Title

intro 4f — Including an observed exogenous variable

Description   Remarks and examples   Also see

Description

All the observed variables in a DSGE model must be modeled as endogenous control variables. This requirement implies that there is no reduced form for the endogenous variables as a function of observed exogenous variables. Any variable that is theoretically exogenous must be modeled.

Mechanically, the solution is to define a control variable that is equal to a state variable that models the exogenous process. We clarify this issue by discussing a model in which the price of oil is theoretically exogenous.

Remarks and examples

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Remarks are presented under the following headings:

The model

We model an economy in which the growth rate of the trade-weighted exchange rate is exogenous and in which it affects inflation. Henceforth, we call the trade-weighted exchange rate just the exchange rate. We begin by adding the growth rate of the exchange rate to the inflation equation in the New Keynesian model of [DSGE] **intro 1**.

$$x_t = E_t(x_{t+1}) - \{r_t - E_t(p_{t+1}) - g_t\} \tag{1}$$

$$r_t = \frac{1}{\beta} p_t + u_t \tag{2}$$

$$p_t = \beta E_t(p_{t+1}) + \kappa x_t + \psi e_t \tag{3}$$

$$u_{t+1} = \rho_u u_t + \epsilon_{t+1} \tag{4}$$

$$g_{t+1} = \rho_g g_t + \xi_{t+1} \tag{5}$$

x_t is the output gap, which is modeled as an unobserved control variable. r_t is the interest rate, which is modeled as an observed control variable. p_t is the inflation rate, which is modeled as an observed control variable. g_t and u_t are first-order autoregressive state variables. e_t is the growth rate of the exchange rate, which we want to model as an observed exogenous variable.

The model in (1)–(5) cannot be solved because there is no equation for how the observed e_t evolves over time. A first-order autoregressive process [AR(1)] is a standard approximation to an exogenous variable, like the growth in exchange rate. We complete the model by adding an AR(1) for the unobserved state variable es_t and an equation linking the unobserved es_t to the observed e_t.

$$x_t = E_t(x_{t+1}) - \{r_t - E_t(p_{t+1}) - g_t\} \tag{6}$$
$$r_t = \frac{1}{\beta}p_t + u_t \tag{7}$$
$$p_t = \beta E_t(p_{t+1}) + \kappa x_t + \psi es_t \tag{8}$$
$$e_t = es_t \tag{8a}$$
$$u_{t+1} = \rho_u u_t + \epsilon_{t+1} \tag{9}$$
$$g_{t+1} = \rho_g g_t + \xi_{t+1} \tag{10}$$
$$es_{t+1} = \rho_e es_t + \eta_{t+1} \tag{11}$$

New (8a) specifies how the unobserved state es_t is transformed into the observed control variable e_t. Equation (11) specifies that the unobserved state es_t is an AR(1) process. Other equations are unchanged.

Parameter estimation

We fit this model using data on the U.S. interest rate `r`, inflation rate `p`, and the growth rate of the exchange rate `e`. The equation `(e = es)` links the observed variable `e` to the unobserved state variable `es`.

```
. use http://www.stata-press.com/data/r15/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
. dsge (x = E(F.x) -(r - E(f.p) - g), unobserved)
>      (r = 1/{beta}*p + u)
>      (p = {beta}*E(F.p) + {kappa}*x + {psi}*es)
>      (e = es)
>      (F.u = {rho_u}*u, state)
>      (F.g = {rho_g}*g, state)
>      (F.es = {rho_e}*es, state)
(setting technique to bfgs)
Iteration 0:   log likelihood = -24249.537
Iteration 1:   log likelihood = -8517.9588  (backed up)
Iteration 2:   log likelihood = -1808.8075  (backed up)
Iteration 3:   log likelihood = -1625.0334  (backed up)
Iteration 4:   log likelihood = -1600.0843  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -1557.8925  (not concave)
Iteration 6:   log likelihood =  -1512.705  (not concave)
Iteration 7:   log likelihood = -1481.8476  (not concave)
Iteration 8:   log likelihood = -1278.0033  (not concave)
Iteration 9:   log likelihood = -1198.5095  (not concave)
Iteration 10:  log likelihood = -1186.0134
Iteration 11:  log likelihood = -1177.2355  (not concave)
Iteration 12:  log likelihood = -1175.3012
Iteration 13:  log likelihood = -1174.3886
Iteration 14:  log likelihood = -1172.7244
Iteration 15:  log likelihood = -1172.3166
Iteration 16:  log likelihood = -1172.0638
Iteration 17:  log likelihood = -1172.0376
Iteration 18:  log likelihood = -1172.0364
Iteration 19:  log likelihood = -1172.0364
```

```
DSGE model

Sample: 1973q2 - 2015q4                          Number of obs     =        171
Log likelihood = -1172.0364
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.5106735	.1022178	5.00	0.000	.3103303	.7110167
kappa	.0901563	.0334284	2.70	0.007	.0246378	.1556747
psi	.01373	.0037684	3.64	0.000	.006344	.021116
rho_u	.7789341	.047395	16.43	0.000	.6860415	.8718266
rho_g	.9597842	.0206193	46.55	0.000	.9193712	1.000197
rho_e	.2372214	.0742225	3.20	0.001	.0917479	.3826948
sd(e.u)	2.206087	.3466166			1.526731	2.885443
sd(e.g)	.703933	.1545767			.4009683	1.006898
sd(e.es)	10.48932	.5671986			9.37763	11.60101

From the estimates for `psi`, it appears that the growth of the exchange rate impacts inflation.

Also see

[DSGE] **intro 2** — Learning the syntax

[DSGE] **intro 4** — Writing a DSGE in a solvable form