finportfolio — Financial portfolio selection+

+This command is part of StataNow.

Description

`finportfolio` calculates portfolio weights given a collection of returns and an optimization objective. Equal weights, minimum variance weights, and maximum Sharpe ratio weights are supported. Weights can be calculated with and without short selling (negative weights).

Quick start

Compute global minimum variance portfolio weights from return variables `r1`, `r2`, `r3`, and `r4`

```
finportfolio minvariance r1 r2 r3 r4
```

Same as above, but require nonnegative weights (no short selling)

```
finportfolio minvariance r1 r2 r3 r4, noshort
```

Find the minimum variance portfolio that hits a target return of 0.05

```
finportfolio minvariance r1 r2 r3 r4, target(0.05)
```

Same as above, but require nonnegative weights

```
finportfolio minvariance r1 r2 r3 r4, target(0.05) noshort
```

Find the global minimum variance portfolio, and sort the assets by portfolio weight in the output

```
finportfolio minvariance r1 r2 r3 r4, sort(ascending)
```

Compute the portfolio return using equal weights for returns `r1` through `r10`

```
finportfolio equal r1-r10
```

Set custom portfolio weights for three return variables

```
matrix myweights = (0.25, 0.5, 0.25)
matrix colnames myweights = r1 r2 r3
finportfolio fixed r1-r3, at(myweights)
```

Find the portfolio weights that maximize the Sharpe (return-to-risk) ratio with risk-free rate `rf`

```
finportfolio maxsharpe r1-r10, rfrate(rf)
```

Menu

Statistics > Financial statistics > Financial portfolio selection

Syntax

`finportfolio` *method varlist* [*if*] [*in*] [, *options*]

method	Description
`minvariance`	minimum variance weights
`maxsharpe`	maximum Sharpe ratio weights
`equal`	equal weights
`fixed`	fixed weights

options	Description
Main	
`rfrate(`*varname*`)`	specify variable containing the risk-free rate
`generate(`*varname*[`, label("`*label*`") replace`]`)`	generate variable containing portfolio returns
`novarlabel`	display variable names rather than variable labels
`sort(`*sortmethod*`)`	specify sort order for assets in the output
minvar_options	options for method `minvariance`
maxsharpe_options	options for method `maxsharpe`
fixed_option	option for method `fixed`

You must `tsset` your data before using `finportfolio`; see [TS] **tsset**.

`rfrate()` may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

`collect` is allowed; see [U] **11.1.10 Prefix commands**.

sortmethod	Description
`ascending`	sort assets by ascending values of weights
`descending`	sort assets by descending values of weights
`alphabetical`	sort by asset label

minvar_options	Description
Main	
`target(#)`	specify target return
`noshort`	disallow short selling (negative weights)
`gamma(#)`	tuning parameter when `noshort` is specified
Maximization	
maximize_options	control the optimization when `noshort` is specified

maxsharpe_options	Description
Main	
`noshort`	disallow short selling (negative weights)
`gamma(#)`	tuning parameter when `noshort` is specified
Maximization	
maximize_options	control the optimization when `noshort` is specified

fixed_option	Description
Main	
* at(*wspec*)	use specified weights

*at() is required.

Options

Options are presented under the following headings:

Options for all methods
Options for finportfolio minvariance
Options for finportfolio maxsharpe
Option for finportfolio fixed

Options for all methods

Main

rfrate(*varname*) specifies a risk-free asset. When rfrate() is specified, the Sharpe ratio is $(r - r^f)/\sigma$, where r is the portfolio return, r^f is the average risk-free rate, and σ is the portfolio standard deviation. When rfrate() is not specified, the risk-free rate is set to 0.

generate(*varname*[, label("*label*") replace]) generates a new variable to store the return series of the portfolio that finportfolio creates. The generated variable is stored in double precision, regardless of the default storage type set by using set type.

label("*label*") specifies the variable label for the newly generated variable.

replace allows an existing variable named *varname* to be replaced.

novarlabel specifies that the variable names be displayed instead of the variable labels. By default, variable labels are used to label assets in the output.

sort(*sortmethod*) specifies how to sort the assets in the portfolio output. By default, assets are listed in the order specified in *varlist*.

sort(ascending) sorts assets by their portfolio weight, from lowest weighted to highest weighted.

sort(descending) sorts assets by their portfolio weight, from highest weighted to lowest weighted.

sort(alphabetical) sorts assets alphabetically by their variable labels or, if the novarlabel option is specified, by their variable names.

Options for finportfolio minvariance

Main

target(#) specifies the desired rate of return in the units of the assets in *varlist* of finportfolio.

noshort specifies that all weights must be positive. This is also known as a no-short-selling condition. By default, negative weights are allowed. noshort may be used with or without a target return.

gamma(#) specifies the penalty term of the barrier function used to compute no-short weights. The smaller gamma is, the more that small, nonzero weights are driven toward zero. The penalty term must be positive.

When a target return is specified, the default is $k \times r^{\text{target}}$ with $k = 0.0001$. When no target return is specified, the default is $k \times (r^{\max} + r^{\min})/2$, where $r^{\max}$ is the maximum average return of the returns in *varlist* and $r^{\min}$ is the minimum average return in *varlist*.

Maximization

maximize_options: difficult, technique(nr), technique(dfp), technique(bfgs), iterate(#), [no]log, trace, gradient, showstep, hessian, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(*init_specs*); see [R] **Maximize**.

These options are allowed only with the noshort option.

Options for finportfolio maxsharpe

Main

noshort specifies that all weights must be positive. This is also known as a no-short-selling condition. By default, negative weights are allowed. noshort may be used with or without a target return.

gamma(#) specifies the penalty term of the barrier function used to compute no-short weights. The larger gamma is, the more that small, nonzero weights are driven towards zero. The penalty term must be positive.

The default is $k \times (r^{\max} + r^{\min})/2$, where $k = 0.0001$, $r^{\max}$ is the maximum average return of the returns in *varlist* and $r^{\min}$ is the minimum average return in *varlist*.

Maximization

maximize_options: difficult, technique(nr), technique(dfp), technique(bfgs), iterate(#), [no]log, trace, gradient, showstep, hessian, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(*init_specs*); see [R] **Maximize**.

These options are allowed only with the noshort option.

Option for finportfolio fixed

Main

at(*wspec*) specifies the fixed weights to be used when method fixed is specified. This option is required with method fixed, and the weights must sum to one.

wspec may be one of

matname [, copy]

[# . . .]

name = # [*name* = # [. . .]]

You can specify the fixed weights in one of three ways.

1. You can specify the name of a Stata row vector containing the weights such as at(myrowvec). If your row vector has column names matching the variables in *varlist*, then at() matches the weights accordingly, regardless of the order. You may omit weights from the row vector myrowvec, and if you do so, the weights corresponding to the omitted variables will be zero.

If you specify the `copy` suboption, for example, `at(myrowvec, copy)`, then the values in `myrowvec` are used as the weights in a direct positional copy. The first entry is the first weight, the second entry is the second weight, and so forth. When `copy` is specified, the length of the row vector `myrowvec` must be equal to the number of variables in *varlist*.

2. You can specify a list of numbers directly, for example, `at(.1 .2 .3 .4)`. The weights in the list are assigned positionally. If the list has fewer elements than the *varlist* of returns, then the remaining weights are set to zero.

3. You can specify a list of variable names and numbers, for example, `at(r1=0.7 r3=0.3)`. The specified variables are given the weight assigned. Weights on other returns, if any, are set to zero.

   The following methods are equivalent for specifying weights of 0.1, 0.3, and 0.6 to returns `r1`, `r2`, and `r3`, respectively:

```
. finportfolio fixed r1 r2 r3, at(0.1 0.3 0.6)

. finportfolio fixed r1 r2 r3, at(r1=0.1 r2=0.3 r3=0.6)

. matrix wgt_default = (0.1, 0.3, 0.6)
. finportfolio fixed r1 r2 r3, at(wgt_default, copy)

. matrix wgt_proper = (0.1, 0.3, 0.6)
. matrix colnames wgt_proper = r1 r2 r3
. finportfolio fixed r1 r2 r3, at(wgt_proper)
```

   If we wish to assign weights only to `r1` and `r2`, we can type any of

```
. finportfolio fixed r1 r2 r3, at(0.1 0.9)

. finportfolio fixed r1 r2 r3, at(r1=0.1 r2=0.9)

. matrix wgt_r1r2 = (0.1, 0.9)
. matrix colnames wgt_r1r2 = r1 r2
. finportfolio fixed r1 r2 r3, at(wgt_r1r2)
```

   If we wish to assign weights only to `r1` and `r3`, we can type any of

```
. finportfolio fixed r1 r2 r3, at(r1=0.1 r3=0.9)

. matrix wgt_r1r3 = (0.1, 0.9)
. matrix colnames wgt_r1r3 = r1 r3
. finportfolio fixed r1 r2 r3, at(wgt_r1r3)
```

Remarks and examples

Remarks are presented under the following headings:

Introduction

Portfolios are combinations of assets. A portfolio is represented by a vector, each entry of which is the fraction of the portfolio that is invested in a particular asset. Portfolios allow an investor to balance the risks and returns of various individual assets.

The `finportfolio` command creates portfolio weights according to various criteria. The simplest portfolio, equal weights, is supported. User-defined weights for tailored portfolios are also supported. The financial literature has introduced several types of portfolios that maximize or minimize some objective. `finportfolio` can find the minimum variance portfolio of a collection of assets, globally or given a target return, with or without short sales. `finportfolio` can also compute portfolio weights that maximize the Sharpe (return-to-risk) ratio, with or without short sales.

For an introduction to portfolio selection, see Hurn et al. (2020, chap. 1). A more detailed discussion of portfolio weights, including minimum variance and maximum Sharpe ratio weights, is in Campbell (2018, chap. 2).

Minimum variance portfolios

The minimum variance method aims to produce the portfolio of assets with the lowest variance. This portfolio can be unconstrained, that is, having global minimum variance, or it can be constructed subject to a specified target return. In the latter case, the portfolio weights calculated are those that reach the target return at the lowest possible variance. In addition, portfolios can be constructed with or without short selling, that is, with or without negative weights.

▷ Example 1: Global minimum variance portfolio

We have stock prices for 25 fictional companies, along with true data on the S&P 500 index and the federal funds rate. We describe some of the variables we will use in our examples.

```
. use https://www.stata-press.com/data/r19/finex
(Fictional stock price data)

. describe datestr datem sp500 fedfunds aaa-cph

Variable      Storage   Display    Value
    name         type    format    label      Variable label
-------------------------------------------------------------------------------
datestr         str11   %11s                  String date
datem           int     %tm                   Monthly date
sp500           double  %10.0g                S&P 500
fedfunds        float   %9.0g                 Federal funds rate
aaa             float   %9.0g                 Apex Adventure Airlines
afh             float   %9.0g                 Apex Film Holdings
ard             float   %9.0g                 Arbor Renewable Dynamics
cph             float   %9.0g                 Coastal Property Holdings
```

The datem variable has been previously declared as the time variable for this time-series dataset by using the tsset command. We confirm this with tsreport.

```
. tsreport
Time variable: datem
─────────────────────────
Starting period =  1955m1
Ending period   = 2019m12
Number of obs   =     780
Number of gaps  =       0
```

The data run from the first month of 1955 to the last month of 2019 without any gaps.

Our dataset contains information on stock prices, but portfolios are constructed on the basis of returns. We use finreturns to generate 25 returns series.

```
. quietly finreturns acme-tks, simple(r_) multiply(100)
```

The simple() option requests simple returns, and the argument r_ specifies that the simple returns be stored in a collection of variables called r_acme through r_tks. The multiply(100) option multiplies all simple returns by 100, so that a simple return of, say, 0.005 becomes 0.5, interpretable as a percentage change. The data are monthly, so the generated variables are monthly percentage returns.

Now we are ready to generate some portfolios using the individual stock returns. Let's use returns from stocks aaa, afh, and ard, stored in respective variables r_aaa, r_afh, and r_ard, to obtain a minimum variance portfolio. We use *varlist* to refer to these three variables.

```
. finportfolio minvariance r_aaa-r_ard
Global minimum variance portfolio
Short selling allowed
Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.0123108
AFH	.3763484
ARD	.6113408

```
Portfolio return    = 0.4277
Portfolio std. dev. = 1.4432
Risk-free rate      = 0.0000
Sharpe ratio        = 0.2964
```

The table reports the minimum variance weights for the stock returns. The minimum variance portfolio puts 61% into ARD, 38% into AFH, and the rest into AAA. This portfolio has a historical return of 0.43% at a monthly rate. The corresponding portfolio standard deviation is 1.44%. Because we did not specify a risk-free rate, the risk-free rate defaults to 0. Finally, the Sharpe (return-to-risk) ratio is about 0.30.

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▷ Example 2: The minimum variance portfolio with a target return

Continuing with example 1, let's look at the performance of the individual stocks in this portfolio.

```
. summarize r_aaa-r_ard
```

Variable	Obs	Mean	Std. dev.	Min	Max
r_aaa	779	.5583062	2.04691	-10.97514	7.364605
r_afh	779	.6601578	2.400168	-10.59864	8.662755
r_ard	779	.282029	1.871475	-5.362448	6.507083

The three stocks have quite different average monthly rates of return: 0.56%, 0.66%, and 0.28%, respectively. The global minimum variance portfolio puts the majority of its weight, 61%, into the stock with the lowest return; this is because that stock also has the lowest variance, and the method is designed to minimize variance. We may be willing to tolerate additional variance above the global minimum if that variance is associated with higher returns. The `target()` option finds the minimum variance portfolio that also hits a specified target return.

The global minimum variance portfolio had an average return of 0.43%. We request a slightly more aggressive portfolio, seeking a 0.5% return monthly. This monthly return corresponds to a 6.16% annual return (annualized using $1.005^{12} - 1$).

```
. finportfolio minvariance r_aaa-r_ard, target(0.5)

Minimum variance portfolio with fixed return
Short selling allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.1620223
AFH	.458066
ARD	.3799117

```
Portfolio return     = 0.5000 (fixed return)
Portfolio std. dev. = 1.5451
Risk-free rate       = 0.0000
Sharpe ratio         = 0.3236
```

The weight on ARD has fallen from 61% to 38%, and the weights on the other two assets have increased, because the portfolio has shifted away from low-return assets to high-return assets to meet the target return.

Below the weights, we can see the portfolio return is exactly 0.50% per month, as requested. The portfolio standard deviation is now 1.55%, an increase from the global minimum of 1.44% we obtained in example 1. This is a general feature: portfolios trade off risk and return, so higher return is associated with higher risk (variance). The Sharpe ratio, which is return divided by standard deviation, has increased, indicating that this new portfolio has increased returns more than it increased risk.

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▷ Example 3: The minimum variance portfolio without short selling

Suppose additional stocks such as CPH were added to the portfolio.

```
. finportfolio minvariance r_aaa-r_cph

Global minimum variance portfolio
Short selling allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12

------------------------
             |    Weight
-------------+----------
         AAA |  .0351662
         AFH |  .3867707
         ARD |  .6259844
         CPH | -.0479213
------------------------

Portfolio return    = 0.4312
Portfolio std. dev. = 1.4419
Risk-free rate      = 0.0000
Sharpe ratio        = 0.2991
```

The fourth stock, CPH, has entered with a negative weight. Negative weights are allowed in the unconstrained minimum variance problem and indicate assets that should be sold short. However, not all portfolios allow short selling. The `noshort` option enforces positive weights on all assets in the portfolio, effectively disallowing short selling.

```
. finportfolio minvariance r_aaa-r_cph, noshort
Iteration 0:  Penalized variance =  2.6140645
Iteration 1:  Penalized variance =  2.2131253
Iteration 2:  Penalized variance =  2.1129517
Iteration 3:  Penalized variance =  2.0984588
Iteration 4:  Penalized variance =  2.0942991
Iteration 5:  Penalized variance =  2.0915686
Iteration 6:  Penalized variance =  2.0855053
Iteration 7:  Penalized variance =  2.0835057
Iteration 8:  Penalized variance =  2.0835037
Iteration 9:  Penalized variance =  2.0835037

Global minimum variance portfolio
Short selling not allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12

------------------------
             |    Weight
-------------+----------
         AAA |  .0131033
         AFH |  .3757663
         ARD |  .6108308
         CPH |  .0002995
------------------------

Portfolio return    = 0.4278
Portfolio std. dev. = 1.4432
Risk-free rate      = 0.0000
Sharpe ratio        = 0.2964
```

Now the stock CPH appears with a positive weight.

The output of finportfolio with noshort now displays an iteration log. The restriction on short selling places a nonlinear constraint on the optimization problem, so iterative methods must be used to find the optimal portfolio.

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▷ Example 4: Target return without short sellling

The target() and noshort options can be combined. There are two important implications when combining these options. First, the portfolio chosen with noshort will have variance at least as high as, and usually higher than, the portfolio chosen without noshort if the unconstrained portfolio involves short selling. Second, without short selling, the target return on the portfolio is bounded above by the highest-return asset in the portfolio.

The four assets from example 3 can be used to illustrate these points. Using returns r_aaa through r_cph, we seek a minimum variance portfolio with a target return of 0.5% monthly:

```
. finportfolio minvariance r_aaa-r_cph, target(0.5)

Minimum variance portfolio with fixed return
Short selling allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12

                 |     Weight
-----------------+-----------
             AAA |   .2272578
             AFH |   .4851069
             ARD |   .4876142
             CPH |  -.1999789

Portfolio return    = 0.5000 (fixed return)
Portfolio std. dev. = 1.5201
Risk-free rate      = 0.0000
Sharpe ratio        = 0.3289
```

The portfolio achieves a standard deviation of 1.52% and puts a negative weight on stock CPH. Seeking the same return again but without short selling, we obtain

```
. finportfolio minvariance r_aaa-r_cph, target(0.5) noshort
Iteration 0:  Penalized variance =  2.7490505
Iteration 1:  Penalized variance =  2.4205764
Iteration 2:  Penalized variance =  2.4077978
Iteration 3:  Penalized variance =  2.4035639
Iteration 4:  Penalized variance =   2.401265
Iteration 5:  Penalized variance =  2.3954566
Iteration 6:  Penalized variance =   2.388044
Iteration 7:  Penalized variance =  2.3880425
Iteration 8:  Penalized variance =  2.3880425

Minimum variance portfolio with fixed return
Short selling not allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.1620458
AFH	.4580245
ARD	.3798646
CPH	.0000651

```
Portfolio return    = 0.5000 (fixed return)
Portfolio std. dev. = 1.5451
Risk-free rate      = 0.0000
Sharpe ratio        = 0.3236
```

which achieves the same 0.5% return, with all positive weights, but a slightly higher portfolio standard deviation of 1.55%.

Seeking an even more aggressive portfolio, we set the monthly return to 0.62% and continue to limit ourselves to positive weights.

```
. finportfolio minvariance r_aaa-r_cph, target(0.62) noshort
Finding initial values...
Iteration 0:  Penalized variance =  4.5106848
Iteration 1:  Penalized variance =  4.4282026
Iteration 2:  Penalized variance =  4.4111366  (backed up)
Iteration 3:  Penalized variance =  4.3989881  (backed up)
Iteration 4:  Penalized variance =  4.3792764
Iteration 5:  Penalized variance =  4.3181569
Iteration 6:  Penalized variance =  4.2592891
Iteration 7:  Penalized variance =  4.2446185
Iteration 8:  Penalized variance =  4.2410388
Iteration 9:  Penalized variance =  4.2406448
Iteration 10: Penalized variance =  4.2405455
Iteration 11: Penalized variance =  4.2405251
Iteration 12: Penalized variance =  4.2405249

Minimum variance portfolio with fixed return
Short selling not allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.3933788
AFH	.6063672
ARD	.000222
CPH	.000032

```
Portfolio return    = 0.6200 (fixed return)
Portfolio std. dev. = 2.0589
Risk-free rate      = 0.0000
Sharpe ratio        = 0.3011
```

More and more weight is being placed on the highest-returning assets in the group, namely, AAA and AFH. The portfolio standard deviation also grows. In going from a target of 0.5% to 0.62%, the Sharpe ratio falls, indicating that risk is being taken on faster than reward to generate the required return.

Increasing the desired return outside the range of observed returns, say, to 0.67% per month, will lead to an error; there is no linear combination of assets with strictly positive coefficients that can generate a return larger than the largest return in the asset group.

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Maximum Sharpe ratio portfolios

The methods discussed previously all minimized the variance of the portfolio subject to some constraints. The next methods maximize the Sharpe ratio. The Sharpe ratio is defined as

$$S_p = \frac{r_p - r^f}{\sigma_p}$$

where S_p is the Sharpe ratio of portfolio p, r_p is the average return on the portfolio, r^f is the risk-free rate, and σ_p is the standard deviation of the portfolio. The numerator is the so-called excess return of the portfolio over the risk-free rate, and the denominator captures risk. An increase in the excess return improves the Sharpe ratio; an increase in portfolio standard deviation reduces the Sharpe ratio.

▷ Example 5: The maximum Sharpe ratio portfolio

We next use our four example stocks again to maximize the Sharpe ratio, allowing short selling.

```
. finportfolio maxsharpe r_aaa-r_cph

Maximum Sharpe ratio portfolio
Short selling allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12

-------------------------
             |     Weight
-------------+-----------
         AAA |   .3103278
         AFH |   .5276325
         ARD |    .427776
         CPH |  -.2657363
-------------------------

Portfolio return    = 0.5297
Portfolio std. dev. = 1.5982
Risk-free rate      = 0.0000
Sharpe ratio        = 0.3315
```

Allowing short selling, the return associated with the maximum Sharpe ratio is 0.53% per month, with a standard deviation of 1.60%; the maximum Sharpe ratio achieved is 0.33. Three of the stocks have a positive weight (AAA, AFH, and ARD). One stock (CPH) has a negative weight.

If we disallow short selling, we obtain

```
. finportfolio maxsharpe r_aaa-r_cph, noshort
Iteration 0:  Penalized Sharpe ratio =  .29721912
Iteration 1:  Penalized Sharpe ratio =  .31860978
Iteration 2:  Penalized Sharpe ratio =  .32122602
Iteration 3:  Penalized Sharpe ratio =   .3218071
Iteration 4:  Penalized Sharpe ratio =   .3230303
Iteration 5:  Penalized Sharpe ratio =  .32347348
Iteration 6:  Penalized Sharpe ratio =  .32347557
Iteration 7:  Penalized Sharpe ratio =  .32347561

Maximum Sharpe ratio portfolio
Short selling not allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12

-------------------------
             |     Weight
-------------+-----------
         AAA |   .1855063
         AFH |   .4702679
         ARD |   .3433929
         CPH |   .0008328
-------------------------

Portfolio return    = 0.5112
Portfolio std. dev. = 1.5780
Risk-free rate      = 0.0000
Sharpe ratio        = 0.3240
```

The maximum Sharpe ratio is now 0.32. Because we have disallowed short selling, the maximum Sharpe ratio is lower than it was in the unconstrained case.

So far, the implied risk-free return rate was 0. We can change this assumption with the `rfrate()` option, which takes a variable containing the return on a risk-free asset. In our data, we have the federal funds rate, `fedfunds`, which is a benchmark interest rate that is generally considered nearly risk free. It is quoted at an annual rate. We create a new variable with the monthly return on the federal funds rate and use it as the risk-free rate. We continue to disallow short selling.

```
. generate double rf = fedfunds / 12

. finportfolio maxsharpe r_aaa-r_cph, rfrate(rf) noshort
Iteration 0:  Penalized Sharpe ratio =  .04936168
Iteration 1:  Penalized Sharpe ratio =  .06507217  (backed up)
Iteration 2:  Penalized Sharpe ratio =  .06670697  (backed up)
Iteration 3:  Penalized Sharpe ratio =  .06937459
Iteration 4:  Penalized Sharpe ratio =  .07818756
Iteration 5:  Penalized Sharpe ratio =  .10066681
Iteration 6:  Penalized Sharpe ratio =  .10396478
Iteration 7:  Penalized Sharpe ratio =  .10463189
Iteration 8:  Penalized Sharpe ratio =  .10506173
Iteration 9:  Penalized Sharpe ratio =  .10541845
Iteration 10: Penalized Sharpe ratio =  .10647209
Iteration 11: Penalized Sharpe ratio =   .1080777
Iteration 12: Penalized Sharpe ratio =  .10808199
Iteration 13: Penalized Sharpe ratio =  .10808199

Maximum Sharpe ratio portfolio
Short selling not allowed

Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.1393517
AFH	.8603745
ARD	.0001329
CPH	.000141

```
Portfolio return    = 0.6459
Portfolio std. dev. = 2.2554
Risk-free rate      = 0.4002 (rf)
Sharpe ratio        = 0.1082
```

Now the goal is to reach the maximum return-to-risk ratio net of the risk-free rate. The optimal weights obtain a Sharpe ratio of 0.11, meaning that for every 1% increase in standard deviation, the optimal portfolio attains an 0.11% increase in monthly return, net of the risk-free rate. The portfolio return is 0.65% monthly, with a standard deviation of 2.26%. This return is close to the maximum return that the constrained portfolio can attain, namely, the return of the best-performing asset, 0.66%.

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Portfolios with fixed weights

The previous two methods minimized or maximized a criterion function to arrive at optimal weights. Two methods are available that use fixed portfolio weights and thus do not perform any optimization: equal weights and fixed user-defined weights.

▷ Example 6: A portfolio with equal weights

An equally weighted portfolio simply assigns equal weights to all assets in the portfolio. Continuing with our four example assets, the equally weighted portfolio is

```
. finportfolio equal r_aaa-r_cph
Equally weighted portfolio
Number of obs = 779
Sample: 1955m2 thru 2019m12

             |    Weight
-------------+----------
         AAA |       .25
         AFH |       .25
         ARD |       .25
         CPH |       .25
------------------------

Portfolio return    = 0.4809
Portfolio std. dev. = 1.6167
Risk-free rate      = 0.0000
Sharpe ratio        = 0.2975
```

Despite its simplicity, the equally weighted portfolio can be useful as a benchmark or to explore portfolio characteristics when a simple allocation rule is made. This equally weighted portfolio achieves a monthly return of 0.48% with a standard deviation of 1.62%, for a Sharpe ratio of just under 0.30. These characteristics can serve as a baseline to judge the higher-return, lower-variance portfolios generated by the minimum variance and maximum Sharpe ratio methods.

◁

▷ Example 7: A portfolio with user-defined weights

Finally, custom weights can be assigned using the `fixed` method. The weights, which must sum to one, may be specified using a Stata matrix and are then applied to the assets in the portfolio.

```
. matrix w = (0.3, 0.4, 0.2, 0.1)
. matrix colnames w = r_aaa r_afh r_ard r_cph
. finportfolio fixed r_aaa-r_cph, at(w)
Fixed weights portfolio
Number of obs = 779
Sample: 1955m2 thru 2019m12

             |    Weight
-------------+----------
         AAA |        .3
         AFH |        .4
         ARD |        .2
         CPH |        .1
------------------------

Portfolio return    = 0.5303
Portfolio std. dev. = 1.6861
Risk-free rate      = 0.0000
Sharpe ratio        = 0.3145
```

We create the row vector in the first line and properly label its columns in the second line. We then specify the row vector name in `finportfolio`'s `at()` option. The custom weights are 30% on the AAA stock, 40% on the AFH stock, 20% on the ARD stock, and 10% on the CPH stock. The return generated from these weights is 0.53% monthly, with a standard deviation of 1.69%.

Fixed weights can also be specified in the form *varname*=#. We can type

```
. finportfolio fixed r_aaa-r_cph, at(r_aaa=0.3 r_afh=0.6 r_cph=0.1)
Fixed weights portfolio
Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.3
AFH	.6
ARD	0
CPH	.1

```
Portfolio return    = 0.6059
Portfolio std. dev. = 2.0311
Risk-free rate      = 0.0000
Sharpe ratio        = 0.2983
```

Notice that, because we did not specify a weight for variable `r_ard`, its weight was set to 0.

◁

▷ Example 8: Generating portfolio returns

The `generate()` option generates a new variable that contains the return from the portfolio. Let's compute the equally weighted return to illustrate this option.

```
. finportfolio equal r_aaa-r_cph, generate(p_eweight, label("Equal"))
Equally weighted portfolio
Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.25
AFH	.25
ARD	.25
CPH	.25

```
Portfolio return    = 0.4809
Portfolio std. dev. = 1.6167
Risk-free rate      = 0.0000
Sharpe ratio        = 0.2975
```

Similarly, we can compute the no-short-sales minimum variance portfolio and generate its returns.

```
. finportfolio minvariance r_aaa-r_cph, noshort
> generate(p_minvar, label("MinVar"))
Iteration 0:  Penalized variance =  2.6140645
Iteration 1:  Penalized variance =  2.2131253
Iteration 2:  Penalized variance =  2.1129517
Iteration 3:  Penalized variance =  2.0984588
Iteration 4:  Penalized variance =  2.0942991
Iteration 5:  Penalized variance =  2.0915686
Iteration 6:  Penalized variance =  2.0855053
Iteration 7:  Penalized variance =  2.0835057
Iteration 8:  Penalized variance =  2.0835037
Iteration 9:  Penalized variance =  2.0835037
Global minimum variance portfolio
Short selling not allowed
Number of obs = 779
Sample: 1955m2 thru 2019m12
```

	Weight
AAA	.0131033
AFH	.3757663
ARD	.6108308
CPH	.0002995

```
Portfolio return     = 0.4278
Portfolio std. dev. = 1.4432
Risk-free rate       = 0.0000
Sharpe ratio         = 0.2964
```

The two portfolios are now stored in the dataset in variables p_eweight and p_minvar. We can summarize them:

```
. summarize p_eweight p_minvar
```

Variable	Obs	Mean	Std. dev.	Min	Max
p_eweight	779	.4809442	1.616726	-9.104359	6.934312
p_minvar	779	.4277795	1.443208	-6.640849	6.721109

The minimum variance portfolio has lower standard deviation and also slightly lower return. Notice that the means and standard deviations of the portfolios match the computations from the finportfolio output.

◁

Stored results

finportfolio stores the following in r():

Scalars	
r(N)	number of observations
r(rfrate)	risk-free rate
r(target)	target return, if target() specified
r(short)	1 if short selling is allowed, 0 otherwise
r(tmin)	minimum time
r(tmax)	maximum time
r(return)	portfolio return
r(var)	portfolio variance
r(sharpe)	Sharpe ratio
r(gamma)	penalty term gamma, if noshort was specified
r(converged)	1 if converged, 0 otherwise
Macros	
r(newvar)	name of variable used to store portfolio returns
r(method)	method for selecting portfolio weights
Matrices	
r(b)	vector of portfolio weights

Methods and formulas

Methods and formulas are presented under the following headings:

Introduction
Minimum variance
Minimum variance without short sales
Maximum Sharpe ratio
Maximum Sharpe ratio without short sales

Introduction

Throughout, the setting is a collection of K returns, $\mathbf{r} = (r_1, r_2, \ldots, r_K)'$. These returns have $K \times K$ covariance matrix $\mathbf{\Sigma}$. A portfolio is a weighted average of returns. A portfolio with weights $\mathbf{w} = (w_1, w_2, \ldots, w_K)'$ has portfolio return

$$r_w = r_1 w_1 + \cdots + r_K w_K = \mathbf{r}'\mathbf{w}$$

and has portfolio variance

$$\sigma_w^2 = \mathbf{w}'\mathbf{\Sigma}\mathbf{w}$$

The optimization-based portfolio selection techniques are based on either the minimization of a loss criterion or the maximization of a gain criterion.

Minimum variance

The minimum variance method finds the portfolio that minimizes portfolio variance, subject to the constraint that the portfolio weights sum to 1. This is also called the global minimum variance portfolio,

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \\ \text{s.t.} \quad & \sum w_i = 1 \\ & \mathbf{r}'\mathbf{w} = r^{\text{target}} \end{aligned}$$

where r^{target} is the target return specified in the `target()` option. This problem has a quadratic objective subject to a linear constraint. The first-order conditions are

$$\begin{pmatrix} \boldsymbol{\Sigma} & \mathbf{E}' \\ \mathbf{E} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{w} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{d} \end{pmatrix}$$

where

$$\mathbf{E} = \begin{pmatrix} \mathbf{r}' \\ \mathbf{1} \end{pmatrix}$$

$$\mathbf{d} = \begin{pmatrix} r^* \\ 1 \end{pmatrix}$$

and $\mathbf{1}$ is a conforming vector of 1s. This is a linear problem; its solution is $\mathbf{w}^*$, the optimal weights.

Minimum variance without short sales

The minimum variance problem can produce solutions with negative portfolio weights, representing assets that the portfolio holder would sell short. Sometimes, a solution is desired that restricts the portfolio to exhibit only nonnegative weights. The formal problem becomes

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \\ \text{s.t.} \quad & \sum w_i = 1 \\ & \mathbf{r}'\mathbf{w} = r^{\text{target}} \\ & w_i \geq 0 \qquad \forall i \end{aligned}$$

The `noshort` option accomplishes this by augmenting the standard objective function with a barrier function,

$$\min_{\mathbf{w}} \quad \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} - \gamma \sum_{i=1}^{K} \ln(w_i)$$

where γ is a tuning parameter. The default value for γ depends on whether a target return is specified. If it is, the default is $\gamma = 0.0001 r^{\text{target}}$. When no target return is specified, $\gamma = 0.0001(r^{\text{min}} + r^{\text{max}})/2$, where r^{min} and r^{max} are, respectively, the minimum and maximum return values. The result is an approximate solution to the inequality-constrained problem.

Maximum Sharpe ratio

The Sharpe ratio is the ratio of portfolio return (net of a risk-free rate, r^f) to portfolio standard deviation,

$$S(\mathbf{w}) = \frac{\mathbf{r}'\mathbf{w} - r^f}{\sigma(\mathbf{w})}$$

The maximum Sharpe ratio method finds the portfolio weights **w** that maximize the Sharpe ratio.

Maximum Sharpe ratio without short sales

The maximum Sharpe ratio method can produce some weights that are negative, representing assets that the portfolio holder would like to sell short. The `noshort` option restricts the portfolio weights to be nonnegative. This is accomplished by augmenting the objective with a barrier function,

$$\max_{\mathbf{w}} \quad \frac{\mathbf{r}'\mathbf{w} - r^f}{\sigma(\mathbf{w})} + \gamma \sum_{i=1}^{K} \ln(w_i)$$

where γ is a tuning parameter, $\gamma = 0.0001(r^{\text{min}} + r^{\text{max}})/2$, and r^{min} and r^{max} are, respectively, the minimum and maximum return values. The solution to the augmented problem guarantees that portfolio weights will all be nonnegative.

References

Campbell, J. Y. 2018. *Financial Decisions and Markets: A Course in Asset Pricing*. Princeton, NJ: Princeton University Press.

Hurn, S., V. L. Martin, P. C. B. Phillips, and J. Yu. 2020. *Financial Econometric Modeling*. Oxford: Oxford University Press.

Also see

[FIN] **finreturns** — Generate financial returns+