**Description**

*nlogit* performs full information maximum-likelihood estimation for nested logit models. These models relax the assumption of independently distributed errors and the independence of irrelevant alternatives inherent in conditional and multinomial logit models by clustering similar alternatives into nests.

By default, *nlogit* uses a parameterization that is consistent with a random utility model (RUM). Before version 10 of Stata, a nonnormalized version of the nested logit model was fit, which you can request by specifying the *nonnormalized* option.

You must use *nlogitgen* to generate a new categorical variable to specify the branches of the nested logit tree before calling *nlogit*.

**Quick start**

*Create variables identifying alternatives at higher levels*

For a three-level nesting structure, create 12alt identifying four alternatives at level two based on the variable balt, which identifies eight bottom-level alternatives

```
 nlogitgen 12alt = balt(12alt1: balt1 | balt2, 12alt2: balt3 | balt4, ///
 12alt3: balt5 | balt6, 12alt4: balt7 | balt8)
```

Same as above, defining 12alt from values of balt rather than from value labels

```
 nlogitgen 12alt = balt(12alt1: 1|2, 12alt2: 3|4, 12alt3: 5|6, ///
 12alt4: 7|8)
```

Create talt identifying top-level alternatives based on the four alternatives in 12alt

```
 nlogitgen talt = 12alt(talt1: 12alt1 | 12alt2, talt2: 12alt3 | 12alt4)
```

*Examine tree structure*

Display three-level nesting structure

```
 nlogittree balt 12alt talt
```

Also report choice frequencies of indicator chosen for each bottom-level alternative

```
 nlogittree balt 12alt talt, choice(chosen)
```

Identify potentially problematic observations before fitting the model by specifying case identifier *casevar*

```
 nlogittree balt 12alt talt, choice(chosen) case(casevar)
```
As above, and generate `prob` indicating problematic observations

```
nlogittree balt l2alt talt, choice(chosen) case(casevar) generate(prob)
```

**Fit nested logit model**

Three-level model with alternative-specific covariate \(x_1\), case-specific covariate \(x_2\) modeling top-level choice, and case-specific covariate \(x_3\) modeling level-two choice

```
nlogit chosen x1 || talt: x2 || l2alt: x3 || balt:, case(casevar)
```

As above, but do not estimate intercepts for bottom-level alternatives

```
nlogit chosen x1 || talt: x2 || l2alt: x3 || balt:, noconstant // case(casevar)
```

As above, but estimate intercepts for top-level alternatives

```
nlogit chosen x1 || talt: x2, estconst || l2alt: x3 // balt:, noconstant case(casevar)
```

Specify the base alternative at each level

```
nlogit chosen x1 || talt: x2, base(talt2) || l2alt: x3, base(l2alt4) // balt:, base(balt8) case(casevar)
```

Use nonnormalized parameterization

```
nlogit chosen x1 || talt: x2 || l2alt: x3 || balt:, case(casevar) // nonnormalized
```

**Menu**

- **nlogit**
  - Statistics > Choice models > Nested logit model > Nested logit model

- **nlogitgen**
  - Statistics > Choice models > Nested logit model > Setup for nested logit model

- **nlogittree**
  - Statistics > Choice models > Nested logit model > Display nested logit tree structure
Syntax

Nested logit regression

```
nlogit depvar [indepvars] [if] [in] [weight] [ || lev1_equation
    [ || lev2_equation ... ]] || altvar: [byaltvarlist], case(varname) [ nlogit_options ]
```

where the syntax of `lev#_equation` is

```
altvar: [byaltvarlist] [ , base(# | lbl) estconst ]
```

Create variable based on specification of branches

```
nlogitgen newaltvar = altvar (branchlist) [ , [ no ] log ]
```

where `branchlist` is

```
branch, branch [ , branch ... ]
```

and `branch` is

```
[label: ] alternative [ || alternative [ || alternative ... ]]
```

Display tree structure

```
nlogittree altvarlist [ if ] [ in ] [ weight ] [ , nlogittree_options ]
```
## nlogit_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>* case(varname)</td>
</tr>
<tr>
<td>base(#</td>
</tr>
<tr>
<td>nonconstant</td>
</tr>
<tr>
<td>nonnormalized</td>
</tr>
<tr>
<td>altwise</td>
</tr>
<tr>
<td>constraints(constraints)</td>
</tr>
<tr>
<td><strong>SE/Robust</strong></td>
</tr>
<tr>
<td>vce(vcetype)</td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
</tr>
<tr>
<td>level(#)</td>
</tr>
<tr>
<td>notree</td>
</tr>
<tr>
<td>nocnsreport</td>
</tr>
<tr>
<td>display_options</td>
</tr>
<tr>
<td><strong>Maximization</strong></td>
</tr>
<tr>
<td>maximize_options</td>
</tr>
<tr>
<td>collinear</td>
</tr>
</tbody>
</table>

* case(varname) is required.

## nlogittree_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main</strong></td>
</tr>
<tr>
<td>choice(depvar)</td>
</tr>
<tr>
<td>case(varname)</td>
</tr>
<tr>
<td>generate(newvar)</td>
</tr>
<tr>
<td>nolabel</td>
</tr>
<tr>
<td>nobranches</td>
</tr>
</tbody>
</table>

byaltvarlist may contain factor variables; see [U] 11.4.3 Factor variables.

bootstrap, by, fp, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

fweights, iweights, and pweights are allowed with nlogit, and fweights are allowed with nlogittree; see [U] 11.1.6 weight. Weights for nlogit must be constant within case.

collinear does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
Options

Specification and options for lev#_equation

`altvar` is a variable identifying alternatives at this level of the hierarchy.

`byaltvarlist` specifies the variables to be used to compute the by-alternative regression coefficients for that level. For each variable specified in the variable list, there will be one regression coefficient for each alternative of that level of the hierarchy. If the variable is constant across each alternative (a case-specific variable), the regression coefficient associated with the base alternative is not identifiable. These regression coefficients are labeled as (base) in the regression table. If the variable varies among the alternatives, a regression coefficient is estimated for each alternative.

`base(# | lbl)`, can be specified in each level equation where it identifies the base alternative to be used at that level. The default is the alternative that has the highest frequency.

If `vce(bootstrap)` or `vce(jackknife)` is specified, you must specify the base alternative for each level that has a `byaltvarlist` or whether the constants will be estimated. Doing so ensures that the same model is fit with each call to `nlogit`.

`estconst` applies to all the level equations except the bottom-level equation. Specifying `estconst` requests that constants for each alternative (except the base alternative) be estimated. By default, no constant is estimated at these levels. Constants can be estimated in only one level of the tree hierarchy. If you specify `estconst` for one of the level equations, you must specify `noconstant` for the bottom-level equation.

Options for nlogit

`case(varname)` specifies the variable that identifies each case. `case()` is required.

`base(# | lbl)`, can be specified in each level equation where it identifies the base alternative to be used at that level. The default is the alternative that has the highest frequency.

If `vce(bootstrap)` or `vce(jackknife)` is specified, you must specify the base alternative for each level that has a `byaltvarlist` or whether the constants will be estimated. Doing so ensures that the same model is fit with each call to `nlogit`.

`noconstant` applies only to the equation defining the bottom level of the hierarchy. By default, constants are estimated for each alternative of `altvar`, less the base alternative. To suppress the constant terms for this level, specify `noconstant`. If you do not specify `noconstant`, you cannot specify `estconst` for the higher-level equations.

`nonnormalized` requests a nonnormalized parameterization of the model that does not scale the inclusive values by the degree of dissimilarity of the alternatives within each nest. Use this option to replicate results from older versions of Stata. The default is to use the RUM–consistent parameterization.

`altwise` specifies that alternativewise deletion be used when omitting observations because of missing values in your variables. The default is to use casewise deletion; that is, the entire group of observations making up a case is omitted if any missing values are encountered. This option does not apply to observations that are excluded by the `if` or `in` qualifier or the by prefix; these observations are always handled alternativewise regardless of whether `altwise` is specified.
constraints(constraints); see [R] Estimation options.

The inclusive-valued/dissimilarity parameters are parameterized as ml ancillary parameters. They are labeled as \[\text{alternative}_\tau\_\text{const}\], where alternative is one of the alternatives defining a branch in the tree. To constrain the inclusive-valued/dissimilarity parameter for alternative a1 to be, say, equal to alternative a2, you would use the following syntax:

\[\text{constraint 1 [a1}_\tau\_\text{cons} = [a2}_\tau\_\text{cons}\]
\n.nlogit ..., constraints(1)

\[\text{SE/Robust}\]

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

If vce(robust) or vce(cluster clustvar) is specified, the likelihood-ratio test for the independence of irrelevant alternatives (IIA) is not computed.

\[\text{Reporting}\]

level(#) ; see [R] Estimation options.

notree specifies that the tree structure of the nested logit model not be displayed. See also nolabel and nobranches below for when notree is not specified.

nocnsreport; see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

\[\text{Maximization}\]

maximize_options: difficult, technique(algorithm_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(initspecs); see [R] Maximize. These options are seldom used.

The technique(bhhh) option is not allowed. The default optimization technique is technique(bfgs).

The following option is available with nlogit but is not shown in the dialog box:

collinear prevents collinear variables from being dropped. Use this option when you know that you have collinear variables and you are applying constraints() to handle the rank reduction. See [R] Estimation options for details on using collinear with constraints().

nlogit will not allow you to specify an independent variable in more than one level equation. Specifying the collinear option will allow execution to proceed in this case, but it is your responsibility to ensure that the parameters are identified.

Specification and options for nlogitgen

newaltvar and altvar are variables identifying alternatives at each level of the hierarchy.

label defines a label to associate with the branch. If no label is given, a numeric value is used.
alternative specifies an alternative, of altvar specified in the syntax, to be included in the branch. It is either a numeric value or the label associated with that value. An example of nlogitgen is

```
.nlogitgen type = restaurant(fast: 1 | 2,
> family: CafeEccell | LosNortenos | WingsNmore, fancy: 6 | 7)
```

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

**Specification and options for nlogittree**

_{altvarlist} is a list of alternative variables that define the tree hierarchy. The first variable must define bottom-level alternatives, and the order continues to the variable defining the top-level alternatives.

choice(_depvar) defines the choice indicator variable and forces nlogittree to compute and display choice frequencies for each bottom-level alternative.

case(_varname) specifies the variable that identifies each case. When both case() and choice() are specified, nlogittree executes diagnostics on the tree structure and identifies observations that will cause nlogit to terminate execution or drop observations.

generate(_newvar) generates a new indicator variable, _newvar, that is equal to 1 for invalid observations. This option requires that both choice() and case() are also specified.

nolabel forces nlogittree to suppress value labels in tree-structure output.

nobranches forces nlogittree to suppress drawing branches in the tree-structure output.

**Remarks and examples**

Remarks are presented under the following headings:

- Introduction
- Data setup and the tree structure
- Estimation
- Testing for the IIA
- Nonnormalized model

**Introduction**

nlogit performs full information maximum-likelihood estimation for nested logit models. These models relax the assumption of independently distributed errors and the IIA inherent in conditional and multinomial logit models by clustering similar alternatives into nests. The nested logit model is a direct generalization of McFadden’s choice model fit by cmclogit. You may want to read [CM] _Intro 5, [CM] _Intro 8, and [CM] cmclogit before continuing.

Although nlogit fits choice models, it is not a cm command, and you do not have to cmset your data. nlogit has its own data requirements for nested alternatives, which are detailed in the following examples.

By default, nlogit uses a RUM parameterization. Before version 10 of Stata, a nonnormalized version of the nested logit model was fit, which you can request by specifying the nonnormalized option. We recommend using the RUM parameterization for new projects because it is based on a sound model of consumer behavior.
McFadden (1977, 1981) showed how this model can be derived from a rational choice framework. Amemiya (1985, chap. 9) contains a nice discussion of how this model can be derived under the assumption of utility maximization. Hensher, Rose, and Greene (2015) provide a lucid introduction to choice models, including nested logit.

Throughout this entry, we consider a model of restaurant choice. We begin by introducing the data.

Example 1: Families choosing a restaurant

We have fictional data on 300 families and their choice of seven local restaurants. Freebirds and Mama’s Pizza are fast food restaurants; Café Eccell, Los Nortenos, and Wings ’N More are family restaurants; and Christopher’s and Mad Cows are fancy restaurants. We want to model the decision of where to eat as a function of household income ($\text{income}$, in thousands of dollars), the number of children in the household ($\text{kids}$), the rating of the restaurant according to a local restaurant guide ($\text{rating}$, coded 0–5), the average meal cost per person ($\text{cost}$), and the distance between the household and the restaurant ($\text{distance}$, in miles). $\text{income}$ and $\text{kids}$ are attributes of the family, $\text{rating}$ is an attribute of the alternative (the restaurant), and $\text{cost}$ and $\text{distance}$ are attributes of the alternative as perceived by the families—that is, each family has its own cost and distance for each restaurant.

We begin by loading the data and listing some of the variables for the first three families:

```
. use https://www.stata-press.com/data/r16/restaurant
. describe
Contains data from https://www.stata-press.com/data/r16/restaurant.dta
obs: 2,100                                     2 Dec 2018 15:07
vars: 8                                         

 variable name type format label
family_id     int %9.0g            family ID
restaurant    byte %12.0g           names choices of restaurants
income        int %9.0g            household income
cost          float %9.0g          average meal cost per person
kids          byte %9.0g           number of kids in the household
rating        byte %9.0g           ratings in local restaurant guide
distance      float %9.0g          distance between home and restaurant
chosen        byte %9.0g           0 no 1 yes
```

Sorted by: family_id
Because each family chose among seven restaurants, there are 7 observations in the dataset for each family. The variable chosen is coded 0/1, with 1 indicating the chosen restaurant and 0 otherwise.

We could fit a conditional logit model to our data. Because income and kids are constant within each family, we would use the cmeclogit command instead of clogit. However, the conditional logit may be inappropriate. That model assumes that the random errors are independent, and as a result, it forces the odds ratio of any two alternatives to be independent of the other alternatives, a property known as the IIA. We will discuss the IIA assumption in more detail later.

Assuming that unobserved shocks influencing a decision maker’s attitude toward one alternative have no effect on his or her attitude toward the other alternatives may seem innocuous, but often this assumption is too restrictive. Suppose that when a family was deciding which restaurant to visit, they were pressed for time because of plans to attend a movie later. The unobserved shock (being in a hurry) would raise the likelihood that the family goes to either fast food restaurant (Freebirds or Mama’s Pizza). Similarly, another family might be choosing a restaurant to celebrate a birthday and therefore be inclined to attend a fancy restaurant (Christopher’s or Mad Cows).

Nested logit models relax the independence assumption and allow us to group alternatives for which unobserved shocks may have concomitant effects. Here we suspect that restaurants should be grouped by type (fast, family, or fancy). The tree structure of a family’s decision about where to eat might look like this:

```
1. 1 Freebirds 1 1 0 1.245553
2. 1 MamasPizza 0 1 1 2.82493
3. 1 CafeEccell 0 1 2 4.21293
4. 1 LosNortenos 0 1 3 4.167634
5. 1 WingsNmore 0 1 2 6.330531
6. 1 Christophers 0 1 4 10.19829
7. 1 MadCows 0 1 5 5.601388
8. 2 Freebirds 0 3 0 4.162657
9. 2 MamasPizza 0 3 1 2.865081
10. 2 CafeEccell 0 3 2 5.37799
11. 2 LosNortenos 1 3 3 4.282864
12. 2 WingsNmore 0 3 2 8.133914
13. 2 Christophers 0 3 4 8.664631
14. 2 MadCows 0 3 5 9.119597
15. 3 Freebirds 1 3 0 2.112586
16. 3 MamasPizza 0 3 1 2.215329
17. 3 CafeEccell 0 3 2 6.978715
18. 3 LosNortenos 0 3 3 5.117877
19. 3 WingsNmore 0 3 2 5.312941
20. 3 Christophers 0 3 4 9.551273
21. 3 MadCows 0 3 5 5.539806
```
At the bottom of the tree are the individual restaurants, indicating that there are some random shocks that affect a family’s decision to eat at each restaurant independently. Above the restaurants are the three types of restaurants, indicating that other random shocks affect the type of restaurant chosen. As is customary when drawing nested logit trees, at the top level is one box, representing the family making the decision.

We use the following terms to describe nested logit models.

- **level**, or decision level, is the level or stage at which a decision is made. The example above has only two levels. In the first level, a type of restaurant is chosen—fast food, family, or fancy—and in the second level, a specific restaurant is chosen.
- **bottom level** is the level where the final decision is made. In our example, this is when we choose a specific restaurant.
- **alternative set** is the set of all possible alternatives at any given decision level.
- **bottom alternative set** is the set of all possible alternatives at the bottom level. This concept is often referred to as the choice set in the economics-choice literature. In our example, the bottom alternative set is all seven of the specific restaurants.
- **alternative** is a specific alternative within an alternative set. In the first level of our example, “fast food” is an alternative. In the second or bottom level, “Mad Cows” is an alternative. Not all alternatives within an alternative set are available to someone making a choice at a specific stage, only those that are nested within all higher-level decisions.
- **chosen alternative** is the alternative from an alternative set that we observe someone having chosen.

**Technical note**

Although the trees in nested logit analysis are often interpreted as implying that the highest-level decisions are made first, followed by decisions at lower levels, and finally the decision among alternatives at the bottom level, no such temporal ordering is implied. See Hensher, Rose, and Greene (2015, chap. 14). In our example, we are not assuming that families first choose whether to attend a fast, family, or fancy restaurant and then choose the particular restaurant; we assume merely that they choose one of the seven restaurants.
Data setup and the tree structure

To fit a nested logit model, you must first create a variable that defines the structure of your nested logit tree.

Example 2: Identifying the first-level set of alternatives

To run `nlogit`, we need to generate a categorical variable that identifies the first-level set of alternatives: fast food, family restaurants, or fancy restaurants. We can do so easily by using `nlogitgen`.

```
.nlogitgen type = restaurant(fast: Freebirds | MamasPizza, family: CafeEccell | LosNortenos | WingsNmore, fancy: Christophers | MadCows)
```

New variable type is generated with 3 groups

```
label list lb_type
lb_type:
   1 fast
   2 family
   3 fancy
```

```
.nlogittree restaurant type, choice(chosen)
tree structure specified for the nested logit model
```

<table>
<thead>
<tr>
<th>type</th>
<th>N</th>
<th>restaurant</th>
<th>N</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast</td>
<td>600</td>
<td>Freebirds</td>
<td>300</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MamasPizza</td>
<td>300</td>
<td>15</td>
</tr>
<tr>
<td>family</td>
<td>900</td>
<td>CafeEccell</td>
<td>300</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LosNortenos</td>
<td>300</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WingsNmore</td>
<td>300</td>
<td>69</td>
</tr>
<tr>
<td>fancy</td>
<td>600</td>
<td>Christophers</td>
<td>300</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MadCows</td>
<td>300</td>
<td>24</td>
</tr>
</tbody>
</table>

```
total 2100 300
```

k = number of times alternative is chosen
N = number of observations at each level

The new categorical variable is `type`, which takes on value 1 (fast) if `restaurant` is Freebirds or Mama’s Pizza; value 2 (family) if `restaurant` is Café Eccell, Los Nortenos, or Wings ’N More; and value 3 (fancy) otherwise. `nlogittree` displays the tree structure.

Technical note

We could also use values instead of value labels of `restaurant` in `nlogitgen`. Value labels are optional, and the default value labels for `type` are `type1`, `type2`, and `type3`. The vertical bar is also optional.

```
use https://www.stata-press.com/data/r16/restaurant, clear
.nlogitgen type = restaurant(1 2 3 4 5, 6 7)
```

New variable type is generated with 3 groups

```
label list lb_type
lb_type:
   1 type1
   2 type2
   3 type3
```
In our dataset, every family was able to choose among all seven restaurants. However, in other applications some decision makers may not have been able to choose among all possible alternatives. For example, two cases may have choice hierarchies of

where the second case does not have the restaurant alternatives Café Eccell or Mad Cows available to them. The only restriction is that the relationships between higher- and lower-level alternative sets be the same for all decision makers. In this two-level example, Freebirds and Mama's Pizza are classified as fast food restaurants for both cases; Café Eccell, Los Norteños, and Wings 'N More are family restaurants; and Christopher's and Mad Cows are fancy restaurants. `nlogit` requires only that hierarchy be maintained for all cases.

**Estimation**

> **Example 3: Fitting the model**

With our `type` variable created that defines the three types of restaurants, we can now examine how the alternative-specific attributes (cost, rating, and distance) apply to the bottom alternative set (the seven restaurants) and how family-specific attributes (income and kid) apply to the alternative set at the first decision level (the three types of restaurants).
. use https://www.stata-press.com/data/r16/restaurant, clear
. qui nlogitgen type = restaurant(fast: Freebirds | MamasPizza,>
> family: CafeEccell | LosNortenos| WingsNmore, fancy: Christophers | MadCows)
. nlogit chosen cost rating distance || type: income kids, base(family) ||
> restaurant:, noconstant case(family_id)

Tree structure specified for the nested logit model

<table>
<thead>
<tr>
<th>type</th>
<th>N</th>
<th>restaurant</th>
<th>N</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast</td>
<td>600</td>
<td>Freebirds</td>
<td>300</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MamasPizza</td>
<td>300</td>
<td>15</td>
</tr>
<tr>
<td>family</td>
<td>900</td>
<td>CafeEccell</td>
<td>300</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LosNortenos</td>
<td>300</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WingsNmore</td>
<td>300</td>
<td>69</td>
</tr>
<tr>
<td>fancy</td>
<td>600</td>
<td>Christophers</td>
<td>300</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MadCows</td>
<td>300</td>
<td>24</td>
</tr>
</tbody>
</table>

Total 2100 300

k = number of times alternative is chosen
N = number of observations at each level

Iteration 0:  log likelihood = -541.93581

(output omitted)

Iteration 17:  log likelihood = -485.47331

RUM-consistent nested logit regression

| Case variable: family_id | Number of obs  | 2,100 |
| Alternative variable: restaurant | Number of cases | 300   |

Alternatives per case: min = 7
avg = 7.0
max = 7

Wald chi2(7) = 46.71
Log likelihood = -485.47331
Prob > chi2 = 0.0000

| chosen         | Coef.    | Std. Err. | z    | P>|z|  | [95% Conf. Interval] |
|----------------|----------|-----------|------|------|---------------------|
| restaurant     |          |           |      |      |                     |
| cost           | -.1843847| .0933975  | -1.97| 0.048| -.3674404 -.0013289|
| rating         | .463694  | .3264935  | 1.42 | 0.156| -.1762215 1.10361  |
| distance       | -.3797474| .1003828  | -3.78| 0.000| -.5764941 -.1830007|
| type equations |          |           |      |      |                     |
| fast           |          |           |      |      |                     |
| income         | -.0266038| .0117306  | -2.27| 0.023| -.0495952 -.0036123|
| kids           | -.0872584| .1385026  | -0.63| 0.529| -.3587184 .1842016 |
| family         |          |           |      |      |                     |
| income         | 0        | (base)    |      |      |                     |
| kids           | 0        | (base)    |      |      |                     |
| fancy          |          |           |      |      |                     |
| income         | .0461827 | .0090936  | 5.08 | 0.000| .0283595 .0640059  |
| kids           | -.3959413| .1220356  | -3.24| 0.001| -.6351267 -.1567559|

Dissimilarity parameters

<table>
<thead>
<tr>
<th>/type</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>fast-tau</td>
<td>1.712878</td>
<td>1.48685</td>
<td>-1.201295</td>
<td>4.627051</td>
<td></td>
</tr>
<tr>
<td>family-tau</td>
<td>2.505113</td>
<td>.9646351</td>
<td>.614463</td>
<td>4.395763</td>
<td></td>
</tr>
<tr>
<td>fancy-tau</td>
<td>4.099844</td>
<td>2.810123</td>
<td>-1.407896</td>
<td>9.607583</td>
<td></td>
</tr>
</tbody>
</table>

LR test for IIA (tau=1): chi2(3) = 6.87  Prob > chi2 = 0.0762
First, let's examine how we called nlogit. The delimiters (||) separate equations. The first equation specifies the dependent variable, chosen, and three alternative-specific variables, cost, rating, and distance. We refer to these variables as alternative specific because they vary among the bottom-level alternatives, the restaurants. We obtain one parameter estimate for each variable. These estimates are listed in the equation subtable labeled restaurant.

For the second equation, we specify the type variable. It identifies the first-level alternatives, the restaurant types. Following the colon after type, we specify two case-specific variables, income and kids. Here we obtain a parameter estimate for each variable for each alternative at this level. That is why we call these variable lists “by-alternative” variables. Because income and kids do not vary within each case, to identify the model, one must specify the alternative set of parameters as zero. We specified the base(family) option with this equation to restrict the parameters for the family alternative.

The variable identifying the bottom-level alternatives, restaurant, is specified after the second equation delimiter. We do not specify any variables after the colon delimiter at this level. Had we specified variables here, we would have obtained an estimate for each variable in each equation. As we will see below, these variables parameterize the constant term in the utility equation for each bottom-level alternative. The noconstant option suppresses bottom-level alternative-specific constant terms.

Near the bottom of the output are the dissimilarity parameters, which measure the degree of correlation of random shocks within each of the three types of restaurants. Dissimilarity parameters greater than one imply that the model is inconsistent with random utility maximization; Hensher, Rose, and Greene (2005, sec. 13.6) discuss this in detail. We will ignore the fact that all our dissimilarity parameters exceed one.

The conditional logit model is a special case of nested logit in which all the dissimilarity parameters are equal to one. At the bottom of the output, we find a likelihood-ratio test of this hypothesis. Here we have mixed evidence of the null hypothesis that all the parameters are one. Equivalently, the property known as the IIA imposed by the conditional logit model holds if and only if all dissimilarity parameters are equal to one. We discuss the IIA in more detail now.

### Testing for the IIA

The IIA is a property of the multinomial and conditional logit models that forces the odds of choosing one alternative over another to be independent of the other alternatives. For simplicity, suppose that a family was choosing only between Freebirds and Mama’s Pizza, and the family was equally likely to choose either of the restaurants. The probability of going to each restaurant is 50%. Now suppose that Bill’s Burritos opens up next door to Freebirds, which is also a burrito restaurant. If the IIA holds, then the probability of going to each restaurant must now be 33.33% so that the family remains equally likely to go to Mama’s Pizza or Freebirds.

The IIA may sometimes be a plausible assumption. However, a more likely scenario would be for the probability of going to Mama’s Pizza to remain at 50% and the probabilities of going to Freebirds and Bill’s Burritos to be 25% each, because the two restaurants are next door to each other and serve the same food. Nested logit analysis would allow us to relax the IIA assumption of conditional logit. We could group Bill’s Burritos and Freebirds into one nest that encompasses all burrito restaurants and create a second nest for pizzerias.

The IIA is a consequence of assuming that the errors are independent and identically distributed (i.i.d.). Because the errors are i.i.d., they cannot contain any alternative-specific unobserved information, and therefore adding a new alternative cannot affect the relationship between a pair of existing alternatives.
In the previous example, we saw that a joint test that the dissimilarity parameters were equal to one is one way to test for IIA. However, that test required us to specify a tree for the nested logit model, and different specifications could lead to conflicting results of the test. Hausman and McFadden (1984) suggest that if part of the choice set truly is irrelevant with respect to the other alternatives, omitting that subset from the conditional logit model will not lead to inconsistent estimates. Therefore, Hausman’s (1978) specification test can be used to test for IIA, and this test will not be sensitive to the tree structure we specify for a nested logit model.

Example 4: Testing the IIA assumption

We want to test the IIA for the subset of family restaurants against the alternatives of fast food and fancy restaurants. To do so, we need to use Stata’s hausman command; see \[R\] hausman.

We first run the estimation on the full bottom alternative set, store the results by using estimates store, and then run the estimation on the bottom alternative set, excluding the alternatives of family restaurants. We then run the hausman test.

```
. generate incFast = (type == 1) * income
. generate incFancy = (type == 3) * income
. generate kidFast = (type == 1) * kids
. generate kidFancy = (type == 3) * kids
. clogit chosen cost rating distance incFast incFancy kidFast kidFancy,
> group(family_id) nolog
Conditional (fixed-effects) logistic regression
Number of obs = 2,100
LR chi2(7) = 189.73
Prob > chi2 = 0.0000
Log likelihood = -488.90834 Pseudo R2 = 0.1625

chosen | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-------|------------------------------------------------------------------
cost | -.1367799 .0358479 -3.82 0.000 -.2070404 -.0665193
distance | -.1977505 .0471653 -4.19 0.000 -.2901927 -.1053082
incFast | -.0390183 .0094018 -4.15 0.000 -.0574455 -.0205911
incFancy | .0407053 .0080405 5.06 0.000 .0249462 .0564644
kidFast | -.2398757 .1063674 -2.26 0.024 -.448352 -.0313994
kidFancy | -.3893862 .1143797 -3.40 0.001 -.6135662 -.1652061
```

. estimates store fullset
. clogit chosen cost rating distance incFast kidFast if type != 2,
> group(family_id) nolog
note: 222 groups (888 obs) dropped because of all positive or
all negative outcomes.
Conditional (fixed-effects) logistic regression
Number of obs = 312
LR chi2(5) = 44.35
Prob > chi2 = 0.0000
Log likelihood = -85.955324 Pseudo R2 = 0.2051

```
chosen | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-------|------------------------------------------------------------------
cost | -.0616621 .067852 -0.91 0.363 -.1946496 .0713254
distance | -.244396 .0995056 -2.46 0.014 -.4394234 -.0493687
incFast | -.0737506 .0177424 -4.16 0.000 -.108529 -.0389721
kidFast | .4105386 .2137051 1.92 0.055 -.0083157 .8293928
```
### Coefficients

<table>
<thead>
<tr>
<th></th>
<th>(b)</th>
<th>(B)</th>
<th>(b-B)</th>
<th>sqrt(diag(V_{b-V_B}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>. fullset</td>
<td></td>
<td>Difference</td>
<td>S.E.</td>
</tr>
<tr>
<td>cost</td>
<td>-.0616621</td>
<td>-.1367799</td>
<td>.0751178</td>
<td>.0576092</td>
</tr>
<tr>
<td>rating</td>
<td>.1659001</td>
<td>.3066622</td>
<td>-.1407621</td>
<td>.2451308</td>
</tr>
<tr>
<td>distance</td>
<td>-.244396</td>
<td>-.1977505</td>
<td>-.0466456</td>
<td>.0876173</td>
</tr>
<tr>
<td>incFast</td>
<td>-.0737506</td>
<td>-.0390183</td>
<td>-.0347323</td>
<td>.015049</td>
</tr>
<tr>
<td>kidFast</td>
<td>.4105386</td>
<td>-.2398757</td>
<td>.6504143</td>
<td>.1853533</td>
</tr>
</tbody>
</table>

b = consistent under Ho and Ha; obtained from clogit
B = inconsistent under Ha, efficient under Ho; obtained from clogit

Test: Ho: difference in coefficients not systematic
\[
\text{chi2}(5) = (b-B)^\prime [(V_{b-V_B})^{-1}] (b-B) = 10.70
\]

Prob>chi2 = 0.0577
(V_{b-V_B} is not positive definite)

Similar to our findings in example 3, the results of the test of the IIA are mixed. We cannot reject the IIA at the commonly used 5% significance level, but we could at the 10% level. Substantively, a significant test result suggests that the odds of going to one of the fancy restaurants versus going to one of the fast food restaurants change if we include the family restaurants in the alternative set and that a nested logit specification may be warranted.

### Nonnormalized model

Previous versions of Stata fit a nonnormalized nested logit model that is available via the nonnormalized option. The nonnormalized version is presented in, for example, Greene (2018, 837–839). Here we outline the differences between the nonnormalized model and the random utility parameterization of the model. Our discussion follows Heiss (2002) and assumes the nested logit tree has two levels, with \( M \) alternatives at the upper level and a total of \( J \) alternatives at the bottom level.

In a RUM framework, by consuming alternative \( j \), decision maker \( i \) obtains utility

\[
U_{ij} = V_{ij} + \epsilon_{ij} = \alpha_j + x_{ij}\beta_j + z_i\gamma_j + \epsilon_{ij}
\]

where \( V_{ij} \) is the deterministic part of utility and \( \epsilon_{ij} \) is the random part. \( x_{ij} \) are alternative-specific variables and \( z_i \) are case-specific variables. The set of errors \( \epsilon_{i1}, \ldots, \epsilon_{iJ} \) are assumed to follow the generalized extreme-value (GEV) distribution, which is a generalization of the type 1 extreme-value distribution that allows for alternatives within nests of the tree structure to be correlated. Let \( \rho_m \) denote the correlation in nest \( m \), and define the dissimilarity parameter \( \tau_m = \sqrt{1 - \rho_m} \). \( \tau_m = 0 \) implies that the alternatives in nest \( m \) are perfectly correlated, whereas \( \tau_m = 1 \) implies independence.

The inclusive value for the \( m \)th nest corresponds to the expected value of the utility that decision maker \( i \) obtains by consuming an alternative in nest \( m \). Denote this value by \( IV_m \),

\[
IV_m = \ln \sum_{j \in B_m} \exp(V_k/\tau_m)
\]  

(1)

where \( B_m \) denotes the set of alternatives in nest \( m \). Given the inclusive values, we can show that the probability that random-utility–maximizing decision maker \( i \) chooses alternative \( j \) is

\[
Pr_j = \frac{\exp(V_j/\tau(j)) \exp(\tau(j)IV(j))}{\sum_m \exp(\tau_mIV_m)}
\]
where \( \tau(j) \) and \( \text{IV}(j) \) are the dissimilarity parameter and inclusive value for the nest in which alternative \( j \) lies. 

In contrast, for the nonnormalized model, we have a latent variable

\[ \tilde{V}_{i,j} = \tilde{\alpha}_j + x_{i,j}\tilde{\beta}_j + z_i\tilde{\gamma}_j \]

and corresponding inclusive values

\[ \tilde{\text{IV}}_m = \ln \sum_{j \in B_m} \exp(\tilde{V}_k) \]  

Equations (1) and (2) represent the key difference between the random utility and the nonnormalized models. By scaling the \( V_{ij} \) within each nest, the RUM parameterization allows utilities to be compared across nests. Without the rescaling, utilities can be compared only for goods within the same nest. Moreover, adding a constant to each \( V_{ij} \) for consumer \( i \) will not affect the probabilities of the RUM, but adding a constant to each \( \tilde{V}_{ij} \) will affect the probabilities from the nonnormalized model. Decisions based on utility maximization can depend only on utility differences and not the scale or zero point of the utility function because utility is an ordinal concept, so the nonnormalized model cannot be consistent with utility maximization.

Heiss (2002) showed that the nonnormalized model can be consistent with a random utility parameterization in the special case where all the variables are specified in the bottom-level equation. Then multiplying the nonnormalized coefficients by the respective dissimilarity parameters results in the coefficients that are consistent with a RUM.

**Technical note**

Degenerate nests occur when there is only one alternative in a branch of the tree hierarchy. The associated dissimilarity parameter of the RUM is not defined. The inclusive-valued parameter of the nonnormalized model will be identifiable if there are alternative-specific variables specified in (1) of the model specification (the \textit{indepvars} in the model syntax). Numerically, you can skirt the issue of nonidentifiable/undefined parameters by setting constraints on them. For the RUM constraint, set the dissimilarity parameter to 1. See the description of \texttt{constraints()} in \textit{Options} for details on setting constraints on the dissimilarity parameters.
nlogit stores the following in **e()**: 

**Scalars**

- **e(N)**: number of observations
- **e(N_case)**: number of cases
- **e(N_ic)**: \( N \) for Bayesian information criterion (BIC)
- **e(N_clust)**: number of clusters
- **e(k_eq)**: number of equations in **e(b)**
- **e(k_eq_model)**: number of equations in overall model test
- **e(k_alt)**: number of alternatives for bottom level
- **e(k_altj)**: number of alternatives for \( j \)th level
- **e(k_indvars)**: number of independent variables
- **e(k_ind2vars)**: number of by-alternative variables for bottom level
- **e(k_ind2varsj)**: number of by-alternative variables for \( j \)th level
- **e(df_m)**: model degrees of freedom
- **e(df_c)**: clogit model degrees of freedom
- **e(ll)**: log likelihood
- **e(ll_c)**: clogit model log likelihood
- **e(chi2)**: \( \chi^2 \)
- **e(chi2_c)**: likelihood-ratio test for IIA
- **e(p)**: \( p \)-value for model Wald test
- **e(p_c)**: \( p \)-value for IIA test
- **e(i_base)**: base index for bottom level
- **e(i_basej)**: base index for \( j \)th level
- **e(levels)**: number of levels
- **e(alt_min)**: minimum number of alternatives
- **e(alt_avg)**: average number of alternatives
- **e(alt_max)**: maximum number of alternatives
- **e(const)**: constant indicator for bottom level
- **e(constj)**: constant indicator for \( j \)th level
- **e(rum)**: 1 if RUM, 0 otherwise
- **e(rank)**: rank of \( e(V) \)
- **e(ic)**: number of iterations
- **e(rc)**: return code
- **e(converged)**: 1 if converged, 0 otherwise
Macros

- `e(cmd)` nlogit
- `e(cmdline)` command as typed
- `e(depvar)` name of dependent variable
- `e(indvars)` by-alternative variables for bottom level
- `e(ind2vars)` by-alternative variables for jth level
- `e(case)` variable defining cases
- `e(altvar)` alternative variable for bottom level
- `e(altvarj)` alternative variable for jth level
- `e(alteqs)` equation names for bottom level
- `e(alteqsj)` equation names for jth level
- `e(alti)` ith alternative for bottom level
- `e(altij)` ith alternative for jth level
- `e(wtype)` weight type
- `e(wexp)` weight expression
- `e(marktype)` casewise or altwise, type of markout
- `e(key_Nic)` cases, key for N for Bayesian information criterion (BIC)
- `e(title)` title in estimation output
- `e(clustvar)` name of cluster variable
- `e(chi2type)` Wald, type of model $\chi^2$ test
- `e(vcetype)` vcetype specified in `vce()`
- `e(opt)` type of optimization
- `e(which)` max or min: whether optimizer is to perform maximization or minimization
- `e(ml_method)` type of ml method
- `e(user)` name of likelihood-evaluator program
- `e(technique)` maximization technique
- `e(datasignature)` the checksum
- `e(datasignaturevars)` variables used in calculation of checksum
- `e(properties)` b V
- `e(ilog)` iteration log (up to 20 iterations)
- `e(gradient)` gradient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(V_modelbased)` model-based variance
- `e(sample)` marks estimation sample

Matrices

- `e(b)` coefficient vector
- `e(Cns)` constraints matrix
- `e(k_altern)` number of alternatives at each level
- `e(k_branch)` number of branches at each alternative of jth level
- `e(stats)` alternative statistics for bottom level
- `e(statsj)` alternative statistics for jth level
- `e(altidx)` alternative indices for jth level
- `e(altind2vars)` indicators for bottom level estimated by-alternative variable—$e(k_{alt}) \times e(k_{ind2vars})$
- `e(altind2varsj)` indicators for jth level estimated by-alternative variable—$e(k_{altj}) \times e(k_{ind2varsj})$

Functions

- `e(sample)` marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:

**Two-level nested logit model**

**Three-level nested logit model**
Two-level nested logit model

Consider our two-level nested logit model for restaurant choice. We define \( T = \{1, 2, 3\} \) to be the set of indices denoting the three restaurant types and \( R_1 = \{1, 2\} \), \( R_2 = \{3, 4, 5\} \), and \( R_3 = \{6, 7\} \) to be the set of indices representing each restaurant within type \( t \in T \). Let \( C_1 \) and \( C_2 \) be the random variables that represent the choices made for the first level, restaurant type, and second level, restaurant, of the hierarchy, where we observe the choices \( C_1 = t, t \in T \), and \( C_2 = j, j \in R_t \). Let \( z_t \) and \( x_{tj} \), for \( t \in T \) and \( j \in R_t \), refer to the row vectors of explanatory variables for the first-level alternatives and bottom-level alternatives for one case, respectively. We write the utilities (latent variables) as \( U_{tj} = z_t \alpha_t + x_{tj} \beta_j + \epsilon_{tj} = \eta_{tj} + \epsilon_{tj} \), where \( \alpha_t \) and \( \beta_j \) are column vectors and the \( \epsilon_{tj} \) are random disturbances. When the \( x_{tj} \) are alternative specific, we can drop the indices from \( \beta \), where we estimate one coefficient for each alternative in \( R_t, t \in T \). These variables are specified in the first equation of the nlogit syntax (see example 3).

When the random-utility framework is used to describe the choice behavior, the alternative that is chosen is the alternative that has the highest utility. Assume for our restaurant example that we choose \( \epsilon \) where it has been shown that the \( t \) given choice of restaurant type \( \tau \) reduces to the multinomial logistic function.

Another formulation of the conditional probability of choosing alternative \( j \) in \( R_t \) given choice \( t \) in \( T \) is the logistic function without this normalization,

\[
F_{R|T}(\epsilon \mid t) = \exp \left[ -\left\{ \sum_{m \in R_t} \exp(\epsilon_{tm}/\tau_t) \right\}^{\tau_t} \right] \tag{3}
\]

where it has been shown that the \( \epsilon_{tj}, j \in R_t \), are exchangeable with correlation \( 1 - \tau_t^2 \), for \( \tau_t \in (0, 1] \) (Kotz and Nadarajah 2000). For example, the probability of choosing Christopher’s, \( j = 6 \) given type \( t = 3 \), is

\[
\Pr(C_2 = 6 \mid C_1 = 3) = \Pr(U_{36} - U_{37} > 0) = \Pr(\epsilon_{37} \leq \epsilon_{36} + \eta_{36} - \eta_{37}) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\epsilon_{36} + \eta_{36} - \eta_{37}} f_{R|T}(\epsilon_{36}, \epsilon_{37}) \, d\epsilon_{37} \right\} \, d\epsilon_{36}
\]

where \( f = \frac{\partial F}{\partial \epsilon_{36} \partial \epsilon_{37}} \) is the joint density function of \( \epsilon \) given \( t \). \( U_{37} \) is the utility of eating at Mad Cows, the other fancy (\( t = 3 \)) restaurant. Amemiya (1985) demonstrates that this integral evaluates to the logistic function

\[
\Pr(C_2 = 6 \mid C_1 = 3) = \frac{\exp(\eta_{36}/\tau_3)}{\exp(\eta_{36}/\tau_3) + \exp(\eta_{37}/\tau_3)} = \frac{\exp(x_{36} \beta_6/\tau_3)}{\exp(x_{36} \beta_6/\tau_3) + \exp(x_{37} \beta_7/\tau_3)}
\]

and in general

\[
\Pr(C_2 = j \mid C_1 = t) = \frac{\exp(x_{tj} \beta_j/\tau_t)}{\sum_{m \in R_t} \exp(x_{tm} \beta_m/\tau_t)} \tag{4}
\]

Letting \( \tau_t = 1 \) in (3) reduces to the product of independent extreme-value distributions, and (4) reduces to the multinomial logistic function.

For the logistic function in (4), we scale the linear predictors by the dissimilarity parameters. Another formulation of the conditional probability of choosing alternative \( j \in R_t \) given choice \( t \in T \) is the logistic function without this normalization,
\[ \Pr(C_2 = j \mid C_1 = t) = \frac{\exp(x_{tj}\beta_j)}{\sum_{m \in R_t} \exp(x_{tm}\beta_m)} \]

and this is what is used in nlogit’s nonnormalized parameterization.

Amemiya (1985) defines the general form for the joint distribution of the \( \epsilon \)'s as
\[
F_{T,R}(\epsilon) = \exp \left\{ - \sum_{k \in T} \theta_k \left( \sum_{m \in R_k} \exp(-\epsilon_{km}/\tau_k) \right)^{\tau_k} \right\}
\]
from which the probability of choice \( t, t \in T \) can be derived as
\[
\Pr(C_1 = t) = \frac{\theta_t \left\{ \sum_{m \in R_t} \exp(\eta_{tm}/\tau_t) \right\}^{\tau_t}}{\sum_{k \in T} \theta_k \left\{ \sum_{m \in R_k} \exp(\eta_{km}/\tau_k) \right\}^{\tau_k}}
\]
nlogit sets \( \theta_t = 1 \). Noting that
\[
\left\{ \sum_{m \in R_t} \exp(\eta_{tm}/\tau_t) \right\}^{\tau_t} = \left\{ \sum_{m \in R_t} \exp \left( \frac{z_t\alpha_t + x_{tm}\beta_m}{\tau_t} \right) \right\}^{\tau_t}
\]
\[
= \exp(z_t\alpha_t) \left\{ \sum_{m \in R_t} \exp(\frac{x_{tm}\beta_m}{\tau_t}) \right\}^{\tau_t}
\]
\[
= \exp(z_t\alpha_t + \tau_t I_t)
\]
we define the inclusive values \( I_t \) as
\[
I_t = \ln \left\{ \sum_{m \in R_t} \exp(x_{tm}\beta_m/\tau_t) \right\}
\]
and we can view
\[
\exp(\tau_t I_t) = \left\{ \sum_{m \in R_t} \exp(x_{tm}\beta_m)^{1/\tau_t} \right\}^{\tau_t}
\]
as a weighted average of the \( \exp(x_{tm}\beta_m) \), for \( m \in R_t \). For the nlogit RUM parameterization, we can express (5) as
\[
\Pr(C_1 = t) = \frac{\exp(z_t\alpha_t + \tau_t I_t)}{\sum_{k \in T} \exp(z_k\alpha_k + \tau_k I_k)}
\]
Next, we define inclusive values for the nonnormalized model to be
\[
\tilde{I}_t = \ln \left\{ \sum_{m \in R_t} \exp(x_{tm}\beta_m) \right\}
\]
and we express \( \Pr(C_1 = t) \) as
\[
\Pr(C_1 = t) = \frac{\exp(z_t\alpha_t + \tau_t \tilde{I}_t)}{\sum_{k \in T} \exp(z_k\alpha_k + \tau_k \tilde{I}_k)}
\]
Equation (5) is consistent with (6) only when \( \eta_{ij} = x_{ij} \beta_j \), so in general the nlogit nonnormalized model is not consistent with the RUM.

Now assume that we have \( N \) cases where we add a third subscript, \( i \), to denote case \( i \), \( i = 1, \ldots, N \). Denote \( y_{itj} \) to be a binary variable indicating the choice made by case \( i \) so that for each \( i \) only one \( y_{itj} \) is 1 and the rest are 0 for all \( t \in T \) and \( j \in R_t \). The log likelihood for the two-level RUM is

\[
\log \ell = \sum_{i=1}^{N} \sum_{k \in T} \sum_{m \in R_k} y_{itj} \log \left\{ \Pr(C_{i1} = k) \Pr(C_{i2} = m | C_{i1} = k) \right\} \\
= \sum_{i=1}^{N} \sum_{k \in T} \sum_{m \in R_k} y_{itj} \left[ \frac{z_{ik} \alpha_k + \tau_k I_{ik}}{\tau_k} - \log \left( \sum_{l \in T} \exp \left( \frac{z_{il} \alpha_l}{\tau_l} + \tau_l I_{il} \right) \right) \right]
\]

The likelihood for the nonnormalized model has a similar form, replacing \( I \) with \( \tilde{I} \) and by not scaling \( x_{ikj} \beta_j \) by \( \tau_k \).

**Three-level nested logit model**

Here we define a three-level nested logit model that can be generalized to the four-level and higher models. As before, let the integer set \( T \) be the indices for the first level of choices. Let sets \( S_t, t \in T \), be mutually exclusive sets of integers representing the choices of the second level of the hierarchy. Finally, let \( R_j, j \in S_t \), be the bottom-level choices. Let \( U_{tjk} = \eta_{tjk} + \epsilon_{tjk}, k \in R_j \), and the distribution of \( \epsilon_{tjk} \) be Gumbel’s multivariate extreme value of the form

\[
F(\epsilon) = \exp \left( -\sum_{t \in T} \left[ \sum_{j \in S_t} \left( \sum_{k \in R_j} \exp(-\eta_{tjk} / \tau_j) \right)^{\frac{\tau_j}{\nu_t}} \right]^{\frac{\nu_j}{\tau_j}} \right)
\]

Let \( C_1, C_2, \) and \( C_3 \) represent the choice random variables for levels 1, 2, and the bottom, respectively. Then the set of conditional probabilities is

\[
\Pr(C_3 = k | C_1 = t, C_2 = j) = \frac{\exp(\eta_{tjk} / \tau_j)}{\sum_{l \in R_j} \exp(\eta_{tlk} / \tau_j)}
\]

\[
\Pr(C_2 = j | C_1 = t) = \frac{\left( \sum_{k \in R_j} \exp(\eta_{tjk} / \tau_j) \right)^{\frac{\tau_j}{\nu_t}}}{\sum_{l \in S_t} \left( \sum_{k \in R_l} \exp(\eta_{tlk} / \tau_l) \right)^{\frac{\tau_l}{\nu_t}}}
\]

\[
\Pr(C_1 = t) = \frac{\left( \sum_{j \in S_t} \left( \sum_{k \in R_j} \exp(\eta_{tjk} / \tau_j) \right)^{\frac{\tau_j}{\nu_t}} \right)^{\frac{\nu_t}{\tau_t}}}{\sum_{l \in T} \left( \sum_{j \in S_t} \left( \sum_{k \in R_j} \exp(\eta_{tlk} / \tau_l) \right)^{\frac{\tau_l}{\nu_l}} \right)^{\frac{\nu_l}{\tau_l}}}
\]
Assume that we can decompose the linear predictor as \( \eta_{tjk} = z_{it} \alpha_t + u_{tj} \gamma_j + x_{tjk} \beta_k \). Here \( z_{it} \), \( u_{tj} \), and \( x_{tjk} \) are the row vectors of explanatory variables for the first, second, and bottom levels of the hierarchy, respectively, and \( \alpha_t \), \( \gamma_j \), and \( \beta_k \) are the corresponding column vectors of regression coefficients for \( t \in T \), \( j \in S_t \), and \( k \in R_j \). We then can define the inclusive values for the first and second levels as

\[
I_{tj} = \log \sum_{k \in R_j} \exp(x_{tjk} \beta_k / \tau_j) \\
J_t = \log \sum_{j \in S_t} \exp(u_{tj} \gamma_j / v_t + \tau_j I_{tj})
\]

and rewrite the probabilities

\[
Pr(C_3 = k \mid C_1 = t, C_2 = j) = \frac{\exp(x_{tjk} \beta_k / \tau_j)}{\sum_{l \in R_j} \exp(x_{tjl} \beta_l / \tau_j)} \\
Pr(C_2 = j \mid C_1 = t) = \frac{\exp(u_{tj} \gamma_j / v_t + \tau_j I_{tj})}{\sum_{l \in S_t} \exp(u_{tl} \gamma_l / v_l + \tau_l I_{tl})} \\
Pr(C_1 = t) = \frac{\exp(z_t \alpha_t + v_t I_t)}{\sum_{l \in T} \exp(z_l \alpha_l + v_l I_l)}
\]

We add a fourth index, \( i \), for case and define the indicator variable \( y_{itjk}, i = 1, \ldots, N, \) to indicate the choice made by case \( i \), \( t \in T \), \( j \in S_t \), and \( k \in R_j \). The log likelihood for the \texttt{nlogit} RUM is

\[
\ell = \sum_{i=1}^{N} \sum_{t \in T} \sum_{j \in S_t} \sum_{k \in R_j} y_{itjk} \left\{ z_{it} \alpha_t + u_{tj} J_{it} - \log \left( \sum_{m \in T} z_{im} \alpha_m + v_m J_{im} \right) + \right. \\
\left. u_{itj} \gamma_j / v_t + \tau_j I_{itj} - \log \left( \sum_{m \in S_t} u_{itm} \gamma_m / v_t + \tau_m I_{itm} \right) + \right. \\
\left. x_{itjk} \beta_k / \tau_k - \sum_{m \in R_t} \exp(x_{itjm} \beta_m / \tau_k) \right\}
\]

and for the nonnormalized \texttt{nlogit} model, the log likelihood is

\[
\ell = \sum_{i=1}^{N} \sum_{t \in T} \sum_{j \in S_t} \sum_{k \in R_j} y_{itjk} \left\{ z_{it} \alpha_t + u_{tj} J_{it} - \log \left( \sum_{m \in T} z_{im} \alpha_m + v_m J_{im} \right) + \right. \\
\left. u_{itj} \gamma_j + \tau_j I_{itj} - \log \left( \sum_{m \in S_t} u_{itm} \gamma_m + \tau_m I_{itm} \right) + \right. \\
\left. x_{itjk} \beta_k - \sum_{m \in R_t} \exp(x_{itjm} \beta_m) \right\}
\]

Extending the model to more than three levels is straightforward, albeit notationally cumbersome.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using \texttt{vce(robust)} and \texttt{vce(cluster clustvar)}, respectively. See [P] \texttt{_robust}, particularly Maximum likelihood estimators and Methods and formulas.
References


Also see

[CM] nlogit postestimation — Postestimation tools for nlogit

[CM] cmclogit — Conditional logit (McFadden’s) choice model

[CM] cmmixlogit — Mixed logit choice model

[CM] cmmprobit — Multinomial probit choice model

[R] clogit — Conditional (fixed-effects) logistic regression

[R] mlogit — Multinomial (polytomous) logistic regression

[R] mprobit — Multinomial probit regression

[R] ologit — Ordered logistic regression

[R] slogit — Stereotype logistic regression

[U] 20 Estimation and postestimation commands