

intro 4e — Including a second-order lag of a control

[Description](#)[Remarks and examples](#)[Also see](#)

Description

Some DSGE models capture delayed effects by including a second-order lag of a control variable and excluding the first-order lag. The second-order lag is a problematic term that does not fit into the form required to solve a structural model for its state-space form. This entry shows how to solve this problem by defining new state variables and rewriting the equations.

Remarks and examples

[stata.com](#)

Remarks are presented under the following headings:

[The model](#)[Parameter estimation](#)

The model

Consider a model in which changes in hours worked take two periods to adjust because next period's hours have already been budgeted. In this model, the second-order lag of changes in hours worked is included and the first-order lag is excluded. Equations (1)–(4) specify such a model of growth in hours worked and of consumption growth.

$$n_t = b_1 n_{t-2} + w_t - \gamma c_t \quad (1)$$

$$c_t = (1 - h)w_t + hE_t c_{t+1} + r_t \quad (2)$$

$$w_{t+1} = \rho w_t + \xi_{t+1} \quad (3)$$

$$r_{t+1} = \epsilon_{t+1} \quad (4)$$

Equation (1) specifies that the growth rate of hours worked n_t depends on a second-order lag of itself, wage growth w_t , and consumption growth c_t . Equation (2) specifies that consumption growth is a linear combination of wage growth, expected future consumption growth $E_t c_{t+1}$, and the interest rate r_t . Equation (3) specifies an autoregressive process for wage growth. Equation (4) specifies that interest rate is just a shock. The control variables are n_t and c_t . The state variables are w_t and r_t .

One cannot solve the model in (1)–(4) for the state-space form because the problematic term $b_1 n_{t-2}$ does not fit into the required form. To accommodate this term, we define two new state variables, one for n_{t-1} and one for n_{t-2} . We define new state variables instead of new control variables because lags of the control are predetermined and thus exogenous. The model with new state variables is

$$n_t = b_1 L2n_t + w_t - \gamma c_t \quad (5)$$

$$c_t = (1 - h)w_t + hE_t c_{t+1} + r_t \quad (6)$$

$$w_{t+1} = \rho w_t + \xi_{t+1} \quad (7)$$

$$r_{t+1} = \epsilon_{t+1} \quad (8)$$

$$Ln_{t+1} = n_t \quad (9)$$

$$L2n_{t+1} = Ln_t \quad (10)$$

Equation (9) defines the new state for n_{t-1} , and (10) defines $L2n_t$ to be the new state for n_{t-2} . The $L2n_t$ in (5) replaces n_{t-2} in (1).

Parameter estimation

We specify n and t as observed control equations. We specify w , r , Ln , and $L2n$ as state equations. We specify that w and r are subject to shocks; the new states to accommodate n_{t-2} are not subject to shocks.

```
. use http://www.stata-press.com/data/r15/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)

. dsge (n = {b1}*L2n + w - {gamma}*c)
>      (c = (1-{h})*w + {h}*E(F.c) + r)
>      (F.w = {rho}*w, state)
>      (F.r = , state)
>      (F.L2n = Ln, state noshock)
>      (F.Ln = n, state noshock)
(setting technique to bfgs)
Iteration 0:  log likelihood = -2325.1996
Iteration 1:  log likelihood = -1277.0146 (backed up)
Iteration 2:  log likelihood = -1193.4512 (backed up)
Iteration 3:  log likelihood = -1189.3181 (backed up)
Iteration 4:  log likelihood = -1188.2629 (backed up)
(switching technique to nr)
Iteration 5:  log likelihood = -1187.9872 (backed up)
Iteration 6:  log likelihood = -1147.3379
Iteration 7:  log likelihood = -1131.5153
Iteration 8:  log likelihood = -1129.0358
Iteration 9:  log likelihood = -1129.0181
Iteration 10: log likelihood = -1129.0181
DSGE model
Sample: 1955q1 - 2015q4                Number of obs   =       244
Log likelihood = -1129.0181
```

	OIM				[95% Conf. Interval]	
	Coef.	Std. Err.	z	P> z		
<hr/>						
/structural						
b1	.1320845	.0608727	2.17	0.030	.0127763	.2513927
gamma	.360925	.1298387	2.78	0.005	.1064457	.6154042
h	.7238121	.0406724	17.80	0.000	.6440957	.8035285
rho	.6177969	.0533568	11.58	0.000	.5132195	.7223743
<hr/>						
sd(e.w)	3.033799	.2423856			2.558732	3.508866
sd(e.r)	1.970288	.1574527			1.661687	2.27889

Looking at the confidence interval for b_1 , we conclude that the second-order lag of hours growth impacts current hours growth.

Also see

[DSGE] [intro 2](#) — Learning the syntax

[DSGE] [intro 4](#) — Writing a DSGE in a solvable form