Description Remarks and examples References Also see

Description

After fitting a DSGE model, we often perform tests of structural parameters, and these tests often place nonlinear restrictions on the parameters. The values and rejection rates of a Wald test for different nonlinear expressions of the same null hypothesis are different. We illustrate this issue, show that likelihood-ratio (LR) tests do not have this problem, and illustrate that you can parameterize your model in terms of invertible transforms of each parameter.

Remarks and examples

Remarks are presented under the following headings:

Wald tests vary with nonlinear transforms LR tests do not vary with nonlinear transforms

Wald tests vary with nonlinear transforms

Performing a statistical test of whether a structural parameter in a DSGE has a specific value is one of the most frequent forms of inference after dsge and dsgenl estimation. The null hypothesis in one of these tests frequently places nonlinear restrictions on the underlying parameters. Two different nonlinear expressions of the same null hypothesis produce different Wald test statistics in finite samples and have different rejection rates. In other words, the Wald test is not invariant to nonlinear transforms of the null hypothesis.

Example 1: Different values from logically equivalent Wald tests

Equations (1)–(5) specify how the observed control variable inflation p_t , the unobserved control variable output growth y_t , and the observed control variable (interest rate) r_t depend on the states z_t and u_t , given the shocks ϵ_t and ξ_t .

$$p_t = \beta E_t(p_{t+1}) + \kappa y_t \tag{1}$$

$$y_t = E_t(y_{t+1}) - \{r_t - E_t(p_{t+1}) - \rho z_t\}$$
(2)

$$r_t = (1/\beta)p_t + u_t \tag{3}$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1} \tag{4}$$

$$u_{t+1} = \delta u_t + \xi_{t+1} \tag{5}$$

We estimate the parameters of this model using the macroeconomic data for the United States in usmacro2.dta.

```
. use https://www.stata-press.com/data/r19/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
 dsge (p = {beta}*F.p + {kappa}*y)
>
       (y = F.y - (r - F.p - \{rhoz\}*z), unobserved)
>
       (r = (1/{beta})*p + u)
>
       (F.u = {rhou}*u, state)
       (F.z = \{rhoz\}*z, state)
>
(setting technique to bfgs)
Iteration 0: Log likelihood = -146218.64
Iteration 1:
              Log likelihood = -5532.4212
                                            (backed up)
              Log likelihood = -1067.4665
Iteration 2:
                                            (backed up)
Iteration 3:
              Log likelihood = -938.92415
                                            (backed up)
              Log likelihood = -885.96401
                                            (backed up)
Iteration 4:
(switching technique to nr)
Iteration 5: Log likelihood = -880.81743
                                            (not concave)
Iteration 6: Log likelihood = -818.95373
                                            (not concave)
Iteration 7: Log likelihood = -776.7714
                                            (not concave)
Iteration 8: Log likelihood = -767.94097
Iteration 9: Log likelihood = -756.2058
Iteration 10: Log likelihood = -753.6899
Iteration 11: Log likelihood = -753.57771
Iteration 12: Log likelihood = -753.57132
Iteration 13: Log likelihood = -753.57131
DSGE model
Sample: 1955q1 thru 2015q4
                                                            Number of obs = 244
Log likelihood = -753.57131
               Coefficient
                            Std. err.
                                            z
                                                 P>|z|
                                                            [95% conf. interval]
/structural
                             .0783486
                                          6.57
                                                 0.000
                                                            .3611066
                                                                        .6682273
        beta
                  .514667
                                          3.50
                                                 0.000
       kappa
                  .165906
                             .0474073
                                                            .0729894
                                                                        .2588225
        rhoz
                 .9545255
                             .0186424
                                         51.20
                                                 0.000
                                                            .9179871
                                                                         .991064
                 .7005481
                             .0452604
                                         15.48
                                                 0.000
                                                            .6118394
                                                                        .7892568
        rhou
                             .304743
      sd(e.u)
                 2.318204
                                                            1.720918
                                                                        2.915489
                                                                        .8709816
      sd(e.z)
                 .6507125
                             .1123843
                                                            .4304433
```

The interest rate equation shown in (3) links the nominal interest rate to the inflation rate. The coefficient on inflation is $1/\beta$. We test whether this parameter is 1.5, a common benchmark value in the literature.

. testnl 1/_b[beta] = 1.5 (1) 1/_b[beta] = 1.5 chi2(1) = 2.24 Prob > chi2 = 0.1342

We do not reject the null hypothesis that $1/\beta$ is 1.5.

If we test the logically equivalent hypothesis that $\beta = 2/3$, the statistic and *p*-value change.

The values of these two logically equivalent Wald tests differ because Wald tests are not invariant to nonlinear transformation. This issue is well known in the literature; see Gregory and Veall (1985) and Phillips and Park (1988) for details. In this example, the inference of failing to reject the null hypothesis remains the same when using a 5% significance level, but this is not true in general. Different formulations of Wald tests can lead to different inferences.

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LR tests do not vary with nonlinear transforms

Example 2: LR tests are invariant to nonlinear transforms

We illustrate this feature by performing LR tests that $\beta = 2/3$ and that $1/\beta = 1.5$. The current estimates are those of the unconstrained model. We repeat these results and store them as unconstrained.

. dsge						
DSGE model						
Sample: 1955q1 thru 2015q4 Number of obs = 244 Log likelihood = -753.57131						
	Coefficient	Std. err.	Z	P> z	[95% conf.	interval]
/structural						
beta	.514667	.0783486	6.57	0.000	.3611066	.6682273
kappa	.165906	.0474073	3.50	0.000	.0729894	.2588225
rhoz	.9545255	.0186424	51.20	0.000	.9179871	.991064
rhou	.7005481	.0452604	15.48	0.000	.6118394	.7892568
sd(e.u)	2.318204	.304743			1.720918	2.915489
sd(e.z)	.6507125	.1123843			.4304433	.8709816

. estimates store unconstrained

Now, we estimate the parameters of the constrained model in which $\beta = 2/3$, store the results as constrained, and perform an LR test of the null hypothesis that $\beta = 2/3$.

```
. constraint 1 _b[beta] = 2/3
. dsge (p = {beta}*F.p + {kappa}*y)
>
       (y = F.y - (r - F.p - \{rhoz\}*z), unobserved)
>
       (r = (1/{beta})*p + u)
>
       (F.u = {rhou}*u, state)
       (F.z = \{rhoz\}*z, state),
>
      constraint(1)
>
(setting technique to bfgs)
Iteration 0: Log likelihood = -119695.1
Iteration 1: Log likelihood = -1425.592
                                            (backed up)
Iteration 2: Log likelihood = -984.57609
                                            (backed up)
Iteration 3: Log likelihood = -948.41524
                                            (backed up)
Iteration 4: Log likelihood = -945.83724
                                            (backed up)
(switching technique to nr)
Iteration 5: Log likelihood = -945.06881
                                            (backed up)
Iteration 6: Log likelihood = -760.71545
Iteration 7: Log likelihood = -755.52634
Iteration 8: Log likelihood = -755.11897
Iteration 9: Log likelihood = -755.11007
Iteration 10: Log likelihood = -755.11003
DSGE model
Sample: 1955q1 thru 2015q4
                                                            Number of obs = 244
Log likelihood = -755.11003
 (1)
       [/structural]beta = .6666667
               Coefficient Std. err.
                                            z
                                                 P>|z|
                                                            [95% conf. interval]
/structural
        beta
                 .6666667
                            (constrained)
       kappa
                 .1076811
                             .0276892
                                          3.89
                                                 0.000
                                                            .0534113
                                                                        .1619509
                 .9538522
                             .0187789
                                         50.79
                                                 0.000
                                                            .9170462
                                                                        .9906581
        rhoz
                                                 0.000
                                                            .6352593
        rhou
                 .7214328
                             .0439669
                                         16.41
                                                                        .8076063
      sd(e.u)
                 1.915459
                             .0867103
                                                             1.74551
                                                                        2.085408
      sd(e.z)
                  .4936797
                              .080513
                                                            .3358771
                                                                        .6514822
```

. estimates store constrained

. lrtest unconstrained constrained

Likelihood-ratio test Assumption: constrained nested within unconstrained LR chi2(1) = 3.08 Prob > chi2 = 0.0794

Note that the value of the LR statistic is 3.08. We now illustrate an LR of the null hypothesis that $1/\beta = 1.5$ produces the same value.

We cannot impose nonlinear restrictions on parameters, so we must begin by reparameterizing the unconstrained model by replacing {beta} with 1/{beta}. To avoid having {beta} mean two different things, we write the model in terms of {gamma}=1/{beta} and estimate the parameters:

```
. dsge (p = 1/{gamma}*F.p + {kappa}*y)
       (y = F.y - (r - F.p - \{rhoz\}*z), unobserved)
>
       (r = (\{gamma\})*p + u)
>
>
       (F.u = {rhou}*u, state)
>
       (F.z = \{rhoz\}*z, state),
>
      from(gamma=2 kappa=0.15 rhou=0.75 rhoz=0.95)
(setting technique to bfgs)
Iteration 0: Log likelihood = -1137.8808
Iteration 1: Log likelihood = -1097.9283
                                            (backed up)
Iteration 2: Log likelihood = -1027.9554
                                            (backed up)
Iteration 3: Log likelihood = -801.19555
                                            (backed up)
              Log likelihood = -784.48041
Iteration 4:
                                            (backed up)
(switching technique to nr)
Iteration 5: Log likelihood = -763.19407
                                            (not concave)
Iteration 6: Log likelihood = -754.49971
                                            (not concave)
Iteration 7: Log likelihood = -754.08362
Iteration 8: Log likelihood = -753.57362
Iteration 9: Log likelihood = -753.57131
Iteration 10: Log likelihood = -753.57131
DSGE model
Sample: 1955q1 thru 2015q4
                                                            Number of obs = 244
Log likelihood = -753.57131
               Coefficient Std. err.
                                                 P>171
                                                            [95% conf. interval]
                                            7.
/structural
       gamma
                 1.943005
                             .2957867
                                          6.57
                                                 0.000
                                                            1.363273
                                                                        2.522736
                 .1659061
                             .0474073
                                          3.50
                                                 0.000
                                                            .0729895
                                                                        .2588226
       kappa
                 .9545256
                                         51.20
                                                 0.000
        rhoz
                             .0186424
                                                            .9179872
                                                                         .991064
        rhou
                 .7005481
                             .0452604
                                         15.48
                                                 0.000
                                                            .6118393
                                                                        .7892568
      sd(e.u)
                 2.318205
                             .3047433
                                                            1.720919
                                                                        2.915491
      sd(e.z)
                 .6507124
                             .1123842
                                                            .4304434
                                                                        .8709813
```

. estimates store unconstrained2

The estimates of the parameters other than gamma and the value of the log likelihood are nearly the same as those for the unconstrained model. The value for gamma = 1.94 is the same as 1/beta = 1/0.514 = 1.95. By tightening the convergence tolerance, we could make these values exactly the same. These values are nearly the same because this example is an instance of a general property of maximum likelihood estimators. Transforming a parameter by an invertible function does not change the log likelihood or the other parameter estimates. In other words, maximum likelihood estimators are invariant to invertible transformations of the parameters; see Casella and Berger (2002, 319) for details.

Having stored the estimates from the unconstrained model, we now estimate the parameters of the constrained model and store these results in constrained2.

```
. constraint 2 _b[gamma] = 1.5
. dsge (p = 1/{gamma}*F.p + {kappa}*y)
>
       (y = F.y - (r - F.p - \{rhoz\}*z), unobserved)
       (r = ({gamma})*p + u)
>
>
       (F.u = {rhou}*u, state)
       (F.z = \{rhoz\}*z, state),
>
       constraint(2)
(setting technique to bfgs)
Iteration 0: Log likelihood = -119695.1
Iteration 1: Log likelihood = -1425.592
                                           (backed up)
                                           (backed up)
Iteration 2: Log likelihood = -984.57609
Iteration 3: Log likelihood = -948.41524
                                           (backed up)
Iteration 4: Log likelihood = -945.83724
                                           (backed up)
(switching technique to nr)
Iteration 5: Log likelihood = -945.06881
                                           (backed up)
Iteration 6: Log likelihood = -760.71545
Iteration 7: Log likelihood = -755.52634
Iteration 8: Log likelihood = -755.11897
Iteration 9: Log likelihood = -755.11007
Iteration 10: Log likelihood = -755.11003
DSGE model
Sample: 1955q1 thru 2015q4
                                                            Number of obs = 244
Log likelihood = -755.11003
 (1)
       [/structural]gamma = 1.5
               Coefficient Std. err.
                                           z
                                                P>|z|
                                                           [95% conf. interval]
/structural
       gamma
                      1.5 (constrained)
                                         3.89
                                                0.000
       kappa
                 .1076811
                            .0276892
                                                           .0534113
                                                                       .1619509
                                                                       .9906581
       rhoz
                 .9538522
                            .0187789
                                        50.79
                                                0.000
                                                           .9170462
       rhou
                 .7214328
                            .0439669
                                        16.41
                                                0.000
                                                           .6352593
                                                                       .8076063
      sd(e.u)
                 1.915459
                            .0867103
                                                            1.74551
                                                                       2.085408
      sd(e.z)
                 .4936797
                             .080513
                                                           .3358771
                                                                       .6514822
```

. estimates store constrained2

The estimates of the parameters other than gamma and the value of the log likelihood are the same as those for the constrained model. This is another instance of the invariance of the maximum likelihood estimator to invertible transformations of the parameters.

Having stored the log likelihoods from the constrained and unconstrained model, we now perform an LR of the null hypothesis that $\gamma = 1.5$.

```
. lrtest unconstrained2 constrained2
Likelihood-ratio test
Assumption: constrained2 nested within unconstrained2
LR chi2(1) = 3.08
Prob > chi2 = 0.0794
```

The LR test statistic and its *p*-value are the same as those reported for the test against the null hypothesis that $\beta = 2/3$, which illustrates that LR tests are invariant to nonlinear transforms.

References

Casella, G., and R. L. Berger. 2002. Statistical Inference. 2nd ed. Pacific Grove, CA: Duxbury.

- Gregory, A. W., and M. R. Veall. 1985. Formulating Wald tests of nonlinear restrictions. *Econometrica* 53: 1465–1468. https://doi.org/10.2307/1913221.
- Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. *Econometrica* 56: 1065–1083. https://doi.org/10.2307/1911359.

Also see

- [DSGE] dsge postestimation Postestimation tools for dsge
- [DSGE] dsgenl postestimation Postestimation tools for dsgenl
- [R] Irtest Likelihood-ratio test after estimation
- [R] test Test linear hypotheses after estimation

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