Colon operators perform element-by-element operations.

Syntax

- \( a :+ b \)  
- \( a :- b \)  
- \( a :* b \)  
- \( a :/ b \)  
- \( a :^ b \)  
- \( a :== b \)  
- \( a :!= b \)  
- \( a :> b \)  
- \( a :>= b \)  
- \( a :< b \)  
- \( a :<= b \)  
- \( a :& b \)  
- \( a :| b \)  

Remarks and examples

Remarks are presented under the following headings:

- C-conformability: element by element
- Usefulness of colon logical operators
- Use parentheses

C-conformability: element by element

The colon operators perform the indicated operation on each pair of elements of \( a \) and \( b \). For instance,

\[
\begin{bmatrix} c & d \\ f & g \\ h & i \end{bmatrix} :* \begin{bmatrix} j & k \\ l & m \\ n & o \end{bmatrix} = \begin{bmatrix} c*j & d*k \\ f*l & g*m \\ h*n & i*o \end{bmatrix}
\]
Also colon operators have a relaxed definition of conformability:

\[
\begin{bmatrix}
  c \\
  f \\
  g
\end{bmatrix}
:\star
\begin{bmatrix}
  j & k \\
  l & m \\
  n & o
\end{bmatrix}
= \begin{bmatrix}
  c \cdot j & c \cdot k \\
  f \cdot l & f \cdot m \\
  g \cdot n & g \cdot o
\end{bmatrix}
\]

\[
\begin{bmatrix}
  c & d \\
  f & g \\
  h & i
\end{bmatrix}
:\star
\begin{bmatrix}
  j \\
  l \\
  n
\end{bmatrix}
= \begin{bmatrix}
  c \cdot j & d \cdot j \\
  f \cdot l & g \cdot l \\
  h \cdot n & i \cdot n
\end{bmatrix}
\]

\[
\begin{bmatrix}
  c \\
  f \\
  h
\end{bmatrix}
:\star
\begin{bmatrix}
  j & k \\
  l & m \\
  n & o
\end{bmatrix}
= \begin{bmatrix}
  c \cdot j & c \cdot k \\
  f \cdot l & f \cdot m \\
  c \cdot n & d \cdot o
\end{bmatrix}
\]

\[
\begin{bmatrix}
  c & d \\
  f & g \\
  h & i
\end{bmatrix}
:\star
\begin{bmatrix}
  j \\
  l \\
  n
\end{bmatrix}
= \begin{bmatrix}
  c \cdot j & d \cdot j \\
  f \cdot l & g \cdot j \\
  c \cdot n & i \cdot j
\end{bmatrix}
\]

The matrices above are said to be c-conformable; the c stands for colon. The matrices have the same number of rows and columns, or one or the other is a vector with the same number of rows or columns as the matrix, or one or the other is a scalar.

C-conformability is relaxed, but not everything is allowed. The following is an error:

\[(c \ d \ e) \ : \star \begin{bmatrix}
  f \\
  g \\
  h
\end{bmatrix}\]

**Usefulness of colon logical operators**

It is worth paying particular attention to the colon logical operators because they can produce pattern vectors and matrices. Consider the matrix

\[
: x = (5, 0 \ 0 \ 2 \ 3, 8)
\]

\[
\begin{array}{ccc}
1 & 5 & 0 \\
2 & 0 & 2 \\
3 & 3 & 8
\end{array}
\]
Which elements of \( x \) contain 0?

\[
\begin{array}{ccc}
1 & 2 \\
1 & 0 & 1 \\
2 & 1 & 0 \\
3 & 0 & 0 \\
\end{array}
\]

How many zeros are there in \( x \)?

\[
\text{sum}(x::==0) = 2
\]

**Use parentheses**

Because of their relaxed conformability requirements, colon operators are not associative even when the underlying operator is. For instance, you expect \((a+b)+c == a+(b+c)\), at least ignoring numerical roundoff error. Nevertheless, \((a+b):+c == a:+(b:+c)\) does not necessarily hold. Consider what happens when

\[
a: 1 \times 4 \\
b: 5 \times 1 \\
c: 5 \times 4
\]

Then \((a:+b):+c\) is an error because \(a:+b\) is not c-conformable.

Nevertheless, \(a:+(b:+c)\) is not an error and in fact produces a \(5 \times 4\) matrix because \(b:+c\) is \(5 \times 4\), which is c-conformable with \(a\).

**Conformability**

\[
a : op b:\n\]

\[
a: r_1 \times c_1 \\
b: r_2 \times c_2, \text{ } a \text{ and } b \text{ c-conformable} \\
\text{result: } \max(r_1, r_2) \times \max(c_1, c_2)
\]

**Diagnostics**

The colon operators return missing and abort with error under the same conditions that the underlying operator returns missing and aborts with error.

**Also see**

[M-2] exp — Expressions

[M-2] intro — Language definition