**ghessenbergd() — Generalized Hessenberg decomposition**

### Description

`ghessenbergd(A, B, H, R, U, V)` computes the generalized Hessenberg decomposition of two general, real or complex, square matrices, `A` and `B`, returning the upper Hessenberg form matrix in `H`, the upper triangular matrix in `R`, and the orthogonal (unitary) matrices in `U` and `V`.

`_ghessenbergd(A, B, U, V)` mirrors `ghessenbergd()`, the difference being that it returns `H` in `A` and `R` in `B`.

`_ghessenbergd_la()` is the interface into the LAPACK routines used to implement the above function; see [M-1] LAPACK. Its direct use is not recommended.

### Syntax

```c
void ghessenbergd(numeric matrix A, B, H, R, U, V)
void _ghessenbergd(numeric matrix A, B, U, V)
```

### Remarks and examples

The generalized Hessenberg decomposition of two square, numeric matrices (`A` and `B`) can be written as

\[
U' \times A \times V = H \\
U' \times B \times V = R
\]

where `H` is in upper Hessenberg form, `R` is upper triangular, and `U` and `V` are orthogonal matrices if `A` and `B` are real or are unitary matrices otherwise.

In the example below, we define `A` and `B`, obtain the generalized Hessenberg decomposition, and list `H` and `Q`.

```plaintext
: A = (6, 2, 8, -1\-3, -4, -6, 4\0, 8, 4, 1\-8, -7, -3, 5)
: B = (8, 0, -8, -1\-6, -2, -6, -1\-7, -6, 2, -6\1, -7, 9, 2)
: ghessenbergd(A, B, H=., R=., U=., V=.)
: H

1    2    3    4
1 -4.735680169 1.363736029 5.097381347 3.889763589
2  9.304479208 -8.594240253 -7.993282943 4.803411217
3  0  4.553169015  3.236266637 -2.147709419
4  0  0  6.997043028 -3.524816722
```

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Conformability

ghessenbergd(A, B, H, R, U, V):

input:
- A: \( n \times n \)
- B: \( n \times n \)

output:
- H: \( n \times n \)
- R: \( n \times n \)
- U: \( n \times n \)
- V: \( n \times n \)

_ghessenbergd(A, B, U, V):

input:
- A: \( n \times n \)
- B: \( n \times n \)

output:
- A: \( n \times n \)
- B: \( n \times n \)
- U: \( n \times n \)
- V: \( n \times n \)

Diagnostics

_ghessenbergd() aborts with error if A or B is a view.

ghessenbergd() and _ghessenbergd() return missing results if A or B contains missing values.

Also see

[M-1] LAPACK — The LAPACK linear-algebra routines
[M-5] gschurd() — Generalized Schur decomposition