285710813    5.563547054   |
2 |             0  -5.952366071   -1.473533834    2.750066482   |
3 |             0             0   -.2015830885    3.882051743   |
4 |             0             0    6.337230739    1.752690714   |
  +-------------------------------------------------------------+

: R
                 1              2              3              4
  +-------------------------------------------------------------+
1 |  4.406836593    2.267479575   -6.745927817    1.720793701   |
2 |             0   10.18086202   -2.253089622     5.74882307   |
3 |             0             0   -12.5704981              0   |
4 |             0             0              0   9.652818299   |
  +-------------------------------------------------------------+

: w
                 1              2                      3                      4
  +-------------------------------------------------------------------------+
1 |  12.9931394  -5.95236607  .36499234+1.63500766i  .36499234-1.63500766i   |
  +-------------------------------------------------------------------------+

: b
                 1              2              3              4
  +-------------------------------------------------------------+
1 |  4.406836593   10.18086202   3.694083258   3.694083258      |
  +-------------------------------------------------------------+

: w:/b
                 1              2                      3                      4
  +-------------------------------------------------------------------------+
1 |  2.94840508  -.584662287  .098804579+.442601735i  .098804579-.442601735i  |
  +-------------------------------------------------------------------------+
```

`m` contains the number of real, generalized eigenvalues

```
: m
  2
```

Conformability

`gschurd`(A, B, T, R, U, V, w, b):

input:

A:	$n \times n$
B:	$n \times n$

output:

T:	$n \times n$
R:	$n \times n$
U:	$n \times n$
V:	$n \times n$
w:	$1 \times n$
b:	$1 \times n$

`_gschurd`(A, B, U, V, w, b):

input:

A:	$n \times n$
B:	$n \times n$

output:

A:	$n \times n$
B:	$n \times n$
U:	$n \times n$
V:	$n \times n$
w:	$1 \times n$
b:	$1 \times n$

`gschurdgroupby`(A, B, f, T, R, U, V, w, b, m):

input:

A:	$n \times n$
B:	$n \times n$
f:	1×1

output:

T:	$n \times n$
R:	$n \times n$
U:	$n \times n$
V:	$n \times n$
w:	$1 \times n$
b:	$1 \times n$
m:	1×1

_gschurdgroupby(*A*, *B*, *f*, *U*, *V*, *w*, *b*, *m*):

input:

A:	$n \times n$
B:	$n \times n$
f:	1×1

output:

A:	$n \times n$
B:	$n \times n$
U:	$n \times n$
V:	$n \times n$
w:	$1 \times n$
b:	$1 \times n$
m:	1×1

Diagnostics

_gschurd() and _gschurdgroupby() abort with error if *A* or *B* is a view.

gschurd(), _gschurd(), gschurdgroupby(), and _gschurdgroupby() return missing results if *A* or *B* contains missing values.

Also see

[M-1] **LAPACK** — Linear algebra package (LAPACK) routines

[M-5] **geigensystem( )** — Generalized eigenvectors and eigenvalues

[M-5] **ghessenbergd( )** — Generalized Hessenberg decomposition

[M-4] **Matrix** — Matrix functions