Description

Many DSGE models, when written in their natural representation based on economic theory, include problematic terms that do not fit into the algebraic form required to solve a DSGE model. This entry shows how to accommodate these problematic terms by defining a new state variable or a new control variable and then rewriting the equations so that they have the required form. We demonstrate how to make the required changes using dsge, but the advice in this entry applies to both dsge and dsgenl.

We assume that you are already familiar with the elements of a DSGE; see [DSGE] **Intro 1**.

Remarks and examples

Remarks are presented under the following headings:

Introduction
Shocks to a control equation
Including a lag of a control variable
Including a lag of a state variable
Including an expectation of a control dated by more than one period ahead
Including a second-order lag of a control variable
Including an observed exogenous variable

Introduction

We can estimate the parameters of a DSGE model only if we can solve for its state-space form, and we can solve a DSGE model for its state-space form only if the equations for control variables and state variables have specific structures. Unfortunately, many DSGE models contain problematic terms that do not fit into these structures without some manipulation. Fortunately, the manipulation is easy, once you know the tricks discussed below.

As we discussed in *Structural and reduced forms of DSGE models* in [DSGE] **Intro 1**, the structural form of a DSGE model can be written as

$$E_t[\mathbf{f}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{y}_t, \boldsymbol{\theta})] = \mathbf{0} \tag{1}$$

and the structural form of a linearized DSGE model can be written as

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1 E_t(\mathbf{y}_{t+1}) + \mathbf{A}_2\mathbf{y}_t + \mathbf{A}_3\mathbf{x}_t \tag{2}$$

$$\mathbf{B}_0\mathbf{x}_{t+1} = \mathbf{B}_1 E_t(\mathbf{y}_{t+1}) + \mathbf{B}_2\mathbf{y}_t + \mathbf{B}_3\mathbf{x}_t + \mathbf{C}\boldsymbol{\epsilon}_{t+1} \tag{3}$$

where $\mathbf{y}_t$ is a vector of control variables, $\mathbf{x}_t$ is a vector of state variables, and $\boldsymbol{\epsilon}_t$ is a vector of shocks. $\mathbf{A}_0$ through $\mathbf{A}_3$ and $\mathbf{B}_0$ through $\mathbf{B}_3$ are matrices of parameters. The entries in $\mathbf{A}_i$ and $\mathbf{B}_j$ are all functions of the structural parameters, which we denote by vector $\boldsymbol{\theta}$. $\mathbf{A}_0$ and $\mathbf{B}_0$ are diagonal matrices, and $\mathbf{A}_2$ has zeros on its diagonal. $\mathbf{C}$ is a selection matrix that determines which state variables are subject to shocks.

For nonlinear DSGE models, (1) specifies the required structure of the equations. Equations can contain current values of state and control variables and can contain one-step-ahead values of state and control variables. All other terms, such as future values dated more than one period ahead or lagged values, are problematic.

For linearized DSGE models, (2) specifies the required structure for control variable equations, and (3) specifies the required structure for state variable equations. Per (2), a control can depend on only three types of terms—expected values of control variables in the next period, current values of other control variables, and current values of state variables. All other terms are problematic. Per (3), the values of state variables in the next period can depend on only four types of terms—expected values of control variables in the next period, current values of control variables, current values of state variables, and shocks. All other terms are problematic.

When written based on economic theory, the structural forms of many DSGE models include problematic terms that do not fit into the required form. These problematic terms are handled by defining a new state variable or a new control variable and rewriting the equations to have the required form. We define a new state variable when the problematic term involves an exogenous, also known as a predetermined, variable. We define a new control variable when the problematic term includes a new endogenous variable. The rewriting process replaces the variable in the problematic term with the new state or the new control variable.

Problematic terms usually violate one of the following four restrictions:

1. Equations for control variables may not contain shocks.

2. Equations may not contain lagged control variables.

3. Equations may not contain lagged state variables.

4. Equations may not contain expectations of control variables dated by more than one period ahead.

All of these restrictions can be overcome by adding a new state variable or a new control variable and rewriting the model so that it has the required form. Here are five examples of the four restrictions and solutions to each one.

1. If you have an equation for a control variable that contains a shock, make this shock a new state variable; see *Shocks to a control equation* below.

2. If you have an equation for a control variable that contains a lagged control variable, make the lagged control variable a new state variable; see *Including a lag of a control variable* below.

3. If you have an equation for a state variable that contains a lag of a state variable, make the lagged state variable a new state variable; see *Including a lag of a state variable* below.

4. If you have an equation for a control variable that contains an expectation of a control variable dated by more than one period ahead, make it a new control variable. We use a new control variable instead of a new state variable because the far-future expectation is endogenous, not predetermined. See *Including an expectation of a control dated by more than one period ahead* below for details.

5. If you have an equation for a control variable that contains a second-order lag of a control variable, make the second-order lagged control variable a new state variable. You will also need to create a new state variable for the first-order lag. See *Including a second-order lag of a control variable* below for details.

A nuanced issue not covered by restrictions 1–4 is that all the observed variables in a DSGE must be endogenous control variables. This structure is usually not a problem, because DSGE models are usually generated by theories that treat everything observed as endogenous. Occasionally, however, models include an observed variable that is exogenous. For example, a small, open economy might not be able to affect its real exchange rate. The solution for this problem is to model the observed variable as a control variable that is equal to an exogenous state variable. See *Including an observed exogenous variable* below for details.

For the most part, this entry discusses how to make models with problematic terms fit into the structure of (1) or (2) and (3). Note that correlated state variables already fit into this structure. This correlation is modeled by off-diagonal elements in $\mathbf{B_3}$, not by elements in $\mathbf{C}$. See [DSGE] **Intro 4g** for an example.

Shocks to a control equation

Sometimes, we observe only a control variable with error. Suppose that the equation for the control variable consumption is

$$c_t = E_t c_{t+1} - r_t + \epsilon_t \tag{4a}$$

where c_t is consumption, r_t is the interest rate, and ϵ_t is a shock. Shocks to a control variable are not allowed, so ϵ_t is a problematic term.

The solution entails three steps. First, we define a new state variable $u_t = \epsilon_t$. We define a new state instead of a new control because the shock is exogenous. Second, we write a state equation for u_t. Recall that the state equation specifies tomorrow's state value in terms of today's state value and tomorrow's shock. The state equation for u_t is

$$u_{t+1} = \epsilon_{t+1}$$

Third, we rewrite (4a), substituting u_t for ϵ_t. The consumption equation becomes

$$c_t = E_t c_{t+1} - r_t + u_t$$

which is in the required form. In the dsge command, this is expressed as

```
. dsge ...                    ///
        (c = F.c - r + u)     ///
        (F.u = 0, state)      ///
        ...
```

See [DSGE] **Intro 4a** for a complete example.

Including a lag of a control variable

Models that mix adaptive and rational expectations contain a lag of the control variable and the expected future value of the control variable. Suppose our model had the following equation for the control variable y_t,

$$y_t = \alpha y_{t-1} + (1 - \alpha) E_t(y_{t+1}) - r_t \tag{4b}$$

where y_t is output and r_t is the interest rate. The problematic term is αy_{t-1} because it involves a lag of the control variable.

The solution entails three steps. First, we define a new state variable $Ly_t = y_{t-1}$. We define a new state instead of new control because the lagged control is predetermined, which makes it exogenous. Second, we write its state equation as

$$Ly_{t+1} = y_t$$

Third, we rewrite (4b), substituting Ly_t for y_{t-1},

$$y_t = \alpha Ly_t + (1 - \alpha)E_t(y_{t+1}) - r_t$$

which is in the required form. To do this using the `dsge` command, we would type

```
. dsge ...                                       ///
       (y = {alpha}*Ly + (1-{alpha})*F.y - r)    ///
       (F.Ly = y, state noshock)                 ///
       ...
```

See [DSGE] **Intro 4b** for a complete example.

Including a lag of a state variable

Lagged state variables are handled in a manner analogous to lagged control variables. Suppose we have an equation for a state variable z_t that contains a lag of itself. For example, this occurs when z_t is a second-order autoregressive process instead of a first-order autoregressive process. Specifically, our model contains the equation

$$z_{t+1} = \rho_1 z_t + \rho_2 z_{t-1} + \epsilon_{t+1} \tag{5}$$

The problematic term is $\rho_2 z_{t-1}$ because it violates the form required by (2).

The solution entails three steps. First, we define a new state variable $Lz_t = z_{t-1}$. We define a new state instead of new control because the lagged state is exogenous. Second, we write its state equation as

$$Lz_{t+1} = z_t$$

Third, we rewrite (5), substituting Lz_t for z_{t-1},

$$z_{t+1} = \rho_1 z_t + \rho_2 Lz_t + \epsilon_{t+1}$$

which is in the required form. In the `dsge` command, we would type

```
. dsge ...                                       ///
       (F.z = {rho1}*z + {rho2}*Lz, state)       ///
       (F.Lz = z, state noshock)                 ///
       ...
```

See [DSGE] **Intro 4c** for a complete example.

Including an expectation of a control dated by more than one period ahead

Many models include an expectation of a control variable dated by more than one period ahead. For example, suppose that the equation for the control variable consumption growth is

$$c_t = (1 - h)w_t + hE_t c_{t+2} + r_t \tag{6}$$

where c_t is consumption growth, w_t is wage growth, and r_t is the interest rate. Only expectations of control variables dated one period ahead are allowed, so $E_t(c_{t+2})$ is a problematic term.

The solution entails three steps. First, we define a new control variable $Fc_t = E_t(c_{t+1})$. The new variable is a control instead of a state because $E_t(c_{t+1})$ is endogenous instead of exogenous. Second, we include this new control equation,

$$Fc_t = E_t(c_{t+1})$$

Third, we substitute Fc_{t+1} for c_{t+2} in (6). The equation becomes

$$c_t = (1-h)w_t + hE_t(Fc_{t+1}) + r_t$$

which is in the required form. Using the `dsge` command, we would type

```
. dsge ...                              ///
      (c = (1-{h})*w + {h}*F.Fc + r)    ///
      (Fc = F.c, unobserved)            ///
      ...
```

See [DSGE] **Intro 4d** for a complete example.

Including a second-order lag of a control variable

Higher-order lags require defining more than one new state variable, as we illustrate here.

Equations for the state variable capital frequently involve "time to build", so that the control variable investment made in time t becomes available only as new capital several periods in the future. Suppose it takes three periods to convert investment x_t into capital. We denote current capital by k_t and write this three-period relationship as

$$k_{t+1} = (1-\delta)k_t + x_{t-2} \qquad (7)$$

which specifies that the capital stock tomorrow is the sum of investment made two periods ago and the capital today that has not depreciated at rate δ.

The term x_{t-2} is problematic because it is a second-order lag of the state variable x_t, which violates the form required by (2).

The solution entails three steps. First, we define the new state variables $L2x_t = x_{t-2}$ and $Lx_t = x_{t-1}$. Second, we write their state equations as

$$L2x_{t+1} = Lx_t$$

and

$$Lx_{t+1} = x_t$$

Third, we rewrite (7) as

$$k_{t+1} = (1-\delta)k_t + L2x_t$$

With the `dsge` command, we could fit this model as

```
. dsge ...                              ///
      (F.k = (1-{delta})*k + L2x)       ///
      (F.L2x = Lx, state noshock)       ///
      (F.Lx = x, state noshock)         ///
      ...
```

For a complete example, see [DSGE] **Intro 4e**.

In general, if the lag $t - k$ appears anywhere in any of your equations, you will need k new state equations.

Including an observed exogenous variable

Sometimes, we want to treat an observed variable as exogenous. Suppose we want to treat the changes in an observed exchange rate e_t as exogenous. The problem is that all the observed variables in a DSGE model must be modeled as endogenous control variables. The solution is to define a control variable that is equal to a state variable that models the exogenous process. So we could set the observed exchange rate e_t to equal the exchange rate state, es_t,

$$e_t = es_t$$

and specify, say, a first-order autoregressive process for es_t,

$$es_{t+1} = \rho_e es_t + \eta_{t+1}$$

To fit this type of model using the dsge command, we would type

```
. dsge ...                          ///
      (e = es)                      ///
      (F.es = {rhoe}*es, state)     ///
      ...
```

See [DSGE] **Intro 4f** for a complete example.

Also see

[DSGE] **Intro 2** — Learning the syntax

[DSGE] **Intro 3** — Classic DSGE examples

[DSGE] **dsge** — Linear dynamic stochastic general equilibrium models