Glossary

autoregressive process. An autoregressive process is a time series in which the current value of a variable is a linear function of its own past values and a white-noise error term. A first-order autoregressive process, denoted as an AR(1) process, is $y_t = \rho y_{t-1} + \epsilon_t$. An AR($p$) model contains p lagged values of the dependent variable.

conditional mean. A conditional mean expresses the mean of one variable as a function of some other variables. A regression function is a conditional mean.

control variable. A control variable is an endogenous variable. Control variables can be observed or unobserved.

In a structural DSGE model, the current value of a control variable depends on the current value of other control variables, on the expected future values of any control variable, and on the current values of state variables. The current value of a control variable is found by solving the model for the state-space form.

covariance stationary. A covariance stationary process is a weakly stationary process.

dynamic forecast. A dynamic forecast uses forecasted values wherever lagged values of the endogenous variables appear in the model, allowing one to forecast multiple periods into the future.

dynamic stochastic general equilibrium model. A dynamic stochastic general equilibrium model is a multivariate time-series model that specifies the structural relationship between state variables and control variables and is typically derived from economic theory.

endogenous variable. An endogenous variable is a variable whose values are determined by the equilibrium of a structural model. The values of an endogenous variable are determined inside the system.

equilibrium. The equilibrium values of variables in a model are the values that satisfy all the model's equations simultaneously.

exogenous variable. An exogenous variable is one whose values change independently of the other variables in a structural model. The values of an exogenous variable are determined outside the system. In a time-series model, an exogenous variable is also a predetermined variable.

expected future value. An expected future value is a forecast of the value of a variable in the future based on current information. In DSGE models, expected future values are computed under rational expectations.

Under rational expectations, $E_t(y_{t+1})$ is the condition mean of y_{t+1} conditional on the complete history of all variables in the model and the structure of the model itself.

forward operator. The forward operator F denotes the value of a variable at time $t + 1$. Formally, $Fy_t = y_{t+1}$, and $F^2 y_t = Fy_{t+1} = y_{t+2}$. A forward operator is also called a lead operator.

identified. Identified is a condition required to estimate the parameters of a model. In other words, only identified parameters can be estimated.

In DSGE models, the parameters are identified when there is a unique parameter vector that maximizes the likelihood function. For a discussion of identification, see [DSGE] **Intro 6**.

impulse–response function. An impulse–response function (IRF) measures the effect of a shock to an endogenous variable on itself or another endogenous variable. The kth impulse–response function of variable i on variable j measures the effect on variable j in period $t + k$ in response to a one-unit shock to variable i in period t, holding everything else constant.

independent and identically distributed. A series of observations is independent and identically distributed (i.i.d.) if each observation is an independent realization from the same underlying distribution. In some contexts, the definition is relaxed to mean only that the observations are independent and have identical means and variances; see Davidson and MacKinnon (1993, 42).

initial values. Initial values specify the starting place for the iterative maximization algorithm used by DSGE.

Kalman filter. The Kalman filter is a recursive procedure for predicting the state vector in a state-space model.

lag operator. The lag operator L denotes the value of a variable at time $t - 1$. Formally, $Ly_t = y_{t-1}$, and $L^2 y_t = Ly_{t-1} = y_{t-2}$.

lead operator. See *forward operator*.

likelihood-ratio (LR) test. The LR test is a classical testing procedure used to compare the fit of two models, one of which, the constrained model, is nested within the full (unconstrained) model. Under the null hypothesis, the constrained model fits the data as well as the full model. The LR test requires one to determine the maximal value of the log-likelihood function for both the constrained and the full models.

linearized model. A linearized model is an approximation to a model that is nonlinear in the variables and nonlinear in the parameters. The approximation is linear in variables but potentially nonlinear in the parameters. In a linearized model, variables are interpreted as unit deviations from steady state.

log-linear model. A log-linear model is an approximation to a model that is nonlinear in the variables and nonlinear in the parameters. In a log-linear model, variables are interpreted as percentage deviations from steady state.

model solution. A model solution is a function for the endogenous variables in terms of the exogenous variables. A model solution is also known as the reduced form of a model.

In DSGE terminology, a model solution expresses the control variables as a function of the state variables alone and expresses the state variables as a function of their values in the previous period and shocks. The reduced form of a DSGE model is also known as the state-space form of the DSGE model.

model-consistent expectation. A model-consistent expectation is the conditional mean of a variable implied by the model under consideration.

For example, under rational expectations the model-consistent expectation of $E_t(y_{t+1})$ is the mean of y_{t+1} implied by the model, conditional on the realizations of variables dated time t or previously.

nonpredetermined variable. A nonpredetermined variable is a variable whose value at time t is determined by the system of equations in the model. Contrast with predetermined variable.

null hypothesis. In hypothesis testing, the null hypothesis typically represents the conjecture that one is attempting to disprove. Often the null hypothesis is that a parameter is zero or that a statistic is equal across populations.

one-step-ahead forecast. See *static forecast*.

policy matrix. The policy matrix in the reduced form of a DSGE model is the matrix that expresses control variables as a function of state variables.

predetermined variable. A predetermined variable is a variable whose value is fixed at time t, given everything that has occurred previously. More technically, the value of a predetermined variable is fixed, given the realizations of all observed and unobserved variables at times $t - 1, t - 2, \ldots$.

rational expectations. A rational expectation of a variable does not deviate from the mean of that variable in a predictable way. More technically, a rational expectation of a variable is the conditional mean of the variable implied by the model.

realization. The realization of a random variable is the value it takes on when drawn.

reduced form. The reduced form of a model expresses the endogenous variables as functions of the exogenous variables.

The reduced form of a DSGE model expresses the control variables as a function of the state variables alone and expresses the state variables as a function of their values in the previous period and shocks. The reduced form of a DSGE model is a state-space model.

saddle-path stable. A saddle-path stable model is a structural model that can be solved for its state-space form. The existence of a saddle-path stable solution depends on the parameter values of the model. For a discussion of saddle-path stability, see [DSGE] **Intro 5**.

shock variable. A shock variable is a random variable whose value is specified as an independently and identically distributed (i.i.d.) random variable. The maximum likelihood estimator is derived under normally distributed shocks but remains consistent under i.i.d. shocks. Robust standard errors must be specified when the errors are i.i.d. but not normally distributed.

state transition matrix. The state transition matrix in the reduced form of a DSGE model is the matrix that expresses how the future values of state variables depend on their current values.

state variable. A state variable is an unobserved exogenous variable.

In DSGE models, a state variable is an unobserved exogenous variable that may depend on its own previous value, the previous values of other state variables, and shocks.

state-space model. A state-space model describes the relationship between an observed time series and an unobservable state vector that represents the "state" of the world. The measurement equation expresses the observed series as a function of the state vector, and the transition equation describes how the unobserved state vector evolves over time. By defining the parameters of the measurement and transition equations appropriately, one can write a wide variety of time-series models in the state-space form.

For DSGE models, the state-space form is the reduced form of the structural model.

The DSGE framework changes the jargon and the structure of state-space models. The measurement equation is the vector of equations for the control variables, and the transition equation is the vector of equations for the state variables. In contrast to the standard state-space model, DSGE models allow a control variable to be unobserved.

static forecast. A static forecast uses actual values wherever lagged values of the endogenous variables appear in the model. As a result, static forecasts perform at least as well as dynamic forecasts, but static forecasts cannot produce forecasts into the future when lags of the endogenous variables appear in the model.

Because actual values will be missing beyond the last historical time period in the dataset, static forecasts can forecast only one period into the future (assuming only first lags appear in the model); thus they are often called one-step-ahead forecasts.

steady-state equilibrium. A steady-state equilibrium is a time-invariant rest point of a dynamic system.

More technically, a steady-state equilibrium is a set of values for the endogenous variables to which the dynamic system will return after an exogenous variable is changed or a random shock occurs. This set of values is time invariant in that it does not depend on the time period in which the change

or shock occurs. Multistep dynamic forecasts converge to these values. A steady-state equilibrium is also known as a long-run equilibrium because it specifies time-invariant values for the endogenous variables to which the dynamic system will return, if left unshocked.

stochastic equation. A stochastic equation, in contrast to an identity, is an equation in a forecast model that includes a random component, most often in the form of an additive error term. Stochastic equations include parameters that must be estimated from historical data.

stochastic trend. A stochastic trend is a nonstationary random process. Unit-root process and random coefficients on time are two common stochastic trends. See [TS] **ucm** for examples and discussions of more commonly applied stochastic trends.

strict stationarity. A process is strictly stationary if the joint distribution of $y_1, \ldots, y_k$ is the same as the joint distribution of $y_{1+\tau}, \ldots, y_{k+\tau}$ for all k and τ. Intuitively, shifting the origin of the series by τ units has no effect on the joint distributions.

structural model. A structural model specifies the theoretical relationship among a set of variables. Structural models contain both endogenous variables and exogenous variables. Parameter estimation and interpretation require that structural models be solved for a reduced form.

trend. The trend specifies the long-run behavior in a time series. The trend can be deterministic or stochastic. Many economic, biological, health, and social time series have long-run tendencies to increase or decrease. Before the 1980s, most time-series analysis specified the long-run tendencies as deterministic functions of time. Since the 1980s, the stochastic trends implied by unit-root processes have become a standard part of the toolkit.

Wald test. A Wald test is a classical testing procedure used to compare the fit of two models, one of which, the constrained model, is nested within the full (unconstrained) model. Under the null hypothesis, the constrained model fits the data as well as the full model. The Wald test requires one to fit the full model but does not require one to fit the constrained model.

weakly stationary. A process is weakly stationary if the mean of the process is finite and independent of t, the unconditional variance of the process is finite and independent of t, and the covariance between periods t and $t - s$ is finite and depends on $t - s$ but not on t or s themselves. Weakly-stationary processes are also known as covariance stationary processes.

white noise. A variable u_t represents a white-noise process if the mean of u_t is zero, the variance of u_t is σ^2, and the covariance between u_t and u_s is zero for all $s \neq t$.

Reference

Davidson, R., and J. G. MacKinnon. 1993. *Estimation and Inference in Econometrics*. New York: Oxford University Press.