Intro 3a — New Keynesian model

Description

This introduction estimates and interprets the parameters of a simple New Keynesian model. In this entry, we demonstrate how to constrain parameters in the model and how to interpret structural parameters, policy matrix parameters, and state transition matrix parameters. We also predict values of both observed control variables and unobserved states.

Remarks and examples

Remarks are presented under the following headings:

The model
Parameter estimation
Policy and transition matrices
One-step-ahead predictions
Estimating an unobserved state

The model

Equations (1)–(5) specify a canonical New Keynesian model of inflation $p_t$, the output gap $x_t$, and the interest rate $r_t$. These are the linearized equations; the model’s nonlinear equations are similar to those in Writing down nonlinear DSGEs in [DSGE] Intro 1.

\[
\begin{align*}
   p_t & = \beta E_t (p_{t+1}) + \kappa x_t \tag{1} \\
   x_t & = E_t (x_{t+1}) - \{ r_t - E_t (p_{t+1}) - g_t \} \tag{2} \\
   r_t & = \psi p_t + u_t \tag{3} \\
   u_{t+1} & = \rho u_t + \epsilon_{t+1} \tag{4} \\
   g_{t+1} & = \rho_g g_t + \xi_{t+1} \tag{5}
\end{align*}
\]

Equation (1) specifies inflation as a linear combination of expected future inflation and the output gap. Equation (2) specifies the output gap as a linear combination of the expected future output gap, the real interest rate, and a state variable $g_t$. Equation (3) specifies the interest rate as a linear combination of inflation and a state variable $u_t$. The state variables are modeled as first-order autoregressive processes. The state variable $u_t$ is the deviation of $r_t$ from its equilibrium value of $\psi p_t$. The state variable $g_t$ is also the deviation of $x_t$ from its equilibrium value.

Three of the parameters have structural interpretation. The parameter $\kappa$ is known as the slope of the Phillips curve and is predicted to be positive. The parameter $\beta$ is the discount factor that represents the degree to which agents discount the future relative to the current period. The parameter $\psi$ measures the degree to which interest rates react to movements in inflation.
Parameter estimation

Not all model parameters are identified. We constrain $\beta$ to be 0.96, a common value in the literature. The remaining parameters are identified.

```plaintext
use https://www.stata-press.com/data/r16/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
constraint 1 _b[beta]=0.96
dsge (p = {beta}*F.p + {kappa}*x)
> (x = F.x - (r - F.p - g), unobserved)
> (r = {psi}*p + u)
> (F.u = {rhou}*u, state)
> (F.g = {rhog}*g, state),
from(psi=1.5) constraint(1)
(setting technique to bfgs)
Iteration 0: log likelihood = -5736.4646
Iteration 1: log likelihood = -1190.0604 (backed up)
Iteration 2: log likelihood = -960.00953 (backed up)
Iteration 3: log likelihood = -928.79225 (backed up)
Iteration 4: log likelihood = -842.40806 (backed up)
(switching technique to nr)
Iteration 5: log likelihood = -810.92149 (backed up)
Iteration 6: log likelihood = -766.17852 (not concave)
Iteration 7: log likelihood = -760.42901
Iteration 8: log likelihood = -756.77337
Iteration 9: log likelihood = -754.05236
Iteration 10: log likelihood = -753.58052
Iteration 11: log likelihood = -753.57142
Iteration 12: log likelihood = -753.57131
Iteration 13: log likelihood = -753.57131
DSGE model
Sample: 1955q1 - 2015q4 Number of obs = 244
Log likelihood = -753.57131
( 1) /structural[beta] = .96

| Coef. | Std. Err. | z    | P>|z|   | [95% Conf. Interval] |
|-------|-----------|------|-------|---------------------|
| beta  | .96       |      |       |                     |
| kappa | 0.0849633 | 0.0287693 | 2.95 | 0.003  | 0.0285764 | 0.1413502 |
| psi   | 1.943007  | 0.2957876 | 6.57 | 0.000  | 1.363274 | 2.52274 |
| rhou  | 0.7005481 | 0.0452603 | 15.48 | 0.000 | 0.6118395 | 0.7892568 |
| rhog  | 0.9545256 | 0.0186424 | 51.20 | 0.000 | 0.9179872 | 0.991064 |
```

The slope of the Phillips curve, $\kappa$, is estimated to be positive. The coefficient on inflation in the interest rate equation is estimated to be almost 1.94, meaning that interest rates are expected to rise almost two for one with increases in inflation.
Policy and transition matrices

Elements of the policy matrix represent the response of a control variable to a one-unit increase in a state variable.

```
. estat policy
Policy matrix

<table>
<thead>
<tr>
<th></th>
<th>Delta-method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef. Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>-.4172517</td>
<td>.039361</td>
<td>-10.60</td>
<td>0.000</td>
<td>-.4943978</td>
</tr>
<tr>
<td>g</td>
<td>.9678148</td>
<td>.2777441</td>
<td>3.48</td>
<td>0.000</td>
<td>.4234464</td>
</tr>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>-1.608212</td>
<td>.4050524</td>
<td>-3.97</td>
<td>0.000</td>
<td>-2.4021</td>
</tr>
<tr>
<td>g</td>
<td>.9529166</td>
<td>.481391</td>
<td>1.98</td>
<td>0.048</td>
<td>.0094076</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>.1892771</td>
<td>.059166</td>
<td>3.20</td>
<td>0.001</td>
<td>.0733139</td>
</tr>
<tr>
<td>g</td>
<td>1.880471</td>
<td>.2615988</td>
<td>7.19</td>
<td>0.000</td>
<td>1.367746</td>
</tr>
</tbody>
</table>
```

An increase in $u$ decreases the extent to which the inflation is above its short-run equilibrium value. This change decreases the output gap and increases interest rate.

An increase in $g$ increases the extent to which the inflation is above its short-run equilibrium value. This change also increases output gap and interest rates.

Because the states are uncorrelated with each other in this example, the elements of the state transition matrix are just the persistence parameters in the model.

```
. estat transition
Transition matrix of state variables

<table>
<thead>
<tr>
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<th>Delta-method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef. Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
<td></td>
</tr>
<tr>
<td>F.u</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>.7005481</td>
<td>.0452603</td>
<td>15.48</td>
<td>0.000</td>
<td>.6118395</td>
</tr>
<tr>
<td>g</td>
<td>1.67e-16</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>F.g</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>0 (omitted)</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>g</td>
<td>.9545256</td>
<td>.0186424</td>
<td>51.20</td>
<td>0.000</td>
<td>.9179872</td>
</tr>
</tbody>
</table>
```

Note: Standard errors reported as missing for constrained transition matrix values.

See [DSGE] Intro 4g for an example in which the state variables depend on each other. The states can also depend on each other when a state variable is specified to depend on control variables, as in [DSGE] Intro 3b, or when the state vector includes lagged control variables, as in [DSGE] Intro 4a.
One-step-ahead predictions

Predictions after dsge depend on the estimated state-space parameters. Below, we obtain one-step-ahead predictions for each of the two observed control variables in the model, and we graph the actual and predicted inflation rates.

```
. predict dep*
   (option xb assumed; fitted values)
. tsline p dep1, legend(col(1))
```

The graph shows that the one-step-ahead predictions closely follow realized inflation.

Estimating an unobserved state

The observed control variables are driven by two unobserved state variables. We can use predict with the state option to estimate the state variables.
Here we estimate the unobserved state $u$ and plot it.

```
predict state1, state
.tsline state1
```

The loose monetary policy in the mid-1970s, where predicted values of $u_t$ are negative, and the subsequent Volcker contraction are apparent in this plot.

Reference


Also see

[DSGE] Intro 1 — Introduction to DSGEs
[DSGE] Intro 3b — New Classical model
[DSGE] Intro 3c — Financial frictions model
[DSGE] Intro 3d — Nonlinear New Keynesian model
[DSGE] dsge — Linear dynamic stochastic general equilibrium models
[DSGE] dsge postestimation — Postestimation tools for dsge