# Title



**gschurd()** — Generalized Schur decomposition


Description | Syntax | Remarks and examples | Conformability
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# Description

`gschurd(`$A$`,` $B$`,` $T$`,` $R$`,` $U$`,` $V$`,` $w$`,` $b$`)` computes the generalized Schur decomposition of two square, numeric matrices, $A$ and $B$, and the generalized eigenvalues. The decomposition is returned in the Schur-form matrix, $T$; the upper-triangular matrix, $R$; and the orthogonal (unitary) matrices, $U$ and $V$. The generalized eigenvalues are returned in the complex vectors $w$ and $b$.

`gschurdgroupby(`$A$`,` $B$`,` $f$`,` $T$`,` $R$`,` $U$`,` $V$`,` $w$`,` $b$`,` $m$`)` computes the generalized Schur decomposition of two square, numeric matrices, $A$ and $B$, and the generalized eigenvalues, and groups the results according to whether a condition on each generalized eigenvalue is satisfied. $f$ is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed below. The number of generalized eigenvalues for which the condition is true is returned in $m$.

`_gschurd()` mirrors `gschurd()`, the difference being that it returns $T$ in $A$ and $R$ in $B$.

`_gschurdgroupby()` mirrors `gschurdgroupby()`, the difference being that it returns $T$ in $A$ and $R$ in $B$.

`_gschurd_la()` and `_gschurdgroupby_la()` are the interfaces into the LAPACK routines used to implement the above functions; see [M-1] **LAPACK**. Their direct use is not recommended.

# Syntax

*void* `gschurd(`$A$`,` $B$`,` $T$`,` $R$`,` $U$`,` $V$`,` $w$`,` $b$`)`

*void* `_gschurd(`$A$`,` $B$`,` $U$`,` $V$`,` $w$`,` $b$`)`

*void* `gschurdgroupby(`$A$`,` $B$`,` $f$`,` $T$`,` $R$`,` $U$`,` $V$`,` $w$`,` $b$`,` $m$`)`

*void* `_gschurdgroupby(`$A$`,` $B$`,` $f$`,` $U$`,` $V$`,` $w$`,` $b$`,` $m$`)`

# Remarks and examples



Remarks are presented under the following headings:


*Generalized Schur decomposition*
*Grouping the results*


## Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices, $\mathbf{A}$ and $\mathbf{B}$, can be written as

$$\mathbf{U}' \times \mathbf{A} \times \mathbf{V} = \mathbf{T}$$

$$\mathbf{U}' \times \mathbf{B} \times \mathbf{V} = \mathbf{R}$$

where $\mathbf{T}$ is in Schur form, $\mathbf{R}$ is upper triangular, and $\mathbf{U}$ and $\mathbf{V}$ are orthogonal if $\mathbf{A}$ and $\mathbf{B}$ are real and are unitary if $\mathbf{A}$ or $\mathbf{B}$ is complex. The complex vectors $\mathbf{w}$ and $\mathbf{b}$ contain the generalized eigenvalues.

If $\mathbf{A}$ and $\mathbf{B}$ are real, $\mathbf{T}$ is in real Schur form and $\mathbf{R}$ is a real upper-triangular matrix. If $\mathbf{A}$ or $\mathbf{B}$ is complex, $\mathbf{T}$ is in complex Schur form and $\mathbf{R}$ is a complex upper-triangular matrix.

In the example below, we define `A` and `B`, obtain the generalized Schur decomposition, and list `T` and `R`.

```
: A = (6, 2, 8, -1\-3, -4, -6, 4\0, 8, 4, 1\-8, -7, -3, 5)
: B = (8, 0, -8, -1\-6, -2, -6, -1\-7, -6, 2, -6\1, -7, 9, 2)
: gschurd(A, B, T=., R=., U=., V=., w=., b=.)
: T
                 1              2              3              4
    +-------------------------------------------------------------+
  1 |  12.99313938    1.746927947    3.931212285   -10.91622337  |
  2 |            0     .014016016    6.153566902    1.908835695  |
  3 |            0   -4.362999645    1.849905717   -2.998194791  |
  4 |            0              0              0   -5.527285433  |
    +-------------------------------------------------------------+
: R
                 1              2              3              4
    +-------------------------------------------------------------+
  1 |  4.406836593    6.869534063   -1.840892081    1.740906311  |
  2 |            0    13.88730687              0   -.6995556735  |
  3 |            0              0    9.409495218   -4.659386723  |
  4 |            0              0              0    9.453808732  |
    +-------------------------------------------------------------+
: w
                1                        2                        3              4
    +----------------------------------------------------------------------------+
  1 |  12.9931394   .409611804+1.83488354i   .024799819-.111092453i   -5.52728543  |
    +----------------------------------------------------------------------------+
: b
                 1              2              3              4
    +-----------------------------------------------------------+
  1 |  4.406836593    4.145676341   .2509986829    9.453808732  |
    +-----------------------------------------------------------+
```

Generalized eigenvalues can be obtained by typing

```
: w:/b
                1                        2                        3              4
    +----------------------------------------------------------------------------+
  1 |  2.94840508   .098804579+.442601735i   .098804579-.442601735i   -.584662287  |
    +----------------------------------------------------------------------------+
```

## Grouping the results

`gschurdgroupby()` reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair `w` and `b`. It also reorders the generalized Schur form `T`, `R`, and orthogonal (unitary) matrices, `U` and `V`, correspondingly.

We must pass `gschurdgroupby()` a pointer to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the following example, we use gschurdgroupby() to put the finite, real, generalized eigenvalues first. One of the arguments to schurdgroupby() is a pointer to the function onlyreal() which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. onlyreal() returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```
: real scalar onlyreal(complex scalar w, real scalar b)
> {
>         if(b==0) return(0)
>         if(Im(w/b)==0) return(1)
>         return(0)
> }
: gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)
```

We obtain

```
: T
                 1             2             3             4
    +---------------------------------------------------------+
  1 |  12.99313938    8.19798168   6.285710813   5.563547054  |
  2 |            0  -5.952366071  -1.473533834   2.750066482  |
  3 |            0             0  -.2015830885   3.882051743  |
  4 |            0             0   6.337230739   1.752690714  |
    +---------------------------------------------------------+

: R
                 1             2             3             4
    +---------------------------------------------------------+
  1 |  4.406836593   2.267479575  -6.745927817   1.720793701  |
  2 |            0   10.18086202  -2.253089622    5.74882307  |
  3 |            0             0   -12.5704981             0  |
  4 |            0             0             0   9.652818299  |
    +---------------------------------------------------------+

: w
               1             2                       3                       4
    +------------------------------------------------------------------------+
  1 |  12.9931394   -5.95236607   .36499234+1.63500766i   .36499234-1.63500766i  |
    +------------------------------------------------------------------------+

: b
                1             2             3             4
    +-------------------------------------------------------+
  1 |  4.406836593   10.18086202   3.694083258   3.694083258  |
    +-------------------------------------------------------+

: w:/b
               1             2                          3                          4
    +------------------------------------------------------------------------------+
  1 |  2.94840508   -.584662287   .098804579+.442601735i   .098804579-.442601735i  |
    +------------------------------------------------------------------------------+
```

m contains the number of real, generalized eigenvalues

```
: m
  2
```

# Conformability

`gschurd(`$A$, $B$, $T$, $R$, $U$, $V$, $w$, $b$`):`

*input*:

| | |
|---|---|
| $A$: | $n \times n$ |
| $B$: | $n \times n$ |

*output*:

| | |
|---|---|
| $T$: | $n \times n$ |
| $R$: | $n \times n$ |
| $U$: | $n \times n$ |
| $V$: | $n \times n$ |
| $w$: | $1 \times n$ |
| $b$: | $1 \times n$ |

`_gschurd(`$A$, $B$, $U$, $V$, $w$, $b$`):`

*input*:

| | |
|---|---|
| $A$: | $n \times n$ |
| $B$: | $n \times n$ |

*output*:

| | |
|---|---|
| $A$: | $n \times n$ |
| $B$: | $n \times n$ |
| $U$: | $n \times n$ |
| $V$: | $n \times n$ |
| $w$: | $1 \times n$ |
| $b$: | $1 \times n$ |

`gschurdgroupby(`$A$, $B$, $f$, $T$, $R$, $U$, $V$, $w$, $b$, $m$`):`

*input*:

| | |
|---|---|
| $A$: | $n \times n$ |
| $B$: | $n \times n$ |
| $f$: | $1 \times 1$ |

*output*:

| | |
|---|---|
| $T$: | $n \times n$ |
| $R$: | $n \times n$ |
| $U$: | $n \times n$ |
| $V$: | $n \times n$ |
| $w$: | $1 \times n$ |
| $b$: | $1 \times n$ |
| $m$: | $1 \times 1$ |

`_gschurdgroupby(`$A$, $B$, $f$, $U$, $V$, $w$, $b$, $m$`):`

*input*:

| | |
|---|---|
| $A$: | $n \times n$ |
| $B$: | $n \times n$ |
| $f$: | $1 \times 1$ |

*output*:

| | |
|---|---|
| $A$: | $n \times n$ |
| $B$: | $n \times n$ |
| $U$: | $n \times n$ |
| $V$: | $n \times n$ |
| $w$: | $1 \times n$ |
| $b$: | $1 \times n$ |
| $m$: | $1 \times 1$ |

## Diagnostics

`_gschurd()` and `_gschurdgroupby()` abort with error if $A$ or $B$ is a view.

`gschurd()`, `_gschurd()`, `gschurdgroupby()`, and `_gschurdgroupby()` return missing results if $A$ or $B$ contains missing values.

## Also see

[M-1] **LAPACK** — Linear algebra package (LAPACK) routines

[M-5] **geigensystem()** — Generalized eigenvectors and eigenvalues

[M-5] **ghessenbergd()** — Generalized Hessenberg decomposition

[M-4] **Matrix** — Matrix functions