cholsolve() — Solve AX=B for X using Cholesky decomposition

Description

cholsolve(A, B) solves AX = B and returns X for symmetric (Hermitian), positive-definite A. cholsolve() returns a matrix of missing values if A is not positive definite or if A is singular.

cholsolve(A, B, tol) does the same thing; it allows you to specify the tolerance for declaring that A is singular; see Tolerance under Remarks and examples below.

_cholsolve(A, B) and _cholsolve(A, B, tol) do the same thing except that, rather than returning the solution X, they overwrite B with the solution, and in the process of making the calculation, they destroy the contents of A.

Syntax

    numeric matrix    cholsolve(numeric matrix A, numeric matrix B)
    numeric matrix    cholsolve(numeric matrix A, numeric matrix B, real scalar tol)
    void             _cholsolve(numeric matrix A, numeric matrix B)
    void             _cholsolve(numeric matrix A, numeric matrix B, real scalar tol)

Remarks and examples

The above functions solve AX = B via Cholesky decomposition and are accurate. When A is not symmetric and positive definite, [M-5] lusolve(), [M-5] qrsolve(), and [M-5] svsolve() are alternatives based on the LU decomposition, the QR decomposition, and the singular value decomposition (SVD). The alternatives differ in how they handle singular A. Then the LU-based routines return missing values, whereas the QR-based and SVD-based routines return generalized (least-squares) solutions.

Remarks are presented under the following headings:

    Derivation
    Relationship to inversion
    Tolerance

Derivation

We wish to solve for X

\[ AX = B \]  \hspace{1cm} (1)
when \( A \) is symmetric and positive definite. Perform the Cholesky decomposition of \( A \) so that we have \( A = GG' \). Then (1) can be written as
\[
GG'X = B
\] (2)
Define
\[
Z = G'X
\] (3)
Then (2) can be rewritten as
\[
GZ = B
\] (4)
It is easy to solve (4) for \( Z \) because \( G \) is a lower-triangular matrix. Once \( Z \) is known, it is easy to solve (3) for \( X \) because \( G' \) is upper triangular.

**Relationship to inversion**

See *Relationship to inversion* in [M-5] `lusolve()` for a discussion of the relationship between solving the linear system and matrix inversion.

**Tolerance**

The default tolerance used is
\[
\eta = \frac{(1e-13)*\text{trace}(\text{abs}(G))}{n}
\]
where \( G \) is the lower-triangular Cholesky factor of \( A: n \times n \). \( A \) is declared to be singular if `cholesky()` (see [M-5] `cholesky()`) finds that \( A \) is not positive definite, or if \( A \) is found to be positive definite, if any diagonal element of \( G \) is less than or equal to \( \eta \). Mathematically, positive definiteness implies that the matrix is not singular. In the numerical method used, two checks are made: `cholesky()` makes one and then the \( \eta \) rule is applied to ensure numerical stability in the use of the result `cholesky()` returns.

If you specify \( tol > 0 \), the value you specify is used to multiply \( \eta \). You may instead specify \( tol \leq 0 \) and then the negative of the value you specify is used in place of \( \eta \); see [M-1] *Tolerance*.

See [M-5] `lusolve()` for a detailed discussion of the issues surrounding solving nearly singular systems. The main point to keep in mind is that if \( A \) is ill conditioned, then small changes in \( A \) or \( B \) can lead to radically large differences in the solution for \( X \).

**Conformability**

`cholsolve(A, B, tol)`:

*input:*

\[
A: n \times n \\
B: n \times k \\
tol: 1 \times 1 \quad \text{(optional)}
\]

*result:*

\[
n \times k
\]

`_cholsolve(A, B, tol)`:

*input:*

\[
A: n \times n \\
B: n \times k \\
tol: 1 \times 1 \quad \text{(optional)}
\]

*output:*

\[
0 \times 0 \\
n \times k
\]
Diagnostics

cholsolve(A, B, ...), and _cholsolve(A, B, ...) return a result of all missing values if A is not positive definite or if A contains missing values.

_cholsolve(A, B, ...) also aborts with error if A or B is a view.

All functions use the elements from the lower triangle of A without checking whether A is symmetric or, in the complex case, Hermitian.

Also see

[M-5] cholesky() — Cholesky square-root decomposition
[M-5] cholinv() — Symmetric, positive-definite matrix inversion
[M-5] lusolve() — Solve AX=B for X using LU decomposition
[M-5] qrsolve() — Solve AX=B for X using QR decomposition
[M-5] solvelower() — Solve AX=B for X, A triangular
[M-5] svsolve() — Solve AX=B for X using singular value decomposition
[M-5] solve_tol() — Tolerance used by solvers and inverters
[M-4] Solvers — Functions to solve AX=B and to obtain A inverse