

**Example 1a** — Mixture of linear regression models

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## Description

In this example, we show how to fit FMMS with covariates, and we illustrate how you might determine the number of latent classes. For an example without covariates and for a conceptual overview of FMMS, see [\[FMM\] fmm intro](#).

## Remarks and examples

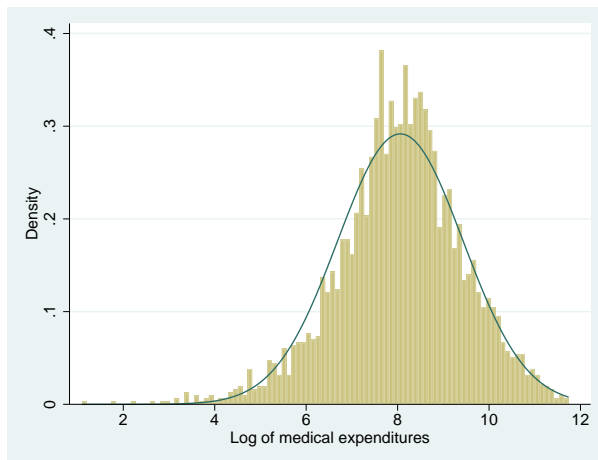
stata.com

Medical expenditures vary greatly from person to person. We believe that some of the variation may be due to having different types of medical care users. We might think of these types as low spenders, average spenders, and high spenders. Because we cannot necessarily tell which group a person belongs to, an FMM may be appropriate for these data.

We use an abbreviated version of `mus03data.dta` from [Cameron and Trivedi \(2022, chap. 3\)](#). `mus03sub.dta` contains information on the log of medical expenditures, `lmedexp`. For brevity, we use only the variables `female`, `age`, `income`, and `totchr`, the last variable recording the number of chronic health problems.

First, let us look at the distribution of medical expenditures.

```
. use https://www.stata-press.com/data/r17/mus03sub
(Abbreviated dataset mus03data from Cameron and Trivedi (2010))
. histogram lmedexp, bin(100) normal
(bin=100, start=1.0986123, width=.10642325)
```



The variable `lmedexp` looks approximately normally distributed. Indeed, it looks as if it may come from a single normal distribution. However, our model includes covariates, and this histogram does not give us an indication of how the regression models may differ across groups. We start by fitting the three-group model, but we will certainly want to check whether a model with a single distribution or with two distributions is a better fit for these data.

## 2 Example 1a — Mixture of linear regression models

```
. fmm 3: regress lmedexp income c.age##c.age totchr i.sex
```

Fitting class model:

```
Iteration 0: (class) log likelihood = -3246.3993
```

```
Iteration 1: (class) log likelihood = -3246.3993
```

Fitting outcome model:

```
Iteration 0: (outcome) log likelihood = -4700.2736
```

```
Iteration 1: (outcome) log likelihood = -4700.2736
```

Refining starting values:

```
Iteration 0: (EM) log likelihood = -7482.765
```

```
Iteration 1: (EM) log likelihood = -7327.5583
```

```
Iteration 2: (EM) log likelihood = -7271.2407
```

```
Iteration 3: (EM) log likelihood = -7254.4109
```

```
Iteration 4: (EM) log likelihood = -7246.0793
```

```
Iteration 5: (EM) log likelihood = -7238.679
```

```
Iteration 6: (EM) log likelihood = -7231.9742
```

```
Iteration 7: (EM) log likelihood = -7226.4046
```

```
Iteration 8: (EM) log likelihood = -7222.1152
```

```
Iteration 9: (EM) log likelihood = -7219.0098
```

```
Iteration 10: (EM) log likelihood = -7216.9001
```

```
Iteration 11: (EM) log likelihood = -7215.5809
```

```
Iteration 12: (EM) log likelihood = -7214.8641
```

```
Iteration 13: (EM) log likelihood = -7214.5912
```

```
Iteration 14: (EM) log likelihood = -7214.6342
```

```
Iteration 15: (EM) log likelihood = -7214.8937
```

```
Iteration 16: (EM) log likelihood = -7215.2936
```

```
Iteration 17: (EM) log likelihood = -7215.7769
```

```
Iteration 18: (EM) log likelihood = -7216.3017
```

```
Iteration 19: (EM) log likelihood = -7216.8377
```

```
Iteration 20: (EM) log likelihood = -7217.3632
```

note: EM algorithm reached maximum iterations.

Fitting full model:

```
Iteration 0: log likelihood = -4734.6429
```

```
Iteration 1: log likelihood = -4733.3724
```

```
Iteration 2: log likelihood = -4732.1323
```

```
Iteration 3: log likelihood = -4731.0186
```

```
Iteration 4: log likelihood = -4729.3225
```

```
Iteration 5: log likelihood = -4727.7218
```

```
Iteration 6: log likelihood = -4727.6741
```

```
Iteration 7: log likelihood = -4727.6738
```

Finite mixture model

Number of obs = 2,955

Log likelihood = -4727.6738

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
1.Class	(base outcome)					
2.Class _cons	1.162296	.292186	3.98	0.000	.5896216	1.73497
3.Class _cons	-1.153202	.3188697	-3.62	0.000	-1.778175	-.5282289

Class: 1  
 Response: lmedexp  
 Model: regress

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
lmedexp						
income	.0059804	.002604	2.30	0.022	.0008768	.0110841
age	.1201823	.2926979	0.41	0.681	-.4534951	.6938597
c.age#c.age	-.0007572	.0019417	-0.39	0.697	-.0045628	.0030483
totchr	.9223744	.0810612	11.38	0.000	.7634974	1.081251
sex						
Female	.0576508	.1453985	0.40	0.692	-.227325	.3426266
_cons	.6300965	10.96433	0.06	0.954	-20.8596	22.11979
var(e.lmed~p)	1.43183	.1533984			1.160642	1.766382

Class: 2  
 Response: lmedexp  
 Model: regress

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
lmedexp						
income	.0023725	.0012209	1.94	0.052	-.0000205	.0047655
age	.2136658	.1075408	1.99	0.047	.0028897	.424442
c.age#c.age	-.0013195	.0007152	-1.84	0.065	-.0027213	.0000823
totchr	.3106586	.0292864	10.61	0.000	.2532583	.3680589
sex						
Female	-.0918924	.0543976	-1.69	0.091	-.1985097	.0147249
_cons	-.9546721	4.017561	-0.24	0.812	-8.828947	6.919602
var(e.lmed~p)	.7966127	.0805009			.6534764	.9711013

Class: 3  
 Response: lmedexp  
 Model: regress

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
lmedexp						
income	.0009315	.0048146	0.19	0.847	-.0085049	.0103679
age	-.2645947	.2637125	-1.00	0.316	-.7814618	.2522724
c.age#c.age	.0015761	.001754	0.90	0.369	-.0018616	.0050138
totchr	.186475	.0647115	2.88	0.004	.0596427	.3133072
sex						
Female	-.1761484	.1371471	-1.28	0.199	-.4449517	.0926549
_cons	20.79524	9.853989	2.11	0.035	1.481775	40.1087
var(e.lmed~p)	.3846891	.0983236			.2331038	.634849

That is a lot of output! Let's start with the part of the output that is probably familiar if you have used `regress`. We have one regression table for each class. The coefficient estimates here are interpreted just as you do the coefficients from a linear regression model. Because the dependent variable is log transformed, we can interpret the coefficients in terms of a percentage change. For example, a one-unit increase in `totchr` results in an 18.6% increase in medical expenditures for class 3, all else held constant. The estimates for each class also include a variance term. So, we see that the first class has much higher variability than the third.

The first table in the output gives the coefficients for the latent class membership, next to `1.Class`, `2.Class`, and `3.Class` at the top of the table. These coefficients can be interpreted in the same manner as you interpret the coefficients from multinomial logistic regression (`mlogit`), which is to say that they are difficult to interpret. However, the postestimation command `estat lcprob` will turn them into probabilities.

```
. estat lcprob, nose
```

Latent class marginal probabilities Number of obs = 2,955

	Margin
Class	
1	.2215875
2	.708474
3	.0699385

We see that individuals in the population fall into the three classes in proportions 0.22, 0.71, and 0.07. Notice that we specified the `nose` option above. `estat lcprob` can be slow because it is time consuming to compute standard errors when there are a lot of covariates in the model. When fitting preliminary models, we might not be concerned about standard errors of the latent class probabilities, so we use the `nose` option to speed things up.

We have estimated that about 22% of observations are in group 1, about 71% are in group 2, and about 7% are in group 3. But, we still do not know which group corresponds to which spending class. If we want to calculate the level of spending for each group, we can use `estat lcmean` to calculate the marginal means for each class; see [FMM] `estat lcmean`.

```
. estat lcmean
```

Latent class marginal means Number of obs = 2,955

		Delta-method				[95% conf. interval]	
		Margin	std. err.	z	P> z		
1	<code>lmedexp</code>	7.185846	.1572402	45.70	0.000	6.877661	7.494031
2	<code>lmedexp</code>	8.143981	.0469051	173.63	0.000	8.052049	8.235914
3	<code>lmedexp</code>	10.15809	.1712913	59.30	0.000	9.822369	10.49382

We see that class 1 corresponds to low spenders, class 2 corresponds to average spenders, class 3 corresponds to high spenders.

Because medical expenditures for class 1 and class 2 are relatively close to each other, compared with class 3, we may be tempted to fit a model with two classes. We may also compare our model with a model with one class, which reduces to a linear regression.

First, we store our estimates from the model with three latent classes with the name `fmm3` by using `estimates store`.

```
. estimates store fmm3
```

Then, we fit a model with two classes and then a model with one class, storing the results of each model in `fmm2` and `fmm1`, respectively.

```
. fmm 2: regress lmedexp income c.age##c.age totchr i.sex
(output omitted)
. estimates store fmm2
. fmm 1: regress lmedexp income c.age##c.age totchr i.sex
(output omitted)
. estimates store fmm1
```

Finally, we use `estimates stats` to compare the models.

```
. estimates stats fmm1 fmm2 fmm3
```

Akaike's information criterion and Bayesian information criterion

Model	N	ll(null)	ll(model)	df	AIC	BIC
fmm1	2,955	.	-4807.386	7	9628.772	9670.711
fmm2	2,955	.	-4758.177	15	9546.354	9636.223
fmm3	2,955	.	-4727.674	23	9501.348	9639.147

Note: BIC uses  $N$  = number of observations. See [R] [BIC note](#).

The Akaike information criterion (AIC) clearly favors the three-component model, whereas the Bayesian information criterion (BIC) marginally favors the two-component model; see [R] [estat ic](#) for more information about the two criteria.

We will proceed with the three-component model.

## □ Technical note

We might be tempted to use a likelihood-ratio test (see [R] [lrtest](#)) to help us decide how many latent classes to fit. However, a model with  $g - 1$  classes with covariates for the mean is not nested in the model extended to  $g$  classes because of the additional equation for the mean of the  $g$ th component. The model with  $g - 1$  classes could be viewed as the model with  $g$  classes with variance components of the  $g$ th class model going to zero. But the parameter value of zero lies on the boundary of the parameter space, and the standard regularity conditions necessary for the likelihood-ratio test do not hold. See [McLachlan and Peel \(2000, 185\)](#) for a detailed explanation. □

## References

- Cameron, A. C., and P. K. Trivedi. 2022. *Microeconometrics Using Stata*. 2nd ed. College Station, TX: Stata Press.
- McLachlan, G. J., and D. Peel. 2000. *Finite Mixture Models*. New York: Wiley.

## Also see

[FMM] **fm intro** — Introduction to finite mixture models

[FMM] **fm regress** — Finite mixtures of linear regression models

[FMM] **estat lmean** — Latent class marginal means

[FMM] **estat lprob** — Latent class marginal probabilities