intro 4b — Including a lag of a control variable

Description

`dsge` does not allow lags of control variables to be included in the model. The structural form of the model that is required so that the model can be solved for its state-space form does not include lags of control variables. This entry shows how to fit a DSGE model that involves a lag of control variables by defining a new state variable and rewriting the equations.

Remarks and examples

Remarks are presented under the following headings:

A model with a lagged endogenous variable

The model in (1)–(5) is similar to many in monetary economics in that it includes inertia in the interest rate because the interest rate depends on its lagged value.

$$p_t = \beta E_t p_{t+1} + \kappa y_t \tag{1}$$
$$y_t = E_t y_{t+1} - (r_t - E_t p_{t+1} - \rho_z z_t) \tag{2}$$
$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(\frac{1}{\beta} p_t + u_t\right) \tag{3}$$
$$z_{t+1} = \rho_z z_t + \epsilon_{t+1} \tag{4}$$
$$u_{t+1} = \rho_u u_t + \xi_{t+1} \tag{5}$$

Equation (1) specifies the structural equation for inflation p_t. Inflation is a linear combination of expected future inflation $E_t(p_{t+1})$ and output growth y_t. Equation (2) specifies the structural equation for output growth. Output growth is a linear combination of expected future output growth $E_t(y_{t+1})$, the interest rate r_t, expected future inflation, and the state z_t. The state z_t is the first-order autoregressive process that drives output growth. Equation (3) specifies the structural equation for the interest rate. The interest rate depends on its own lagged value, the inflation rate, and the state u_t. The state u_t is the first-order autoregressive process that drives the interest rate. The control variables in this model are p_t, y_t, and r_t. The state variables are u_t and z_t.

The term involving r_{t-1} in (3) is problematic because the lag of a control variable does not fit into the structure required to solve the model for the state-space form. We accommodate this term by defining a new state variable Lr_t that equals r_{t-1} and replacing r_{t-1} in (3) with this new state variable. We define a new state instead of new control because the lagged control is predetermined, which makes it exogenous. These changes yield the model in (6)–(11).

$$p_t = \beta E_t p_{t+1} + \kappa y_t \tag{6}$$
$$y_t = E_t y_{t+1} - (r_t - E_t p_{t+1} - \rho_z z_t) \tag{7}$$
$$r_t = \rho_r L r_t + (1 - \rho_r) \left(\frac{1}{\beta} p_t + u_t \right) \tag{8}$$
$$L r_{t+1} = r_t \tag{9}$$
$$z_{t+1} = \rho_z z_t + \epsilon_{t+1} \tag{10}$$
$$u_{t+1} = \rho_u u_t + \xi_{t+1} \tag{11}$$

Parameter estimation

We will estimate the parameters in the model in (6)–(11) using data on U.S. interest rate `r` and inflation `p`.

```
. use http://www.stata-press.com/data/r15/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
. dsge (p = {beta}*E(F.p) + {kappa}*y)
>      (y = E(F.y) -(r - E(f.p) - {rhoz}*z), unobserved)
>      (r = {rhor}*lr + (1-{rhor})*((1/{beta})*p + u))
>      (F.lr = r, state noshock)
>      (F.u = {rhou}*u, state)
>      (F.z = {rhoz}*z, state)
(setting technique to bfgs)
Iteration 0:   log likelihood = -158313.11
Iteration 1:   log likelihood = -5780.6489  (backed up)
Iteration 2:   log likelihood = -1028.4602  (backed up)
Iteration 3:   log likelihood =  -1000.851  (backed up)
Iteration 4:   log likelihood =  -873.3804  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -855.55061  (not concave)
Iteration 6:   log likelihood =   -799.322  (not concave)
Iteration 7:   log likelihood = -784.44106  (not concave)
Iteration 8:   log likelihood = -776.37874  (not concave)
Iteration 9:   log likelihood =  -771.3587  (not concave)
Iteration 10:  log likelihood = -767.02524  (not concave)
Iteration 11:  log likelihood = -764.16952  (not concave)
Iteration 12:  log likelihood = -761.74111  (not concave)
Iteration 13:  log likelihood = -759.86071  (not concave)
Iteration 14:  log likelihood = -758.21296
Iteration 15:  log likelihood = -755.96039
Iteration 16:  log likelihood = -754.36723
Iteration 17:  log likelihood = -753.19361
Iteration 18:  log likelihood = -753.07315
Iteration 19:  log likelihood = -753.07026
Iteration 20:  log likelihood = -753.07026
```

```
DSGE model

Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -753.07026

------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
/structural  |
        beta |   .5026369   .0791865     6.35   0.000     .3474342    .6578397
       kappa |   .1760193   .0511049     3.44   0.001     .0758554    .2761831
        rhoz |   .9591408   .0181341    52.89   0.000     .9235986     .994683
        rhor |  -.0939028   .0939661    -1.00   0.318    -.278073     .0902673
        rhou |   .7094625   .0447807    15.84   0.000     .6216939    .7972311
-------------+----------------------------------------------------------------
     sd(e.u) |   2.324311   .3236236                       1.69002    2.958602
     sd(e.z) |   .6111539   .1084981                      .3985014    .8238063
------------------------------------------------------------------------------
```

Ironically, the inertia term for the interest rate `rhor` was not needed in this model. The autoregressive state u_t is probably modeling both the inertia and the persistence in the shocks to the interest rate.

Also see

[DSGE] **intro 2** — Learning the syntax

[DSGE] **intro 4** — Writing a DSGE in a solvable form