gschurd() — Generalized Schur decomposition

Description

gschurd(A, B, T, R, U, V, w, b) computes the generalized Schur decomposition of two square, numeric matrices, A and B, and the generalized eigenvalues. The decomposition is returned in the Schur-form matrix, T; the upper-triangular matrix, R; and the orthogonal (unitary) matrices, U and V. The generalized eigenvalues are returned in the complex vectors w and b.

gschurdgroupby(A, B, f, T, R, U, V, w, b, m) computes the generalized Schur decomposition of two square, numeric matrices, A and B, and the generalized eigenvalues, and groups the results according to whether a condition on each generalized eigenvalue is satisfied. f is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed below. The number of generalized eigenvalues for which the condition is true is returned in m.

_gschurd() mirrors gschurd(), the difference being that it returns T in A and R in B.

_gschurdgroupby() mirrors gschurdgroupby(), the difference being that it returns T in A and R in B.

_gschurd_la() and _gschurdgroupby_la() are the interfaces into the LAPACK routines used to implement the above functions; see [M-1] LAPACK. Their direct use is not recommended.

Syntax

void gschurd(A, B, T, R, U, V, w, b)
void _gschurd(A, B, U, V, w, b)
void gschurdgroupby(A, B, f, T, R, U, V, w, b, m)
void _gschurdgroupby(A, B, f, U, V, w, b, m)

Remarks and examples

Remarks are presented under the following headings:

Generalized Schur decomposition
Grouping the results

Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices, A and B, can be written as

\[ U' \times A \times V = T \]
\[ U' \times B \times V = R \]
where $T$ is in Schur form, $R$ is upper triangular, and $U$ and $V$ are orthogonal if $A$ and $B$ are real and are unitary if $A$ or $B$ is complex. The complex vectors $w$ and $b$ contain the generalized eigenvalues.

If $A$ and $B$ are real, $T$ is in real Schur form and $R$ is a real upper-triangular matrix. If $A$ or $B$ is complex, $T$ is in complex Schur form and $R$ is a complex upper-triangular matrix.

In the example below, we define $A$ and $B$, obtain the generalized Schur decomposition, and list $T$ and $R$.

```python
A = (6, 2, 8, -1, -3, -4, -6, 4, 0, 8, 4, 1, -8, -7, 3, 5)
B = (8, 0, -8, -1, -6, -2, -6, -1, -7, -6, 2, -6, -1, 5, 9, 2)
gschurd(A, B, T=., R=., U=., V=., w=., b=.)
```

$T$

<table>
<thead>
<tr>
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<th>4</th>
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<tbody>
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$R$

<table>
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<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
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$w$

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</tr>
</thead>
<tbody>
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$b$

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</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

Generalized eigenvalues can be obtained by typing

```python
:w:/b
```

$w:/b$

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<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.94840508</td>
<td>.098804579+.442601735i</td>
<td>.098804579-.442601735i</td>
<td>-.584662287</td>
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</tbody>
</table>

**Grouping the results**

gschurdgroupby() reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair $w$ and $b$. It also reorders the generalized Schur form $T$, $R$, and orthogonal (unitary) matrices, $U$ and $V$, correspondingly.

We must pass gschurdgroupby() a pointer to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.
In the following example, we use `gschurdgroupby()` to put the finite, real, generalized eigenvalues first. One of the arguments to `schurdgroupby()` is a pointer to the function `onlyreal()` which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. `onlyreal()` returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```plaintext
: real scalar onlyreal(complex scalar w, real scalar b)
  > if(b==0) return(0)
  > if(Im(w/b)==0) return(1)
  > return(0)
```

We obtain

```plaintext
: gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)
```

We obtain

<p>| | | | |</p>
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2.94840508</td>
<td>-.584662287</td>
<td>.098804579+.442601735i</td>
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<tr>
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<tr>
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<td>6.337230739</td>
<td>1.752690714</td>
<td>9.652818299</td>
</tr>
</tbody>
</table>
```

```plaintext
m contains the number of real, generalized eigenvalues
```

```plaintext
: m
2
```
Conformability

gschurd(\(A, B, T, R, U, V, w, b\)):

\begin{align*}
\text{input:} & \\
A & : \ n \times n \\
B & : \ n \times n \\
\text{output:} & \\
T & : \ n \times n \\
R & : \ n \times n \\
U & : \ n \times n \\
V & : \ n \times n \\
w & : \ 1 \times n \\
b & : \ 1 \times n
\end{align*}

_gschurd(\(A, B, U, V, w, b\)):

\begin{align*}
\text{input:} & \\
A & : \ n \times n \\
B & : \ n \times n \\
\text{output:} & \\
A & : \ n \times n \\
B & : \ n \times n \\
U & : \ n \times n \\
V & : \ n \times n \\
w & : \ 1 \times n \\
b & : \ 1 \times n
\end{align*}

gschurdgroupby(\(A, B, f, T, R, U, V, w, b, m\)):

\begin{align*}
\text{input:} & \\
A & : \ n \times n \\
B & : \ n \times n \\
f & : \ 1 \times 1 \\
\text{output:} & \\
T & : \ n \times n \\
R & : \ n \times n \\
U & : \ n \times n \\
V & : \ n \times n \\
w & : \ 1 \times n \\
b & : \ 1 \times n \\
m & : \ 1 \times 1
\end{align*}
_{gschurdgroupby}(A, B, f, U, V, w, b, m):

**input:**
- $A$: $n \times n$
- $B$: $n \times n$
- $f$: $1 \times 1$

**output:**
- $A$: $n \times n$
- $B$: $n \times n$
- $U$: $n \times n$
- $V$: $n \times n$
- $w$: $1 \times n$
- $b$: $1 \times n$
- $m$: $1 \times 1$

**Diagnostics**

_{gschurd()} and _{gschurdgroupby()} abort with error if $A$ or $B$ is a view.

_{gschurd()}, _{gschurd()}, _{gschurdgroupby()}, and _{gschurdgroupby()} return missing results if $A$ or $B$ contains missing values.

**Also see**

[M-1] **LAPACK** — The LAPACK linear-algebra routines

[M-5] **geigensystem()** — Generalized eigenvectors and eigenvalues

[M-5] **ghessenbergd()** — Generalized Hessenberg decomposition

[M-4] **Matrix** — Matrix functions