## Description

Colon operators perform element-by-element operations.

### Syntax

- \( a :+ b \) addition
- \( a :\ - b \) subtraction
- \( a :* b \) multiplication
- \( a :/ b \) division
- \( a :^ b \) power
- \( a :== b \) equality
- \( a :!= b \) inequality
- \( a :> b \) greater than
- \( a :>= b \) greater than or equal to
- \( a :< b \) less than
- \( a :<= b \) less than or equal to
- \( a :\& b \) and
- \( a :| b \) or

### Remarks and examples

Remarks are presented under the following headings:

- **C-conformability: element by element**
- **Usefulness of colon logical operators**
- **Use parentheses**

#### C-conformability: element by element

The colon operators perform the indicated operation on each pair of elements of \( a \) and \( b \). For instance,

\[
\begin{bmatrix}
c & d \\
f & g \\
h & i \\
\end{bmatrix} :*
\begin{bmatrix}
j & k \\
l & m \\
n & o \\
\end{bmatrix}
= 
\begin{bmatrix}
c \times j & d \times k \\
f \times l & g \times m \\
h \times n & i \times o \\
\end{bmatrix}
\]
Also colon operators have a relaxed definition of conformability:

\[
\begin{bmatrix}
  c \\
  f \\
  g
\end{bmatrix} :* \begin{bmatrix}
  j & k \\
  l & m \\
  n & o
\end{bmatrix} = \begin{bmatrix}
  c * j & c * k \\
  f * l & f * m \\
  g * n & g * o
\end{bmatrix}
\]

\[
\begin{bmatrix}
  c & d \\
  f & g \\
  h & i
\end{bmatrix} :* \begin{bmatrix}
  j \\
  l \\
  n
\end{bmatrix} = \begin{bmatrix}
  c * j & d * j \\
  f * l & g * l \\
  h * n & i * n
\end{bmatrix}
\]

\[
\begin{bmatrix}
  c & d \\
  f & g \\
  h & i
\end{bmatrix} :* \begin{bmatrix}
  j & k \\
  l & m \\
  n & o
\end{bmatrix} = \begin{bmatrix}
  c * j & d * k \\
  c * l & d * m \\
  c * n & d * o
\end{bmatrix}
\]

\[
\begin{bmatrix}
  c & d \\
  f & g \\
  h & i
\end{bmatrix} :* \begin{bmatrix}
  j \\
  l \\
  n
\end{bmatrix} = \begin{bmatrix}
  c * j & c * k \\
  c * l & c * m \\
  c * n & c * o
\end{bmatrix}
\]

The matrices above are said to be c-conformable; the c stands for colon. The matrices have the same number of rows and columns, or one or the other is a vector with the same number of rows or columns as the matrix, or one or the other is a scalar.

C-conformability is relaxed, but not everything is allowed. The following is an error:

\[
(c d e) :* \begin{bmatrix}
  f \\
  g \\
  h
\end{bmatrix}
\]

**Usefulness of colon logical operators**

It is worth paying particular attention to the colon logical operators because they can produce pattern vectors and matrices. Consider the matrix

\[
: x = (5, 0 \ \ 0 \ \ 2 \ \ 3, 8)
\]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>
Which elements of \( x \) contain 0?

\[
\begin{array}{ccc}
  & 1 & 2 \\
1 & 0 & 1 \\
2 & 1 & 0 \\
3 & 0 & 0 \\
\end{array}
\]

How many zeros are there in \( x \)?

\[
\text{sum}(x::==0) \\
2
\]

**Use parentheses**

Because of their relaxed conformability requirements, colon operators are not associative even when the underlying operator is. For instance, you expect \((a+b)+c==a+(b+c)\), at least ignoring numerical roundoff error. Nevertheless, \((a:+b):+c==a:+(b:+c)\) does not necessarily hold. Consider what happens when

\[
\begin{align*}
a & : 1 \times 4 \\
b & : 5 \times 1 \\
c & : 5 \times 4
\end{align*}
\]

Then \((a:+b):+c\) is an error because \(a:+b\) is not c-conformable.

Nevertheless, \(a:+(b:+c)\) is not an error and in fact produces a \(5 \times 4\) matrix because \(b:+c\) is \(5 \times 4\), which is c-conformable with \(a\).

**Conformability**

\[
a : \text{op} b:
\]

\[
\begin{align*}
a & : r_1 \times c_1 \\
b & : r_2 \times c_2, \quad a \text{ and } b \text{ c-conformable} \\
\text{result} & : \max(r_1, r_2) \times \max(c_1, c_2)
\end{align*}
\]

**Diagnostics**

The colon operators return missing and abort with error under the same conditions that the underlying operator returns missing and aborts with error.

**Also see**

[M-2] exp — Expressions

[M-2] Intro — Language definition