intro 8 — Wald tests vary with nonlinear transforms

Description

After fitting a DSGE model, we often perform tests of structural parameters, and these tests often place nonlinear restrictions on the parameters. The values and rejection rates of a Wald test for different nonlinear expressions of the same null hypothesis are different. We illustrate this issue, show that likelihood-ratio (LR) tests do not have this problem, and illustrate that you can parameterize your model in terms of invertible transforms of each parameter.

Remarks and examples

Remarks are presented under the following headings:

Wald tests vary with nonlinear transforms

Performing a statistical test of whether a structural parameter in a DSGE has a specific value is one of the most frequent forms of inference after `dsge` estimation. The null hypothesis in one of these tests frequently places nonlinear restrictions on the underlying parameters. Two different nonlinear expressions of the same null hypothesis produce different Wald test statistics in finite samples and have different rejection rates. In other words, the Wald test is not invariant to nonlinear transforms of the null hypothesis. The LR test, on the other hand, is invariant to nonlinear transforms of the null hypothesis.

▷ Example 1: Different values from logically equivalent Wald tests

Equations (1)–(5) specify how the observed control variable inflation p_t, the unobserved control variable output growth y_t, and the observed control variable (interest rate) r_t depend on the states z_t and u_t, given the shocks ϵ_t and ξ_t.

$$p_t = \beta E_t(p_{t+1}) + \kappa y_t \tag{1}$$
$$y_t = E_t(y_{t+1}) - \{r_t - E_t(p_{t+1}) - \rho z_t\} \tag{2}$$
$$r_t = (1/\beta)p_t + u_t \tag{3}$$
$$z_{t+1} = \rho z_t + \epsilon_{t+1} \tag{4}$$
$$u_{t+1} = \delta u_t + \xi_{t+1} \tag{5}$$

We estimate the parameters of this model using the macroeconomic data for the United States in `usmacro2.dta`.

```
. use http://www.stata-press.com/data/r15/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
. dsge (p = {beta}*E(F.p) + {kappa}*y)
>      (y = E(F.y) -(r - E(F.p) - {rhoz}*z), unobserved)
>      (r = (1/{beta})*p + u)
>      (F.u = {rhou}*u, state)
>      (F.z = {rhoz}*z, state)
(setting technique to bfgs)
Iteration 0:   log likelihood = -146218.64
Iteration 1:   log likelihood = -5532.4212  (backed up)
Iteration 2:   log likelihood = -1067.4665  (backed up)
Iteration 3:   log likelihood = -938.92415  (backed up)
Iteration 4:   log likelihood = -885.96401  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -880.81744  (not concave)
Iteration 6:   log likelihood = -819.11157  (not concave)
Iteration 7:   log likelihood = -775.50717  (not concave)
Iteration 8:   log likelihood = -766.59529
Iteration 9:   log likelihood = -756.78683  (not concave)
Iteration 10:  log likelihood = -754.40674
Iteration 11:  log likelihood = -753.85297
Iteration 12:  log likelihood = -753.57309
Iteration 13:  log likelihood = -753.57131
Iteration 14:  log likelihood = -753.57131
DSGE model
Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -753.57131
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.5146674	.0783489	6.57	0.000	.3611064	.6682284
kappa	.1659057	.0474074	3.50	0.000	.0729889	.2588224
rhoz	.9545256	.0186424	51.20	0.000	.9179872	.991064
rhou	.7005483	.0452604	15.48	0.000	.6118395	.7892571
sd(e.u)	2.318203	.3047438			1.720916	2.915489
sd(e.z)	.6507118	.1123845			.4304422	.8709814

The interest rate (3) links the nominal interest rate to the inflation rate. The coefficient on inflation is $1/\beta$. We test whether this parameter is 1.5, a common benchmark value in the literature.

```
. testnl 1/_b[beta] = 1.5
  (1)  1/_b[beta] = 1.5
               chi2(1) =        2.24
           Prob > chi2 =        0.1342
```

We do not reject the null hypothesis that $1/\beta$ is 1.5.

If we test the logically equivalent hypothesis that $\beta = 2/3$, the statistic and p-value change.

```
. test _b[beta] =2/3
 ( 1)  [/structural]beta = .6666667
           chi2(  1) =    3.76
         Prob > chi2 =    0.0524
```

The values of these two logically equivalent Wald tests differ because Wald tests are not invariant to nonlinear transformation. This issue is well known in the literature; see Gregory and Veall (1985) and Phillips and Park (1988) for details. In this example, the inference of failing to reject the null hypothesis remains the same when using a 5% significance level, but this is not true in general. Different formulations of Wald tests can lead to different inferences.

◁

LR tests do not vary with nonlinear transforms

LR tests are invariant to nonlinear transforms.

▷ Example 2: LR tests are invariant to nonlinear transforms

We illustrate this feature by performing LR tests that $\beta = 2/3$ and that $1/\beta = 1.5$. The current estimates are those of the unconstrained model. We repeat these results and store them as `unconstrained`.

```
. dsge
DSGE model
Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -753.57131
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.5146674	.0783489	6.57	0.000	.3611064	.6682284
kappa	.1659057	.0474074	3.50	0.000	.0729889	.2588224
rhoz	.9545256	.0186424	51.20	0.000	.9179872	.991064
rhou	.7005483	.0452604	15.48	0.000	.6118395	.7892571
sd(e.u)	2.318203	.3047438			1.720916	2.915489
sd(e.z)	.6507118	.1123845			.4304422	.8709814

```
. estimates store unconstrained
```

Now, we estimate the parameters of the constrained model in which $\beta = 2/3$, store the results as `constrained`, and perform an LR test of the null hypothesis that $\beta = 2/3$.

```
. constraint 1 _b[beta] = 2/3

. dsge (p = {beta}*E(F.p) + {kappa}*y)
>      (y = E(F.y) -(r - E(F.p) - {rhoz}*z), unobserved)
>      (r = (1/{beta})*p + u)
>      (F.u = {rhou}*u, state)
>      (F.z = {rhoz}*z, state),
>      constraint(1)
(setting technique to bfgs)
Iteration 0:   log likelihood =  -119695.1
Iteration 1:   log likelihood =  -1425.592  (backed up)
Iteration 2:   log likelihood = -984.57609  (backed up)
Iteration 3:   log likelihood = -948.41524  (backed up)
Iteration 4:   log likelihood = -945.83724  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -945.06881  (backed up)
Iteration 6:   log likelihood = -764.18274  (not concave)
Iteration 7:   log likelihood =  -756.7209
Iteration 8:   log likelihood = -755.12606
Iteration 9:   log likelihood = -755.11016
Iteration 10:  log likelihood = -755.11003
Iteration 11:  log likelihood = -755.11003

DSGE model

Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -755.11003
 ( 1)  [/structural]beta = .6666667
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf. Interval]	
/structural						
beta	.6666667	(constrained)				
kappa	.1076807	.0276892	3.89	0.000	.0534109	.1619506
rhoz	.9538519	.0187789	50.79	0.000	.9170458	.9906579
rhou	.7214333	.043967	16.41	0.000	.6352595	.807607
sd(e.u)	1.915459	.0867103			1.74551	2.085408
sd(e.z)	.4936815	.0805138			.3358773	.6514856

```
. estimates store constrained

. lrtest unconstrained constrained

Likelihood-ratio test                                 LR chi2(1)  =      3.08
(Assumption: constrained nested in unconstrained)     Prob > chi2 =    0.0794
```

Note that the value of the LR statistic is 3.08. We now illustrate an LR of the null hypothesis that $1/\beta = 1.5$ produces the same value.

We cannot impose nonlinear restrictions on parameters, so we must begin by reparameterizing the unconstrained model by replacing `{beta}` with `1/{beta}`. To avoid having `{beta}` mean two different things, we write the model in terms of `{gamma}=1/{beta}` and estimate the parameters:

```
. dsge (p = 1/{gamma}*E(F.p) + {kappa}*y)
>       (y = E(F.y) -(r - E(F.p) - {rhoz}*z), unobserved)
>       (r = ({gamma})*p + u)
>       (F.u = {rhou}*u, state)
>       (F.z = {rhoz}*z, state),
>      from(gamma=2 kappa=0.15 rhou=0.75 rhoz=0.95)
(setting technique to bfgs)
Iteration 0:   log likelihood = -1137.8808
Iteration 1:   log likelihood = -1097.9283  (backed up)
Iteration 2:   log likelihood = -1027.9554  (backed up)
Iteration 3:   log likelihood = -801.19555  (backed up)
Iteration 4:   log likelihood = -784.48041  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -763.19407  (not concave)
Iteration 6:   log likelihood = -754.50175  (not concave)
Iteration 7:   log likelihood =  -754.0848
Iteration 8:   log likelihood = -753.57364
Iteration 9:   log likelihood = -753.57131
Iteration 10:  log likelihood = -753.57131

DSGE model

Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -753.57131
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
gamma	1.943005	.2957866	6.57	0.000	1.363274	2.522736
kappa	.1659061	.0474073	3.50	0.000	.0729895	.2588226
rhoz	.9545256	.0186424	51.20	0.000	.9179872	.991064
rhou	.7005481	.0452604	15.48	0.000	.6118393	.7892568
sd(e.u)	2.318205	.3047432			1.720919	2.91549
sd(e.z)	.6507123	.1123842			.4304434	.8709812

```
. estimates store unconstrained2
```

The estimates of the parameters other than `gamma` and the value of the log likelihood are nearly the same as those for the `unconstrained` model. The value for `gamma` $= 1.94$ is the same as `1/beta` $= 1/0.514 = 1.95$. By tightening the convergence tolerance, we could make these values exactly the same. These values are nearly the same because this example is an instance of a general property of maximum likelihood estimators. Transforming a parameter by an invertible function does not change the log likelihood or the other parameter estimates. In other words, maximum likelihood estimators are invariant to invertible transformations of the parameters; see Casella and Berger (2002, 319) for details.

Having stored the estimates from the unconstrained model, we now estimate the parameters of the constrained model and store these results in constrained2.

```
. constraint 2 _b[gamma] = 1.5
. dsge (p = 1/{gamma}*E(F.p) + {kappa}*y)
>      (y = E(F.y) -(r - E(F.p) - {rhoz}*z), unobserved)
>      (r = ({gamma})*p + u)
>      (F.u = {rhou}*u, state)
>      (F.z = {rhoz}*z, state),
>      constraint(2)
(setting technique to bfgs)
Iteration 0:   log likelihood =  -119695.1
Iteration 1:   log likelihood =  -1425.592  (backed up)
Iteration 2:   log likelihood = -984.57609  (backed up)
Iteration 3:   log likelihood = -948.41524  (backed up)
Iteration 4:   log likelihood = -945.83724  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -945.06881  (backed up)
Iteration 6:   log likelihood = -764.18274  (not concave)
Iteration 7:   log likelihood =  -756.7209
Iteration 8:   log likelihood = -755.12606
Iteration 9:   log likelihood = -755.11016
Iteration 10:  log likelihood = -755.11003
Iteration 11:  log likelihood = -755.11003
DSGE model
Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -755.11003
 ( 1)  [/structural]gamma = 1.5
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
gamma	1.5	(constrained)				
kappa	.1076807	.0276892	3.89	0.000	.0534109	.1619506
rhoz	.9538519	.0187789	50.79	0.000	.9170458	.9906579
rhou	.7214333	.043967	16.41	0.000	.6352595	.807607
sd(e.u)	1.915459	.0867103			1.74551	2.085408
sd(e.z)	.4936815	.0805138			.3358773	.6514856

```
. estimates store constrained2
```

The estimates of the parameters other than gamma and the value of the log likelihood are the same as those for the constrained1 model. This is another instance of the invariance of the maximum likelihood estimator to invertible transformations of the parameters.

Having stored the log likelihoods from the constrained and unconstrained model, we now perform an LR of the null hypothesis that $\gamma = 1.5$.

```
. lrtest unconstrained2 constrained2
Likelihood-ratio test                                 LR chi2(1)  =      3.08
(Assumption: constrained2 nested in unconstrained2)   Prob > chi2 =    0.0794
```

The LR test statistic and its p-value are the same as those reported for the test against the null hypothesis that $\beta = 2/3$, which illustrates that LR tests are invariant to nonlinear transforms.

◁

References

Casella, G., and R. L. Berger. 2002. *Statistical Inference*. 2nd ed. Pacific Grove, CA: Duxbury.

Gregory, A. W., and M. R. Veall. 1985. Formulating Wald tests of nonlinear restrictions. *Econometrica* 53: 1465–1468.

Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. *Econometrica* 56: 1065–1083.

Also see

[DSGE] **dsge postestimation** — Postestimation tools for dsge

[R] **lrtest** — Likelihood-ratio test after estimation

[R] **test** — Test linear hypotheses after estimation